BER Improvement in a TDMA/FDMA Cellular System Using Antenna Array

Mehrzad Biguesh[1], Benoit Champagne[2], Shahrokh Valaee[1] and Alex Stéphenne[3]

[1] Sharif University of Technology, Tehran, Iran
[2] Dept. of ECE, McGill University, Montréal, Canada
[3] INRS-Télécommunications, Montréal, Canada
POC: champagne@ece.mcgill.ca

Abstract

We propose a novel strategy based on Capon's method for multiple signal extraction in TDMA/FDMA cellular mobile communications. In this method, called SF-Capon, each intracell cochannel (IC) user is alternatively asked to decrease its transmitted power for a few bits. This enables the estimation of a so-called signal-free (SF) correlation matrix, which is then used in connection with Capon's method to compute an improved beamforming weight vector, with reduced sensitivity to DOA error and angular spread. The superiority of the proposed SF-Capon strategy over standard Capon's and data independent least-squares beamforming is demonstrated via simulation in a fast Rayleigh fading channel.

1. Introduction

In FDMA/TDMA cellular mobile systems, antenna arrays can be used at the base station (BS) to increase capacity by allowing multiple mobile users in a cell to share the same time slot and frequency channel (we shall refer to such users as intracell cochannel (IC) users). In this method, sometimes called space division multiple access (SDMA), the receiving antenna array produces a distinct beamforming pattern for each of the IC users, so that their information signals may be extracted separately [5]. Ideally, the pattern should have relative nulls in the directions of interfering IC users and a main lobe in the direction of the desired IC user. The same concepts may be applied to the transmitting array.

Capon's method is a popular approach for deriving beamforming weight vectors with such spatial properties. It is based on minimizing the beamformer output power subject to a constraint of unit gain in a desired look direction, which is equivalent to maximizing output signal-to-interference and noise power ratio (SINR) for a point source [7]. Capon's method assumes exact knowledge of the direction of arrival (DOA) of the desired signal and treats energy arriving from other directions as undesirable. In the present mobile radio application, this poses serious difficulties. Indeed, only estimates of the DOAs of the IC user signals are available; furthermore, local scattering around the mobiles results in angular spreading as seen from the receiving array. Such discrepancies from the assumed model lead to desired signal cancellation and increase in sidelobe levels; this in turns leads to a significant increase in the bit error rate (BER) at the receiver output.

Here, our interest is in using antenna arrays at the BS (uplink) for multiple signal extraction in TDMA/FDMA cellular mobile systems. We propose a new strategy for extracting signals based on Capon's beamformer which is applicable in such systems. In this method, which we call SF-Capon, each IC user is alternatively asked to decrease its transmitted power for a short time duration (a few bits) while the other IC users are sending their own signal. This enables the computation of a so-called signal-free (SF) correlation matrix for each IC user. Based on Capon's method, this correlation matrix is then used to compute a beamforming weight vector that minimizes the array output interference-plus-noise power. For DOA tracking of IC users, we use a modified form of the method in [1] that facilitates the implementation of the required intracell hand-off protocol [4].

The new method is investigated via computer simulations of multiple narrowband moving sources in a fast Rayleigh fading channel with angular spread. The results clearly show that it is capable of tracking and extracting the individual signal of each IC mobile user. When compared to standard Capon's and conventional least-squares beamformers, the proposed strategy leads to a significant reduction in the bit error rate (BER).

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2. Background material

2.1. Received signal model

To simplify the discussion, the following narrowband model is used for the received signals [2]:

$$\begin{aligned} \mathbf{x}(m) &= \mathbf{A}(m)\mathbf{s}(m) + \mathbf{n}(m) && (1) \\ \mathbf{s}(m) &= [s_1(m)\ s_2(m)\ \cdots\ s_p(m)]^T && (2) \\ \mathbf{A}(m) &= [\mathbf{a}(\theta_1(m))\ \ \mathbf{a}(\theta_2(m))\ \ \cdots\ \ \mathbf{a}(\theta_p(m))] && (3) \end{aligned}$$

