Multi-User Beamforming-Aided AF Relaying: A Low-Complexity Adaptive Design Approach

Jiaxin Yang*, Yunlong Cai†, Benoit Champagne*, and Lajos Hanzo‡
*Department of Electrical and Computer Engineering, McGill University, Montreal, Quebec, Canada
†Department of Information Science and Electronic Engineering, Zhejiang University, Hangzhou, P. R. China
‡School of Electronics and Computer Science, University of Southampton, Southampton, U.K.
Emails: jiaxin.yang@mail.mcgill.ca; ylcai@zju.edu.cn; benoit.champagne@mcgill.ca; lh@ecs.soton.ac.uk

***Abstract*—The problem of cooperative multiple-input multiple-output (MIMO) amplify-and-forward (AF) relaying design for multi-user networks is studied, where each source transmits the data to its unique destination with the assistance of multiple relays equipped with antenna arrays. We aim for jointly optimizing the beamforming weights of different relays in order to minimize the total *received power* at all the destinations, subject to a global relays' power constraint, while a set of linear constraints are imposed to preserve the desired signals at each destination. In contrast to prior contributions, which optimize the relaying weights in a *batch processing* mode, we propose a low-complexity adaptive update approach by designing a recursive algorithm for the relay beamforming weights. A relay power control method is also proposed, which can readily be incorporated into the proposed adaptive framework. The efficacy of the adaptive scheme is demonstrated by simulations in terms of its steady-state performance and tracking capability.**

I. Introduction

Wireless relaying is considered as a cost-effective technique for capacity enhancement and coverage extension in current cellular standards, e.g., 3GPP Long Term Evolution (LTE) and LTE-Advanced [1]. Over the past decade, various relaying transmission techniques have been developed, among which amplify-and-forward (AF) relaying combined with multiple-input multiple-output (MIMO) is particularly promising. Despite its low processing signal complexity, it is capable of supporting concurrent communication of multiple source-destination pairs based on the same spectral resources by exploiting the spatial dimensions provided by the antenna array. In the literature, relay transceiver optimization has been extensively studied in multi-user networks based on diverse design objectives, such as the sum mean square error (SMSE) minimization [2], sum interference leakage minimization [3], sum rate maximization [4], etc. In these contributions, the relay transceivers are usually optimized in a *batch processing* mode by relying on the state-of-the-art optimization tools.

To elaborate a little further, the efficacy of the batch processing mode relies on an implicit assumption concerning the channel state information (CSI), namely that the underlying wireless channels remain approximately constant during the interval between two consecutive relay transceiver updates. However, due to the relatively high computational complexity entailed by the optimization process, in time-varying environments this approach may not promptly react to the changes in the wireless channels, which will cause a significant performance erosion. Motivated by these limitations, a low-complexity adaptive relaying algorithm is desirable, which is capable of promptly updating the relay transceiver weights.

Although adaptive algorithms have been successfully applied both in transmit and receive beamforming [5] and multi-user detection [6], the design of adaptive algorithm for AF relaying still remains largely unexplored. In [7], the authors propose an adaptive relaying algorithm relying on the classic Kalman filter by treating the relays' beamforming weight vector as the Kalman filter's state vector to be estimated. However, this method requires the *a priori* knowledge of the state transition model, which is in practice challenging to obtain. Furthermore, due to the difficulty of incorporating the quadratic power constraint on the state vector of the Kalman filter, no efficient relay power control scheme has been taken into consideration in [7].

Against this background, we design an adaptive algorithm for a multi-user MIMO relaying network, where K source-destination pairs communicate concurrently in a pairwise manner with the assistance of M multi-antenna AF relays. The design problem of the required relay beamforming weights is formulated, where the total received signals' power at all the destinations is minimized, subject to a global relays' power constraint, while satisfying a set of *linear constraints* capable of preserving all the desired signals at each destination. Subsequently, a recursive algorithm is proposed for updating the relay beamforming weights, which relies on a recursive least square (RLS)-type update. Finally, we conceive a relay power control technique intrinsically amalgamated with the proposed RLS algorithm for satisfying the relays' power constraint at each update. Simulation results confirm the fast convergence of the proposed adaptive relaying algorithm and its capability of tracking the changes in the wireless channels.

The rest of the paper is organized as follows. Section II introduces the model of the multi-user relaying network and formulates the design problem. In Section III, the RLS-type relay beamforming weight update algorithm is proposed. Simulation results are presented in Section IV. Finally, we conclude the paper in Section V.

The work of J. Yang and B. Champagne was supported by a CRD grant from the National Sciences and Engineering Research Council of Canada (NSERC) and InterDigital Canada. The work of Y. Cai was supported by the National Natural Science Foundation of China under Grant 61471319.

