Separable Dimension Subspace Method for Joint Signal Frequencies, DOAs and Sensor Mutual Coupling Estimation

Jian Mao[1,2], Benoit Champagne[1], Mairtin O'Droma[2], and Kevin Kwiat[3]

[1]Dept. of Electrical and Computer Engineering
McGill University, Montreal, Quebec H3A 2A7, Canada
mao@tsp.ece.mcgill.ca, champagne@ece.mcgill.ca

[2]Dept.of Electronic and Computer Engineering
University of Limerick, Limerick, Ireland *
jain.mao, mairtin.odroma@ul.ie

[3]Air Force Research Lab
AFRL/IFGA, Rome, NY 13441-4505, USA
kwiatk@rl.af.mil

Abstract

To extract the frequencies and direction of arrivals (DOAs) of multiple sources from experimental data collected by a sensor array is a multiple parameter estimation problem. Some important algorithms for spatial-temporal processing have been developed in the past decades. A practical problem, not often considered, is that the different sensors in the array affect each other through mutual coupling. This effect varies with frequencies and degrades the performance of algorithms. Thus, a separable dimension subspace method to simultaneously estimate signal frequencies, direction of arrivals (DOAs) and sensor mutual coupling is proposed in this paper.

1. Introduction

The estimation of the frequencies and direction of arrivals (DOA's) of multiple signals by using an antenna array has attractive and important applications in various areas, such as radar and communication systems. In particular, for next generation mobile communication systems, due to an endless quest for increased capacity and improved quality, frequency and DOA estimation become a requirement. Many effective algorithms for simultaneously estimating the frequencies and DOA of multiple signals have been developed in the recent past [1, 3, 4]. Most of them assume that the array response is completely known. However, many factors, such as mutual coupling among the different array sensors, will alter the array response in practical applications. Particularly, in the presence of multiple sources with different frequencies, the sensor mutual coupling will vary with frequency and thus severely affect frequency and DOA estimation accuracy. In Reference [5], a joint signal frequencies, DOAs and array model errors estimation algorithm was proposed. Due to simulated annealing process employed, the proposed algorithm has high computation cost. In this paper, a separable dimension subspace method to simultaneously estimate signal frequencies, DOAs and mutual coupling parameters of antenna array is presented. With separable dimension processing, a spatial and temporal estimation problem is separated. i.e, the frequencies are first estimated by using a subspace method and then the DOAs and mutual coupling parameters are estimated for the each estimated frequency. In this way, the computational complexity of the proposed method is relatively small.

2. Problem Formulation

Consider an array composed of M sensors and each sensor output being fed to a tap-delay-line with m taps. The delay between adjacent taps is t_0. Specially, let $\mathbf{X}_l(t)(l = 1, 2, \cdots, M)$ denotes the output of the l-th sensor at time t, and let $\mathbf{X}_l(t) = [x_l(t), \cdots, x_l(t-(m-1)t_0)]^T$. Assume

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that p narrow-band sources with different frequencies and directions impinge on the array and the coming signals can be divided into D groups according to their wavelength, i.e,

$$p = \sum_{k=1}^{D} p_k \qquad (D \leq p) \tag{1}$$

where p_k is the number of signals, from different directions, in the $k-th$ group, which wavelength is λ_k $(k = 1, 2, \cdots, D)$.

The l-th sensor $(l = 1, 2, \cdots, M)$ output vector may be defined as:

$$\begin{aligned} &\mathbf{X}_l(t) \\ &= \sum_{k=1}^{D}\sum_{i=1}^{M}(C_{l,i}(\omega_k)\mathbf{a}(\omega_k)\mathbf{b}_i(\boldsymbol{\theta}_k))\mathbf{S}_k(t) + \mathbf{N}_l(t); \\ &\qquad (k = 1, 2, \cdots, D, i = 1, 2, \cdots, M) \end{aligned} \tag{2}$$

where
$\mathbf{b}_i(\boldsymbol{\theta}_k) = [\exp(-j\omega_k\tau_i(\theta_{k,1})) \cdots \exp(-j\omega_k\tau_i(\theta_{k,p_k}))]$ is a $1 \times p_k$ spatial steering vector;
$\mathbf{a}(\omega_k) = [1 \ \exp(-j\omega_k t_0) \cdots \exp(-j\omega_k(m-1)t_0)]^T$ is an $m \times 1$ temporal steering vector; $C_{l,i}(\omega_k)$ is the $(l,i)-th$ entry in the mutual coupling matrix (MCM) which represents the mutual coupling effect from other sensors;

$\mathbf{S}_k(t) = [S_{k,1}(t) \cdots S_{k,p_k}(t)]^T$ is a $p_k \times 1$ signal vector from the k-th group of narrowband sources and $\mathbf{N}_l = [N_l(t) \cdots N_l(t-(m-1)t_0)]^T$ is an $m \times 1$ additive noise vector for the l-th sensor.