Here: $\mathbf{x}(m)$ is the m'th snapshot for the L-antenna array, $s_i(m)$ is the i'th received signal from (distinct) direction θ_i, $\mathbf{n}(m)$ is a noise vector which we assume to be spatially and temporally white and uncorrelated with the signals, p is the number of IC users that we want to track, T represents transposition and finally, $\mathbf{a}(\theta)$ is the array steering vector for direction θ. For instance, in the case of an arbitrary planar geometry, the l'th entry of $\mathbf{a}(\theta)$ may be expressed as

$$a_l(\theta) = g_l(\theta)e^{j(2\pi r_l/\lambda)\cos(\theta-\phi_l)} \overset{\text{def}}{=} g_l(\theta)e^{j\nu_l \cos(\theta-\phi_l)} \tag{4}$$

where the pair (r_l, ϕ_l) represents the position of the l'th antenna in polar coordinates, $g_l(\theta)$ is the corresponding radiation pattern and λ is the signal wavelength.

In wireless cellular systems, the actual received signals do not assume the simplified form (1)-(3). Indeed, as a result of local scattering around the mobile, each user signal is actually spread in angle around its nominal DOA θ_i. That is, the received signal of each source is a combination of a multitude point sources that are randomly delayed and scaled replicas of the same signal. Such a spatially distributed source may be modeled mathematically via a so-called angular signal density [6]. In addition, source motion results in Doppler spread and fast Rayleigh fading [3].

2.2. Minimum Variance (Capon) beamformer

For narrowband signals, the beamformer output is computed simply by weighting and summing the received samples, i.e.:

$$y(m) = \mathbf{w}^H\mathbf{x}(m) \tag{5}$$

where $\mathbf{w} = [w_1\ \cdots\ w_L]^T$ is the vector of complex weights w_l and H represents Hermitian transposition. With a sophisticated choice of weights, it is possible to extract the desired signal when the number of signal sources is less than the number of array elements (i.e. $p < L$).

One method for cancelling undesired signal sources is Capon's method [7, 2]. In the latter, the desired weight vector is obtained by minimizing the output power of the array subject to a constraint of unit gain in the direction of the i'th signal. The beamformer output power at time m may be expressed as

$$P_{out} = E\{|y(m)|^2\} = \mathbf{w}_i^H\mathbf{R}\mathbf{w}_i \tag{6}$$

where $\mathbf{R} = E\{\mathbf{x}(m)\mathbf{x}^H(m)\}$ is the $L \times L$ correlation matrix of the received signal vector. Thus, Capon's weight vector for the i'th signal, denoted $\mathbf{w}_i$, may be formulated as

$$\min_{\mathbf{w}_i}\{\mathbf{w}_i^H\mathbf{R}\mathbf{w}_i\} \qquad \text{subject to} \qquad \mathbf{w}_i^H\mathbf{a}(\theta_i) = 1 \tag{7}$$

where θ_i is the assumed direction of the i'th signal source. Solution of this optimization problem by use of Lagrange multipliers gives the following result:

$$\mathbf{w}_i = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_i)}{\mathbf{a}(\theta_i)^H\mathbf{R}^{-1}\mathbf{a}(\theta_i)} \tag{8}$$

2.3. Tracking algorithm

Tracking of mobile users is one of the most important aspects for practical application of antenna arrays in mobile communications [5]. For multiple point-target tracking, a robust algorithm was proposed in [1] with a moderate computational complexity. For a planar array geometry with omnidirectional elements, it may be summarized as follows

$$\hat{\mathbf{A}}(k-1) = [\mathbf{a}(\hat{\theta}_1(k-1))\ \ \cdots\ \ \mathbf{a}(\hat{\theta}_p(k-1))] \tag{9}$$

$$\mathbf{W}(k) = [\hat{\mathbf{A}}(k-1)^H\hat{\mathbf{A}}(k-1)]^{-1}\hat{\mathbf{A}}(k-1)^H \tag{10}$$

$$\hat{\mathbf{s}}(k) = \mathbf{W}(k)^H\mathbf{x}(k) \tag{11}$$

$$\tilde{\mathbf{A}}(k) = \hat{\mathbf{A}}(k-1) + \mu_1[\mathbf{x}(k) - \hat{\mathbf{A}}(k-1)\hat{\mathbf{s}}(k)]\hat{\mathbf{s}}^H(k) \tag{12}$$