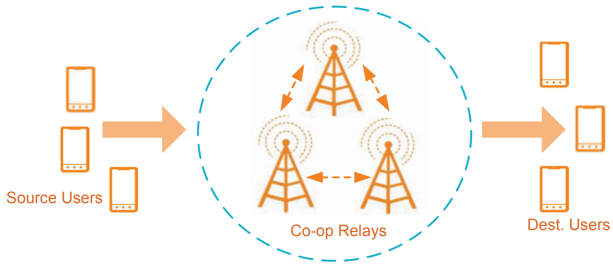


Fig. 1. Multi-user relaying sub-network where multiple sources transmit data to their corresponding destinations assisted by cooperative MIMO relays.

II. System Model and Problem Formulation

Consider a multi-user relaying sub-network, where M MIMO-aided relays, denoted as $\mathtt{R}_m$ for $m \in \mathcal{M} \triangleq \{1, 2, \cdots, M\}$, assist the one-way communication between K pairs of source and destination nodes, as depicted in Fig. 1. Each source-destination pair is assigned a unique index $k \in \mathcal{K} \triangleq \{1, 2, \cdots, K\}$, denoted as $\mathtt{S}_k$ and $\mathtt{D}_k$, respectively, where $\mathtt{S}_k$ only communicates with its paired $\mathtt{D}_k$. We assume that both $\mathtt{S}_k$ and $\mathtt{D}_k$ are user terminals with limited signal processing capabilities and low power budgets, therefore equipped with a single antenna. By contrast, the relay $\mathtt{R}_m$ equipped with $N_{\mathrm{R},m}$ antennas can be a station with more computing power. We consider half-duplex AF protocol, where the data transmission is completed in two slots for each time instance. It is assumed that no direct links are available between source and destination users due to the severe attenuation.

A narrowband flat-fading channel model is considered, where $\mathbf{h}_{m,k} \in \mathbb{C}^{N_{\mathrm{R},m} \times 1}$ denotes the $\mathtt{S}_k$–$\mathtt{R}_m$ channel and $\mathbf{g}_{k,m} \in \mathbb{C}^{N_{\mathrm{R},m} \times 1}$ denotes the Hermitian transpose of the $\mathtt{R}_m$–$\mathtt{D}_k$ channel. It is assumed that the channels $\{\mathbf{h}_{m,k}\}$ and $\{\mathbf{g}_{m,k}\}$ are mutually statistically independent. Let $s_k(n)$ denote the information symbol transmitted by $\mathtt{S}_k$ at the n^{th} time instance, which is modeled as a zero-mean random variable, with a variance of $\mathrm{E}\left\{|s_k(n)|^2\right\} = \sigma_{\mathrm{S}}^2$. Let $\mathbf{n}_{\mathrm{R},m}(n) \in \mathbb{C}^{M \times 1}$ be the temporally and spatially white, additive noise vector at $\mathtt{R}_m$, with a zero mean and covariance matrix of $\mathrm{E}\left\{\mathbf{n}_{\mathrm{R},m}(n)\mathbf{n}_{\mathrm{R},m}^H(n)\right\} = \sigma_{\mathrm{R},m}^2 \mathbf{I}_{N_{\mathrm{R},m}}$. During the first transmission slot at the n^{th} time instance, $\mathtt{R}_m$ receives the following signal:

$$\mathbf{z}_m(n) = \sum_{k=1}^{K} \mathbf{h}_{m,k} s_k(n) + \mathbf{n}_{\mathrm{R},m}(n). \tag{1}$$

Each $\mathtt{R}_m$ applies a linear transformation, represented by matrix $\mathbf{W}_m(n) \in \mathbb{C}^{N_{\mathrm{R},m} \times N_{\mathrm{R},m}}$ to $\mathbf{z}_m(n)$ and forwards the resultant signal

$$\mathbf{r}_m(n) = \mathbf{W}_m(n)\mathbf{z}_m(n) \tag{2}$$

to $\mathtt{D}_k$ for all k. The relays within the cluster obey the following global power constraint[1]:

$$\sum_{m=1}^{M} P_{\mathrm{R},m} \leq P_{\mathrm{R}}^{\max}, \tag{3}$$

where $P_{\mathrm{R},m}$ denotes the average power of $\mathtt{R}_m$, as given by

$$P_{\mathrm{R},m} = \mathrm{Tr}\left(\mathbf{W}_m(n)\left(\sum_{k=1}^{K} \mathbf{h}_{m,k}\mathbf{h}_{m,k}^H + \sigma_{\mathrm{R},m}^2 \mathbf{I}\right)\mathbf{W}_m^H(n)\right). \tag{4}$$