$$\Delta\hat{\theta}_i(k) = \frac{\sum_{l=1}^{L} \nu_l \sin(\hat{\theta}_i(k-1) - \phi_l)\mathrm{Im}\{\ln(\tilde{\mathbf{A}}_{l,i}^*(k)\hat{\mathbf{A}}_{l,i}(k-1))\}}{\sum_{l=1}^{L}(\nu_l \sin(\hat{\theta}_i(k-1) - \phi_l))^2}$$

$$\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \Delta\hat{\theta}_i(k) \tag{13}$$

where: k is the discrete-time index, [2] μ_1 is a real positive constant less than one, Im{.} represents the imaginary part of a complex number and $\hat{\theta}_i(k)$ is the DOA estimate of the i'th signal source.

As discussed in [1], the beamforming pattern related to $\mathbf{w}_i(k)$, the i'th column of $\mathbf{W}(k)$ in (10), inserts deep nulls in the direction of all users other than the i'th user. In fact, $\mathbf{W}(k)$ is the well-known data independent least-square (DILS) beamforming matrix for extraction of multiple signals with assumed DOAs $\hat{\theta}_i(k-1)$ [7].

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[2] Note that the sampling times for DOA estimation, as indexed by k, may be different from the sampling times associated to beamforming, as indexed by m.

3 The SF-Capon strategy

3.1. Basic principle

The DILS beamformer is data independent in the sense that its weights are independent of data statistics [7]. It does not take into account the noise level, the power of the interfering sources and the angular spread. Thus, this kind of beamformer is not statistically optimum and will not result in maximum SINR at the array output.

The standard Capon's method is very sensitive to the assumed DOA of the desired signal (i.e. θ_i in (8)), i.e. a small estimation error in that parameter may result in significant signal cancellation. This method is also known to behave poorly in the presence of correlated interference. For example, in the case of point sources, Capon's method will process coherent multipath components in such a way as to cancel the desired signal [7]. As we have observed in our simulation experiments (see Section 4), a similar type of problem occurs when the desired signal source is distributed (i.e. angular spread). In all these cases, the signal cancellation is accompanied by huge sidelobes and/or mainlobe offset in the beampattern. Thus, the use of Capon's method in cellular communication systems will result in an increase in the output noise as well as in the level of cochannel interference. We note that all these problems with the standard Capon's method may be attributed to the presence of the desired signal when computing the correlation matrix $\mathbf{R}$.

In cellular communication systems, all the received user signals have more or less the same average power as a result of power control management so that the above problems are *a priori* unavoidable. However, in such systems, if a particular user stops its signal transmission temporarily (or reduces its transmitted power significantly) while the other users are sending their own signal without any change, it will be possible to compute a signal-free correlation matrix (SFCM) that only contains the interference and noise components. In effect, this procedure may be realized for each IC users alternately; accordingly, we denote the SFCM for the i'th IC user at the k'th snapshot as $\mathbf{R}_{SF,i}(k)$. Now, to avoid the above problems with standard Capon's method that are related to the presence of a signal component in the correlation matrix $\mathbf{R}$, we may instead use the SFCM of the i'th IC user, i.e. $\mathbf{R}_{SF,i}(k)$, when computing the weight vector for that user. Thus, the weight vector for extracting the i'th user signal with this philosophy based on (8) is as

$$\mathbf{w}_i^{SF} = \frac{\mathbf{R}_{SF,i}^{-1}\mathbf{a}(\theta_i)}{\mathbf{a}(\theta_i)^H\mathbf{R}_{SF,i}^{-1}\mathbf{a}(\theta_i)} \tag{14}$$

We refer to this multiple beamforming strategy as signal-free Capon (SF-Capon). The weight vector $\mathbf{w}_i^{SF}$ found in this way minimizes the output noise and interference power while providing a unit gain in the estimated direction of the desired signal. Again, since the SFCM contains no desired signal component, the resulting weigh vector $\mathbf{w}_i^{SF}$ will not suffer from the previously mentioned problems. Thus, assuming that the interfering IC users are sufficiently separated in DOA from the desired user (typically $\geq$ than the resolution of an L-element antenna array), the SF-Capon strategy will produce a well-shaped beampattern with properly directed mainlobe and relatively low sidelobe levels, which are desirable attributes for mobile communications.