Let $n_{\mathrm{D},k}(n)$ denote the temporally white additive noise at $\mathtt{D}_k$ with a zero mean and variance $\sigma_{\mathrm{D},k}^2$. During the second transmission slot at the n^{th} time instant, the destination $\mathtt{D}_k$ receives the following signal:

$$\begin{aligned} y_{\mathrm{D},k}(n) &= \sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{z}_m(n) + n_{\mathrm{D},k}(n) \\ &= \mathcal{S}_k(n) + \mathcal{I}_k(n) + n_{\mathrm{D},k}(n), \end{aligned} \tag{5}$$

where $\mathcal{S}_k(n)$ and $\mathcal{I}_k(n)$ denote the desired and interference signals affecting $\mathtt{D}_k$ at time n, respectively, as given by

$$\mathcal{S}_k(n) = \sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{h}_{m,k} s_k(n) \tag{6}$$

$$\begin{aligned} \mathcal{I}_k(n) = &\sum_{l=1, l\neq k}^{K} \sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{h}_{m,l} s_l(n) \\ &+ \sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{n}_{\mathrm{R},m}(n). \end{aligned} \tag{7}$$

In this paper, we aim for jointly designing the AF matrices across different relays, i.e., $\{\mathbf{W}_m(n)\}_{m=1}^M$, in order to simultaneously optimize the signal's reception quality at each $\mathtt{D}_k$. Motivated by the recent findings in interference alignment [8], an effective technique of meeting this objective is to minimize the total *interference leakage* of the sub-network, subject to the relays' power constraint, while imposing a set of *linear signal preservation constraints* at each $\mathtt{D}_k$. Mathematically, the problem can be formulated as minimizing $\sum_{k=1}^{K} \mathrm{E}\left\{|\mathcal{I}_k(n)|^2\right\}$ subject to the power constraint (3) and the linear constraints $\sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{h}_{m,k} = \nu_k,\ k \in \mathcal{K}$, where $\{\nu_k\}$ denotes a set of predefined constants. The latter constraints have the effect of preserving the desired signal s_k at each $\mathtt{D}_k$.

It has been shown in [9] that under the above-mentioned signal preservation constraints, minimizing the total interference leakage is equivalent to minimizing the total received power. Using (5) and (6), the latter can be formulated as

$$\min_{\{\mathbf{W}_m(n)\}} \quad \sum_{k=1}^{K} \mathrm{E}\left\{\left|\sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{z}_m(n)\right|^2\right\} \tag{8a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \mathbf{g}_{k,m}^H \mathbf{W}_m(n)\mathbf{h}_{m,k} = \nu_k,\ k \in \mathcal{K} \tag{8b}$$

$$\sum_{m=1}^{M} P_{\mathrm{R},m} \leq P_{\mathrm{R}}^{\max}. \tag{8c}$$

Next, we shall derive an efficient adaptive algorithm, for the on-line update of the relaying matrices on a per-symbol basis.

[1] We consider the scenario where the relays are connected to a cluster head, which is capable of dynamically allocating different power factors to different relays, which motivates the use of a global power constraint.

III. Recursive Adaptive Relaying Algorithm

In this section, we derive a RLS-type adaptive algorithm for solving the relaying design problem (8). Let us define the vectorized version of $\mathbf{W}_m(n)$, which we refer to as the *relaying beamforming vector*, as $\mathbf{w}_m(n) = \mathrm{vec}(\mathbf{W}_m(n))$. To facilitate the derivation of the adaptive algorithm based on the real-time received signals, we replace the expectation in (8a) by an exponentially weighted time average, and after some manipulations, we arrive at the following optimization problem relying on the weighted least-square (LS) objective function:

$$\min_{\{\mathbf{w}_m(n)\}} \quad \sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \left| \sum_{m=1}^{M} \mathbf{w}_m^H(n) \mathbf{r}_{k,m}(n) \right|^2 \tag{9a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \mathbf{c}_{k,m}^H \mathbf{w}_m(n) = \nu_k, \ k \in \mathcal{K} \tag{9b}$$

$$\sum_{m=1}^{M} \mathbf{w}_m^H(n) \mathbf{P}_m \mathbf{w}_m(n) \leq P_{\mathrm{R}}^{\max}, \tag{9c}$$

where $0 < \gamma \leq 1$ denotes the RLS forgetting factor. In obtaining (9), we exploit the properties of the Kronecker product $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{B})$ and $\mathrm{Tr}\left(\mathbf{ABA}^H\right) = \mathrm{vec}(\mathbf{A})^H (\mathbf{B} \otimes \mathbf{I})\,\mathrm{vec}(\mathbf{A})$. We also introduce $\mathbf{r}_{k,m}(n) \triangleq \mathbf{z}_m^*(n) \otimes \mathbf{g}_{k,m}$, $\mathbf{c}_{k,m}(n) \triangleq \mathbf{h}_{m,k}^* \otimes \mathbf{g}_{k,m}$ and $\mathbf{P}_m \triangleq \left(\sum_{k=1}^{K} \mathbf{h}_{m,k}\mathbf{h}_{m,k}^H + \sigma_{\mathrm{R},m}^2 \mathbf{I}_{N_{\mathrm{R},m}}\right) \otimes \mathbf{I}_{N_{\mathrm{R},m}}$. In order to transform the quadratic power constraint (9c) into a norm-based constraint, we define the new beamforming vector $\bar{\mathbf{w}}_m(n) = \mathbf{P}_m^{\frac{1}{2}} \mathbf{w}_m(n)$ and subsequently the concatenated vector $\bar{\mathbf{w}}(n) \triangleq \left[\bar{\mathbf{w}}_1(n)^T, \cdots, \bar{\mathbf{w}}_M(n)^T\right]^T$. Then (9) can be equivalently rewritten in the following more compact form:

$$\min_{\bar{\mathbf{w}}(n)} \quad \sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \left| \bar{\mathbf{w}}^H(n) \mathbf{r}_k(n) \right|^2 \tag{10a}$$

$$\text{s.t.} \quad \mathbf{C}^H \bar{\mathbf{w}}(n) = \boldsymbol{\nu} \tag{10b}$$

$$\|\bar{\mathbf{w}}(n)\|^2 \leq P_{\mathrm{R}}^{\max}, \tag{10c}$$

where we have $\boldsymbol{\nu} = [\nu_1, \cdots, \nu_K]^T \in \mathbb{C}^{K \times 1}$, $\mathbf{C} \triangleq [\mathbf{c}_1, \cdots, \mathbf{c}_K] \in \mathbb{C}^{\bar{N} \times K}$, $\mathbf{r}_k(n) \triangleq \left[\mathbf{r}_{k,1}(n)^T \mathbf{P}_1^{-\frac{1}{2}}, \cdots, \mathbf{r}_{k,M}(n)^T \mathbf{P}_M^{-\frac{1}{2}}\right]^T \in \mathbb{C}^{\bar{N} \times 1}$ and $\mathbf{c}_k \triangleq \left[\mathbf{c}_{k,1}^T \mathbf{P}_1^{-\frac{1}{2}}, \cdots, \mathbf{c}_{k,M}^T \mathbf{P}_M^{-\frac{1}{2}}\right]^T \in \mathbb{C}^{\bar{N} \times 1}$, where $\bar{N} \triangleq \sum_{m=1}^{M} N_{\mathrm{R},m}^2$.

In order to eliminate the linear constraints (10b), we invoke the classic generalized sidelobe canceller (GSC) structure [10], where $\bar{\mathbf{w}}(n)$ can be decomposed into two components:

$$\bar{\mathbf{w}}(n) = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a(n), \tag{11}$$

with $\mathbf{w}_q = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\boldsymbol{\nu}$ representing the fixed beamformer (FBF), which lies in the range space of $\mathbf{C}^H$, and has the effect of maintaining the linear constraints (10b). Still referring to (11), $\mathbf{B}$ is the blocking matrix (BM), which satisfies $\mathbf{B}^H\mathbf{C} = \mathbf{0}$. Due to the assumption of having independent fading channels, $\mathbf{C}$ has full column rank almost surely, and therefore, $\mathbf{B}$ has a rank of $(\bar{N} - K)$ and dimensions of $\bar{N} \times (\bar{N} - K)$. Substituting (11) into (10), we arrive at:

$$\min_{\mathbf{w}_a(n)} \quad \sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \left| \mathbf{w}_q^H \mathbf{r}_k(i) - \mathbf{w}_a^H(n) \bar{\mathbf{r}}_k(i) \right|^2 \tag{12a}$$

$$\text{s.t.} \quad \|\mathbf{w}_a(n)\|^2 \leq \beta, \tag{12b}$$

where $\beta \triangleq P_{\mathrm{R}} - \|\mathbf{w}_q\|^2$ and $\bar{\mathbf{r}}_k(i) \triangleq \mathbf{B}^H \mathbf{r}_k(i)$.

We proceed by expressing the Lagrangian of (12) as

$$\mathcal{L}(\mathbf{w}_a(n), \lambda) = \sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \left| \mathbf{w}_q^H \mathbf{r}_k(i) - \mathbf{w}_a^H(n) \bar{\mathbf{r}}_k(i) \right|^2 + \lambda \left(\mathbf{w}_a^H(n) \mathbf{w}_a(n) - \beta \right), \tag{13}$$

where λ denotes the Lagrange multiplier associated with (12b). To solve (13), we equate the gradient $\nabla_{\mathbf{w}_a^*(n)} \mathcal{L}(\mathbf{w}_a(n), \lambda)$ to zero and obtain

$$\sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \left(\bar{\mathbf{r}}_k(i) \bar{\mathbf{r}}_k^H(i) \mathbf{w}_a(n) - \bar{\mathbf{r}}_k(i) \mathbf{r}_k^H(i) \mathbf{w}_q \right) + \lambda \mathbf{w}_a(n) = \mathbf{0}, \tag{14}$$

which can be equivalently expressed as

$$\left(\mathbf{Q}(n) + \lambda \mathbf{I} \right) \mathbf{w}_a(n) = \mathbf{d}(n), \tag{15}$$

where $\mathbf{Q}(n)$ and $\mathbf{d}(n)$ denote the so-called sample correlation matrix and the sample cross-correlation vector, respectively, which are given by

$$\mathbf{Q}(n) \triangleq \sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \bar{\mathbf{r}}_k(i) \bar{\mathbf{r}}_k^H(i) \tag{16}$$

$$\mathbf{d}(n) \triangleq \sum_{i=1}^{n} \gamma^{n-i} \sum_{k=1}^{K} \bar{\mathbf{r}}_k(i) \mathbf{r}_k^H(i) \mathbf{w}_q. \tag{17}$$

Further rearranging (15) leads to

$$\mathbf{w}_a(n) = \left(\mathbf{I} + \lambda \mathbf{Q}^{-1}(n) \right)^{-1} \bar{\mathbf{w}}_a(n), \tag{18}$$

where $\bar{\mathbf{w}}_a(n) \triangleq \mathbf{Q}^{-1}(n)\mathbf{d}(n)$ can be viewed as a tentative beamforming vector at the n^{th} time instant in the absence of the power constraint (12b), i.e., $\lambda = 0$. It is observed that in (18), $\mathbf{w}_a(n)$ is a product of $\bar{\mathbf{w}}_a(n)$ and the term $\left(\mathbf{I} + \lambda \mathbf{Q}^{-1}(n)\right)^{-1}$. This particular structure suggests an efficient on-line update solution, which consists of two steps. In the first step, we update the tentative beamforming vector $\bar{\mathbf{w}}_a(n)$. If the power constraint is not satisfied for $\bar{\mathbf{w}}_a(n)$, i.e., if $\|\bar{\mathbf{w}}_a(n)\|^2 > \beta$, then in the second step, we compute the optimal Lagrange multiplier λ and adjust $\bar{\mathbf{w}}_a(n)$ in order to meet the power constraint. Next, we elaborate further on these steps.

A. Tentative Update of the Beamforming Vector

To derive a tentative update for $\bar{\mathbf{w}}_a(n)$, we have to compute the inverse of the sample correlation matrix, i.e., $\mathbf{Q}^{-1}(n)$. Note that $\mathbf{Q}(n)$ in (16) consists of a summation of K rank-one modification terms, which motivates the employment of a so-called incremental approach. Let us now express $\mathbf{Q}(n)$ in the time-recursive form of:

$$\mathbf{Q}(n) = \gamma \mathbf{Q}(n-1) + \sum_{k=1}^{K} \bar{\mathbf{r}}_k(n)\bar{\mathbf{r}}_k^H(n). \tag{19}$$

We then introduce a set of intermediate variables $\{\mathbf{P}_k(n)\}_{k=0}^{K}$ which is useful for updating $\mathbf{Q}(n)$ in an incremental manner. Specifically, let $\mathbf{P}_0(n) = \gamma\mathbf{Q}(n-1)$, and $\mathbf{P}_k(n)$ is updated from $\mathbf{P}_{k-1}(n)$ with the aid of a rank-one modification term as the user index k is incremented, i.e.

$$\mathbf{P}_k(n) = \mathbf{P}_{k-1}(n) + \bar{\mathbf{r}}_k(n)\bar{\mathbf{r}}_k^H(n), \; k = 1, \cdots, K, \tag{20}$$

and finally we have $\mathbf{Q}(n) = \mathbf{P}_K(n)$. The update in (20) now involves only a single rank-one modification term, motivating the employment of the Sherman-Morrison formula for sequentially updating $\mathbf{Q}^{-1}(n)$ with the aid of low-complexity multiplications, as shown in Table I.

TABLE I
UPDATE $\mathbf{Q}^{-1}(n)$ FROM $\mathbf{Q}^{-1}(n-1)$ IN AN INCREMENTAL FORM

$\mathbf{P}_0^{-1}(n) \leftarrow \gamma^{-1}\mathbf{Q}^{-1}(n-1)$
For $k = 1, \cdots, K$
$\boldsymbol{\theta}_k(n) = \dfrac{\mathbf{P}_{k-1}^{-1}(n)\bar{\mathbf{r}}_k(n)}{1 + \bar{\mathbf{r}}_k^H(n)\mathbf{P}_{k-1}^{-1}(n)\bar{\mathbf{r}}_k(n)}$
$\mathbf{P}_k^{-1}(n) = \mathbf{P}_{k-1}^{-1}(n) - \boldsymbol{\theta}_k(n)\bar{\mathbf{r}}_k^H(n)\mathbf{P}_{k-1}^{-1}(n)$
End For
$\mathbf{Q}^{-1}(n) \leftarrow \mathbf{P}_K^{-1}(n)$

Then, borrowing from the classic RLS [10], $\bar{\mathbf{w}}_a(n)$ can be updated according to

$$\bar{\mathbf{w}}_a(n) = \mathbf{w}_a(n-1) + \mathbf{Q}^{-1}(n)\sum_{k=1}^{K}\bar{\mathbf{r}}_k(n)e_k^*(n), \tag{21}$$

where the residual error at the k^{th} destination is given by

$$e_k(n) = \mathbf{w}_q^H\mathbf{r}_k(n) - \mathbf{w}_a^H(n-1)\bar{\mathbf{r}}_k(n). \tag{22}$$

Note that (21) gives the exact optimal LS solution in the absence of the power constraint (12b) ($\lambda = 0$).