3.2. Analytical result for simple situation

Consider that there is only one point signal source in the environment, with true DOA θ_1. The correlation matrix $\mathbf{R}$ of the received array signal $\mathbf{x}(m)$, assuming unit power white noise, is then

$$\mathbf{R} = \mathbf{I} + \gamma\mathbf{a}(\theta_1)\mathbf{a}(\theta_1)^H \tag{15}$$

where the received signal power γ may be identified with the SNR. For simplicity we assume omnidirectional array elements with unit gain (i.e. $g(\theta) = 1$ in (4)) so that $\mathbf{a}(\theta_1)^H\mathbf{a}(\theta_1) = L$. In this case $\mathbf{R}^{-1}$ is given by

$$\mathbf{R}^{-1} = \mathbf{I} - \frac{\gamma}{1+\gamma L}\mathbf{a}(\theta_1)\mathbf{a}(\theta_1)^H \tag{16}$$

Firstly, suppose that by mistake we have taken $\hat{\theta}_1 = \theta_1 + \Delta\theta$ as the desired signal DOA in (8). With Capon's method, the resulting weight vector (8) for extracting this signal takes the form:

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{a}(\hat{\theta}_1)}{\mathbf{a}(\hat{\theta}_1)^H\mathbf{R}^{-1}\mathbf{a}(\hat{\theta}_1)} \tag{17}$$

With this weight vector, the contribution of the signal coming from direction θ_1 at the beamformer output power is $\gamma|\mathbf{w}^H\mathbf{a}(\theta_1)|^2$, while the noise contribution is simply $\mathbf{w}^H\mathbf{w}$. Thus the array gain will be:

$$AG = \frac{|\mathbf{w}^H\mathbf{a}(\theta_1)|^2}{\mathbf{w}^H\mathbf{w}} = \frac{|\mathbf{a}(\hat{\theta}_1)^H\mathbf{R}^{-1}\mathbf{a}(\theta_1)|^2}{\mathbf{a}(\hat{\theta}_1)^H\mathbf{R}^{-2}\mathbf{a}(\hat{\theta}_1)} \tag{18}$$

Now consider the SF-Capon strategy, in which the signal power is set to zero when computing the correlation matrix. The resulting SFCM in this case is $\mathbf{R}_{SF,1} = \mathbf{I}$ (simply set $\gamma = 0$ in (15)). By analogy with (18), the array gain for SF-Capon will thus be

$$AG_{SF} = \frac{|\mathbf{a}(\hat{\theta}_1)^H\mathbf{a}(\theta_1)|^2}{L} \tag{19}$$

Using (16), (18) and (19) it can be verified that

$$\frac{AG_{SF}}{AG} \geq 1 + (\gamma L)^2\sin^2\phi \tag{20}$$

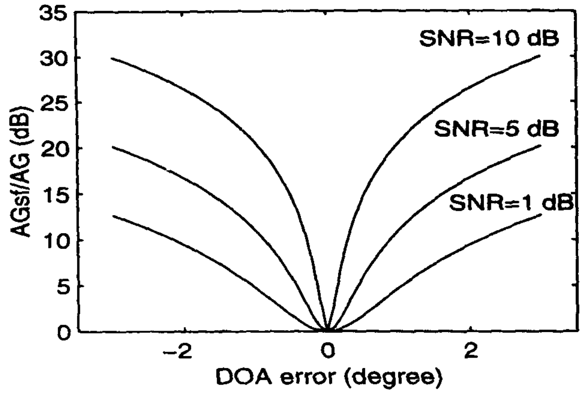

Figure 1. Ratio of AG_{SF}/AG versus DOA error for different values of input SNR.

where ϕ is the angle between the two vectors $\mathbf{a}(\theta_1)$ and $\mathbf{a}(\hat{\theta}_1)$, i.e. $\cos\phi = |\mathbf{a}(\theta_1)^H\mathbf{a}(\hat{\theta}_1)|/L$.

To illustrate the improvement in array gain resulting from SF-Capon, Fig. 1 shows the ratio of AG_{SF}/AG versus $\Delta\theta$ for a UCA with $L = 13$ omnidirectional elements.