B. Incorporating the Relays' Power Constraint

The tentative beamforming vector $\bar{\mathbf{w}}_a(n)$ of (21) does not necessarily satisfy the power constraint of (12b). Then, inspired by the variable loading technique developed in traditional adaptive filtering theory [11], a new relay power control method is proposed which can readily be incorporated into our RLS-type algorithm. Recall that

$$\mathbf{w}_a(n) = \underbrace{\left(\mathbf{I} + \lambda\mathbf{Q}^{-1}(n)\right)^{-1}}_{\triangleq \boldsymbol{\mathcal{F}}(\lambda)}\bar{\mathbf{w}}_a(n). \tag{23}$$

We then "linearize" the term $\boldsymbol{\mathcal{F}}(\lambda)$ using its first-order Taylor series expansion around $\lambda = 0$, leading to:

$$\boldsymbol{\mathcal{F}}(\lambda) \approx \boldsymbol{\mathcal{F}}(0) + \boldsymbol{\mathcal{F}}'(0)\lambda, \tag{24}$$

where $\boldsymbol{\mathcal{F}}(0) = \mathbf{I}$ and $\boldsymbol{\mathcal{F}}'(\lambda)$ can be obtained using the basic identity of the derivative of an inverse $\frac{\partial \mathbf{Y}^{-1}}{\partial x} = -\mathbf{Y}^{-1}\frac{\partial \mathbf{Y}}{\partial x}\mathbf{Y}^{-1}$, yielding:

$$\begin{aligned}\boldsymbol{\mathcal{F}}'(\lambda) &= -\left(\mathbf{I} + \lambda\mathbf{Q}^{-1}(n)\right)^{-1}\mathbf{Q}^{-1}(n)\left(\mathbf{I} + \lambda\mathbf{Q}^{-1}(n)\right)^{-1} \\ \boldsymbol{\mathcal{F}}'(0) &= -\mathbf{Q}^{-1}(n).\end{aligned} \tag{25}$$

Substituting (24) into (23), $\mathbf{w}_a(n)$ can be approximated as

$$\mathbf{w}_a(n) \approx \bar{\mathbf{w}}_a(n) - \lambda\mathbf{v}_a(n), \tag{26}$$

where $\mathbf{v}_a(n) \triangleq \mathbf{Q}^{-1}(n)\bar{\mathbf{w}}_a(n)$. If the power of the tentative beamforming vector $\bar{\mathbf{w}}_a(n)$ violates the power constraint, i.e., $\|\bar{\mathbf{w}}_a(n)\|^2 > \beta$, we have to compute the Lagrange multiplier λ by substituting (26) back into (12b) and obtain the following quadratic equation for λ:

$$\|\mathbf{v}_a(n)\|^2\lambda^2 - 2\,\mathrm{Re}(\mathbf{v}_a^H(n)\bar{\mathbf{w}}_a(n))\lambda + \|\bar{\mathbf{w}}_a(n)\|^2 - \beta = 0. \tag{27}$$

Its solution is given in the following proposition:

Proposition 1: Let us define $a = \|\mathbf{v}_a(n)\|^2$, $b = -2\,\mathrm{Re}(\mathbf{v}_a^H(n)\overline{\mathbf{w}}_a(n))$ and $c = \|\bar{\mathbf{w}}_a(n)\|^2 - \beta$. If $c > 0$, i.e., the power constraint (12b) is violated, the optimal value of λ is given by

$$\lambda = \frac{-b - \mathrm{Re}(\sqrt{b^2 - 4ac})}{2a}. \tag{28}$$

Proof: See, e.g., [11] and omitted here for brevity. ∎

With the above closed-form of λ, the relay power control can be readily incorporated into the RLS-type update developed in the previous subsection. The overall adaptive relaying algorithm with power control will be presented in the next subsection.

C. Algorithm Summary and Discussion

The proposed adaptive relaying algorithm can be implemented in a centralized manner at the head node within the relay cluster. Upon collecting all the CSIs $\{\mathbf{g}_{k,m}, \mathbf{h}_{m,k}\}_{m,k}$, the relaying beamforming vector $\mathbf{w}_a(n)$ is updated at each time instant n after the signal observations $\{\mathbf{z}_m(n)\}$ have been gleaned from all the relays. This centralized scheme is now summarized in Algorithm 1.

Complexity: The computational complexity of Algorithm 1 is given in Table II in terms of the number of complex-valued addition and multiplication operations required for computing each step. It is then readily seen that the total complexity order is $\mathcal{O}(K\bar{N}^2)$, where we recall that $\bar{N} = \sum_{m=1}^{M} N_{\mathrm{R},m}^2$.

We also compare the complexity of the proposed algorithm to that of the Kalman filter-based algorithm of [7] and to a batch-solver relying on the second-order cone programming (SOCP) in Table III, where $N = \sum_{m=1}^{M} N_{\mathrm{R},m}$. The batch-solver follows an approach similar to that in [3], which first reformulates (8) as an SOCP and then solves it for the optimal $\{\mathbf{W}_m^*(n)\}$ using a state-of-the-art optimization tool, such as

Algorithm 1 Time-Recursive Adaptive Relaying Algorithm Relying on Power Control

Initialization: $\mathbf{w}_a(0) = \mathbf{0}$, $\delta > 0$, $\mathbf{Q}^{-1}(0) = \frac{1}{\delta}\mathbf{I}$. Set $n = 1$.

for $n = 1, 2, \cdots$ **do**

1. *Compute Error*:

$$e_k(n) = \mathbf{w}_q^H \mathbf{r}_k(n) - \mathbf{w}_a^H(n-1)\bar{\mathbf{r}}_k(n),$$

2. *Update of* $\mathbf{Q}^{-1}(n)$: Use the incremental approach given in Table I
3. *Tentative Update of* $\bar{\mathbf{w}}_a(n)$:

$$\bar{\mathbf{w}}_a(n) = \mathbf{w}_a(n-1) + \mathbf{Q}^{-1}(n)\sum_{k=1}^{K}\bar{\mathbf{r}}_k(n)e_k^*(n),$$

4. *Relay Power Control*:

$$\mathbf{w}_a(n) = \begin{cases} \bar{\mathbf{w}}_a(n) & \text{if } \|\bar{\mathbf{w}}_a(n)\|^2 \le \beta \\ \bar{\mathbf{w}}_a(n) - \lambda\mathbf{v}_a(n) & \text{if } \|\bar{\mathbf{w}}_a(n)\|^2 > \beta \end{cases},$$

where λ is given in Proposition 1.

5. Broadcast $\mathbf{W}_m(n) = \mathtt{MAT}\{\mathbf{w}_m(n)\}$ to $\mathtt{R}_m$ for all m

end for

TABLE II
COMPUTATIONAL COMPLEXITY ORDER OF ALGORITHM 1

Step No.	Additions	Multiplications
①	$\mathcal{O}(K\bar{N})$	$\mathcal{O}(K\bar{N})$
②	$\mathcal{O}(K\bar{N}^2)$	$\mathcal{O}(K\bar{N}^2)$
③	$\mathcal{O}(\bar{N}(\bar{N}-K))$	$\mathcal{O}((\bar{N}-1)(\bar{N}-K))$
④	$\mathcal{O}(\bar{N}-K)$	$\mathcal{O}(\bar{N}-K)$
Total	$\mathcal{O}(K\bar{N}^2)$	$\mathcal{O}(K\bar{N}^2)$

`MOSEK` [12]. The batch-solver imposes a significantly higher complexity and therefore it is usually implemented in an off-line fashion. For time-varying channels, the batch-solver has to re-compute the optimal relaying weights for each time instant, which can be computationally excessive and hence impractical. However, the proposed algorithm provides an efficient on-line implementation, which is capable of tracking the fluctuation of wireless channels at a much lower complexity. It is also noted that the Kalman filter-based algorithm of [7] exhibits a higher complexity, especially when the number of users K becomes large.

IV. Simulation Experiments and Discussions

In all our simulations, we consider a relaying sub-network consisting of $K = 2$ $\mathtt{S}$–$\mathtt{D}$ pairs and $M = 2$ relays equipped with an identical number of antennas N_{R}. A flat Rician fading channel model having a K-factor of 0.1 is employed for all the $\mathtt{S}_k$–$\mathtt{R}_m$ and $\mathtt{R}_m$–$\mathtt{D}_k$ channels. The global power budget of all the relays is normalized to $P_{\mathrm{R}}^{\max} = 1$. The noise variances are identical at all $\mathtt{R}_m$ and set to $\sigma_{\mathrm{R},m} = \sqrt{\frac{0.1}{N_{\mathrm{R}}}}$.

TABLE III
COMPARISON OF COMPLEXITY ORDER OF DIFFERENT METHODS

Algorithms	Complexity
Proposed	$\mathcal{O}(K\bar{N}^2)$
Kalman filter [7]	$\mathcal{O}(K(\bar{N}^2 + K^2))$
SOCP-based batch solver	$\mathcal{O}(K^{1.5}(N+K)(N^2+K)^2)$

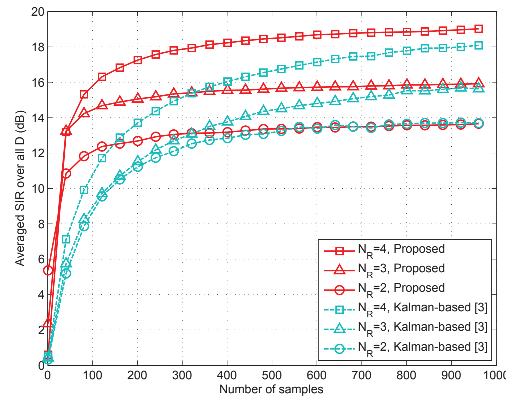


Fig. 2. Convergence and steady-state performance of the proposed time-recursive adaptive relaying algorithm with different numbers of relay antennas N_{R}.