3.3. Implementation considerations

For the estimation of the SFCM $\mathbf{R}_{SF,i}$, the TDMA/FDMA signaling structure shown in Fig.2 is assumed. The data transmitted in a given frequency channel is divided into consecutive frames of length T, each frame being made up of 8 time slots. IC users share the same time slots and BS separately extracts the signals by beamforming. During each time slot there is a reduced transmitted power (RTP) region for each user, which does not overlap with other IC users RTP regions.

Based on this configuration, $\mathbf{R}_{SF}$ may be estimated recursively as follows ($0 < \mu < 1$):

$$\mathbf{R}_{SF,i}(k) = \mu\mathbf{R}_{SF,i}(k-1) + (1-\mu)\mathbf{x}(kT+\tau_i)\mathbf{x}^H(kT+\tau_i) \tag{21}$$

where τ_i denotes the relative position of the i^{th} IC user's RTP.

With all beamforming methods and SDMA strategies we should note an important problem. If the difference in DOA between two users becomes less than a predefined threshold and the two users settle in a common zone, there will be a need of hand-off for one of these users to another time slot and/or frequency channel. The precise value of the threshold depends on the spatial resolution of the L-element antenna array and the angular spread of the signal sources. Not implementing the hand-off when the threshold has been reached would lead to an undesirable beampattern, which in turn would increase interference and noise.

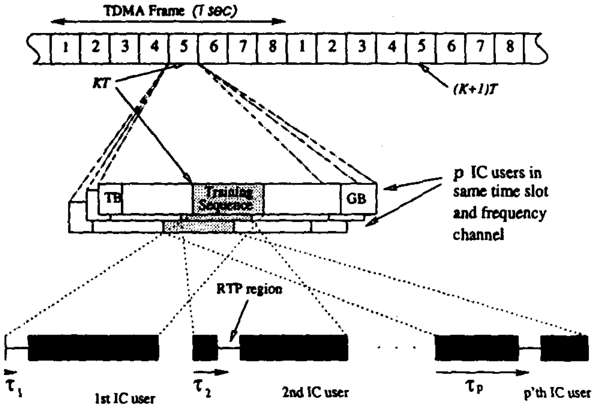

Figure 2. Suggested TDMA/FDMA signaling method.

If we use the tracking method (9)-(13) directly in a mobile environment, large fluctuations in DOA estimation will occur, resulting in frequent and unnecessary hand-offs. To avoid this problem, we propose the following change to (13):

$$\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \mu_2\Delta\hat{\theta}_i(k) \tag{22}$$

The added μ_2 ($0 < \mu_2 < 1$) may be viewed as a relaxation parameter that helps to smooth the DOA tracker.

4 Simulation Results

For computer simulations we assume that there is a uniform circular array (UCA) with $L = 13$ omnidirectional elements at the BS of the cell. The distance between elements of this array is $\lambda/2$ and $\lambda = 0.3$ meter (1 GHz). We assume that the p mobile IC users are moving with speed of 100 km/h at a distance of 400 meters from the UCA, and that the received power from these users is the same. For simplicity, the effect of other cells is not considered.

In our channel simulations, the scatterers are uniformly distributed within a circular region of radius $R_m = 100\lambda$ around the mobile units [3] so that the angular spread of the received signal is 8.6°. DOA tracking of the signal sources is accomplished with the method exposed in Section 2.3 (with $\mu_1 = 0.4$ in (12)), properly modified as in (22). For computing the SFCM $\mathbf{R}_{SF,i}$ we use (21) with $\mu = 0.95$ and $T = 4.6$ msec; thus we update $\hat{\theta}_i(k)$, $\mathbf{R}_{SF,i}(k)$ and $\mathbf{w}_i(k)$ every 4.6 msec. For initialization of the tracking we add a uniformly distributed error between $-2.5°$ and $+2.5°$ (i.e. $\Delta\theta_i(0) \sim \mathcal{U}[-2.5°; 2.5°]$) to the true DOA. To avoid the common zone problem, we assume that hand-off will occur via system management if the angular separation of two users becomes less than 10°.

Assuming that there is only one user per cell (i.e. $p = 1$), Fig. 3 shows the true DOA path and the DOA estimation of

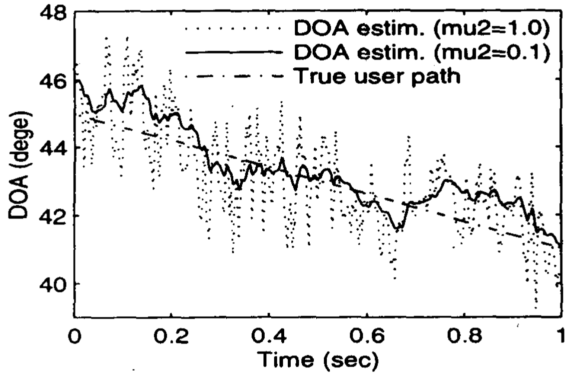

Figure 3. True DOA and DOA estimation of mobile source.

this user using (9)-(12) and (22) with $\mu_2 = 0.1$ and 1. With $\mu_2 = 1$, the curve shows very rapid fluctuations and large deviations in DOA estimation. In this case the DOA difference of two neighboring users will sometime fall below the predefined threshold by mistake and an unnecessary handoff will occur. Besides, these deviations will degrade the performance of the beamforming methods. In contrast, the curve corresponding to $\mu_2 = 0.1$ is smoother and shows smaller deviations from the true path. Note however that fluctuations of the DOA estimate are unavoidable in a mobile environment, due to the distributed nature of the source (multipath effect). We use (9)-(12) and (22) with $\mu_2 = 0.1$ for tracking in our subsequent simulations.

Fig. 4 shows the BER for standard Capon, DILS and SF-Capon beamformers when there are $p = 1, 2, 3, 4$ IC users in the cell; BPSK modulation is assumed. It is seen that standard Capon does not work (for the reasons explained previously). For $p \geq 2$, the performance of SF-Capon significantly exceeds that of DILS. For example in the case $p = 3$ for a BER of 3×10^{-3}, SF-Capon requires an input SNR of 9.5 dB while DILS requires 14.4 dB. Investigation of the beampatterns reveals that SF-Capon produces deep, wide nulls in the direction of the interfering spread signal sources while DILS produces sharp nulls in these directions, which is not adequate for this type of interference.

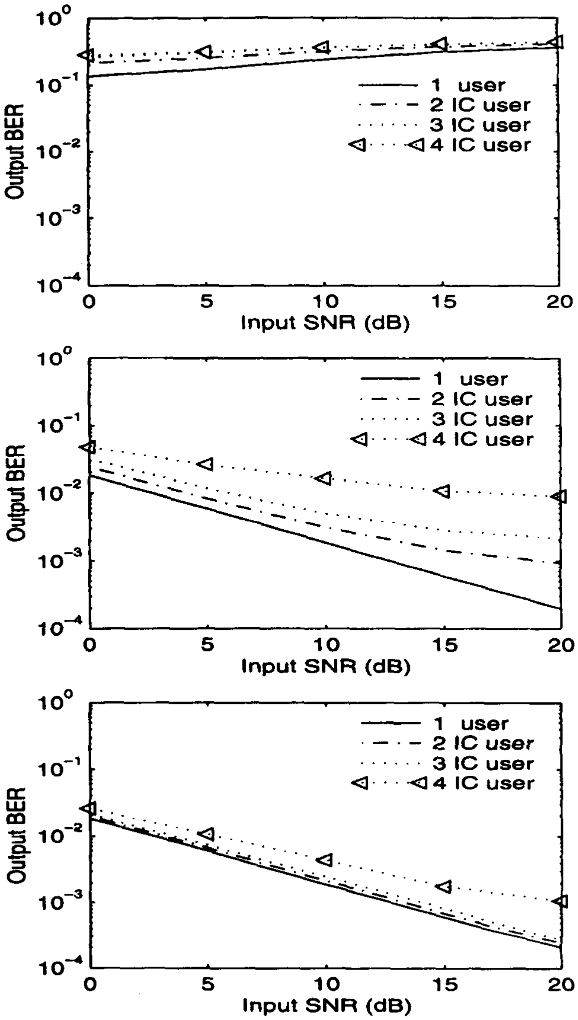

Figure 4. Output BER versus input SNR for different number of IC-users: Capon's method (top), DILS method (middle), SF-Capon's method (bottom).

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