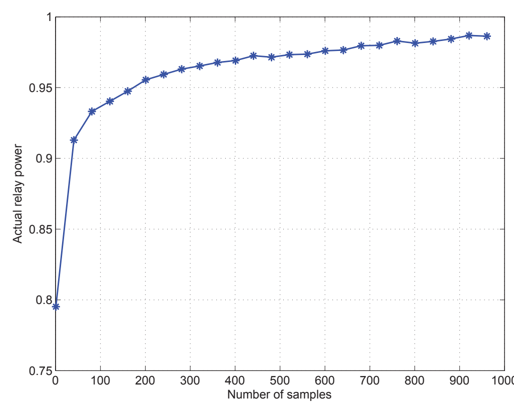


Fig. 3. Total transmitted power of all the relays using the proposed power control scheme.

A. Convergence Behavior for Stationary Channels

We first evaluate the performance of the proposed adaptive algorithm for stationary wireless channels. In Fig. 2, the signal-to-interference-leakage ratio (SIR) averaged over all $\mathtt{D}$'s is shown as a function of the number of received samples with different numbers of relay antennas. We compare the performance of our proposed algorithm to that of the Kalman filter-based algorithm using scaled projection power control [7]. It is observed that in all cases, the SIRs achieved by our proposed algorithm can converge within a reasonable number of recursions. Higher SIR can be obtained upon increasing N_{R} due to the higher spatial multiplexing gain exploited.

TABLE IV
PARAMETERS OF FADING CHANNELS

Doppler spectrum	Jakes
Rician K-factor	0.1
Sampling interval	$0.5\mu s$
Sampling frequency	2MHz
Carrier frequency	2GHz
Doppler spread	10Hz (velocity 1.5m/s)
Coherence time	0.05s
Spatial correlation	No
Transmission block	2500 samples

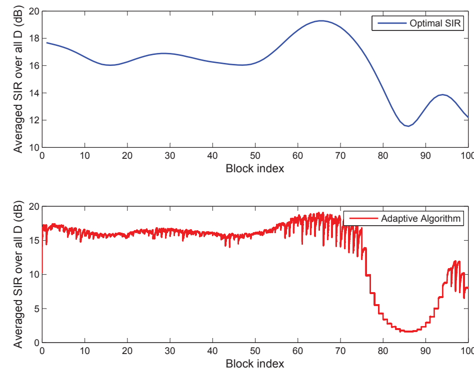
Although both algorithms attain a similar level of the steady-state SIR, our proposed algorithm exhibits a significantly faster convergence rate than the algorithm of [7]. To demonstrate the efficacy of the relay power control scheme, we explicitly show the averaged total transmission of all the relays in Fig. 3. It is shown that the proposed power control scheme is capable of strictly enforcing the power constraint at each update.

B. Tracking Performance for Time-Varying Channels

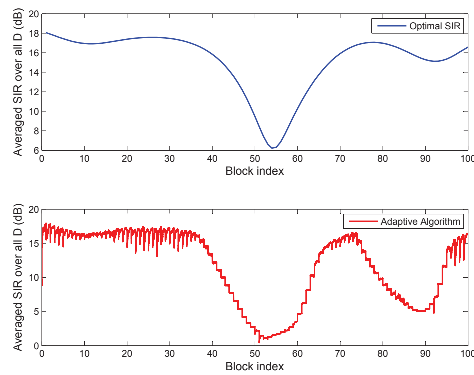
In Fig. 4, the tracking performance of the proposed adaptive algorithm is shown for time-varying fading channels, whose parameters are listed in Table IV. The blue curves labeled "optimal SIR" are obtained by using the SOCP-based batch-solver described in Section III–C, which serve as a benchmark. The SIR achieved by the proposed algorithm is plotted for two independent channel realizations. We observe that at some points the SIR significantly drops due to deep fading, e.g., around the 85^{th} and the 55^{th} blocks in the first and second realizations, respectively. However, it is capable of rapidly converging back to satisfactory performance, hence clearly demonstrating its capability of tracking the fluctuation of the underlying wireless channels.

V. CONCLUSIONS

A cooperative MIMO AF relaying design was conceived for multi-user networks. The beamforming weights of all the relays are jointly optimized by minimizing the total received power subject to a global power constraint and a set of linear signal preservation constraints. A low-complexity adaptive approach based on RLS-type update was proposed. A power control scheme relying on the variable loading technique was also proposed, which can readily be incorporated into the adaptive framework. Simulation results validate the rapid convergence rate of the proposed adaptive algorithm along with its tracking capability in non-stationary wireless environments.



(a)



(b)

Fig. 4. Tracking performance of the proposed algorithm over a time-varying wireless channel. (a) First channel realization; (b) Second channel realization.

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