Therefore, the array data model can be described as follows:

$$\begin{aligned} \mathbf{X}(t) &= \left[\mathbf{X}^T{}_1(t)\mathbf{X}^T{}_2(t)\cdots\mathbf{X}^T{}_M(t)\right]^T \\ &= \sum_{k=1}^{D}(\mathbf{C}(\omega_k)\mathbf{B}(\boldsymbol{\theta}_k)) \otimes \mathbf{a}(\omega_k)\mathbf{S}_k(t) + \mathbf{N}(t) \end{aligned} \tag{3}$$

where

$$\mathbf{B}(\boldsymbol{\theta}_k) = \begin{bmatrix} \mathbf{b}_1(\boldsymbol{\theta}_k) \\ \vdots \\ \mathbf{b}_M(\boldsymbol{\theta}_k) \end{bmatrix}$$

is a $M \times p_k$ spatial steering matrix, $\mathbf{N}(t) = \left[\mathbf{N}^T{}_1(t)\,\mathbf{N}^T{}_2(t)\cdots\mathbf{N}^T{}_M(t)\right]^T$ is $Mm \times 1$ additive white Gaussian noise, Symbol $\otimes$ denotes Kronecker product and $\mathbf{C}(\omega_k)$ $(k = 1, 2, \cdots, D)$ is $M \times M$ mutual coupling matrix. For a uniform linear array or circular array, $\mathbf{C}(\omega_k)$ is either a Toeplitz matrix or a circular matrix, respectively [6].

Based on the available samples $\{\mathbf{X}(t)\}_{t=1}^{N}$, the problem is to estimate the frequencies, DOAs and mutual coupling matrix simultaneously with subspace methods.

3 Subspace Method

Assume that the number of signals p and the number of frequency groups, D, are known.

Define

$$\mathbf{r}'_h = \frac{1}{M}\sum_{l=1}^{M} E[x_l(t-(h-1)t_0)\mathbf{X}_l^H(t)] \tag{4}$$

$$(h = 1, 2, \cdots, D)$$

$$\eta'_l = \frac{1}{m}\sum_{h=1}^{m} \mathrm{E}[y_l(t-(h-1)t_0)\mathbf{Y}^H(t-(h-1)t_0)] \tag{5}$$

$$(l = 1, 2, \cdots, p)$$

where

$$\begin{aligned} &y_l(t-(h-1)t_0) = (\mathbf{X}_l(t))_h = x_l(t-(h-1)t_0), \\ &\mathbf{Y}(t-(h-1)t_0) \\ &= [y_1(t-(h-1)t_0)y_2(t-(h-1)t_0)\cdots y_M(t-(h-1)t_0)]^T \\ &= [x_1(t-(h-1)t_0)x_2(t-(h-1)t_0)\cdots x_M(t-(h-1)t_0)]^T \end{aligned}$$

and $E(\cdot)$ denotes statistical expectation.

$\{\mathbf{r}'_h\}_{h=1}^{D}$ and $\{\eta'_l\}_{l=1}^{p}$ are frequency vector and direction vector. Their range spaces spanned by the column of these vectors which are contained or are equal to the range spaces of $\mathbf{a}(\omega)$and $\mathbf{B}(\boldsymbol{\theta})$, respectively. We have

$$\Re(\mathbf{r}'_h) \subseteq \Re(\mathbf{a}(\omega)), \quad \Re(\eta'_l) \subseteq \Re(\mathbf{B}(\boldsymbol{\theta}))$$

and

$$\aleph(\mathbf{r}'_h) \perp \Re(\mathbf{a}(\omega)), \quad \aleph(\eta'_l) \perp \Re(\mathbf{B}(\boldsymbol{\theta}))$$

where $\Re(\cdot)$ and $\aleph(\cdot)$denote the range space and null space. As is well known, the subspace methods are based on the above geometrical observations.Therefore,we may use correlation processing in spatial and temporal dimension respectively to get the estimates of $\{\mathbf{r}'_h\}_{h=1}^{D}$ and $\{\eta'_l\}_{l=1}^{p}$ and then compute their null subspaces. Finally, the frequencies, DOAs and mutual coupling matrix can be finally estimated with subspace methods by searching corresponding spaces.

4 Separable Dimension Subspace Algorithm

Step 1. Frequency Estimation

(1) Estimate of frequency vectors $\mathbf{r}'_\mathbf{h}$ and $\mathbf{r_h}$:

$$\widehat{\mathbf{r}}'_h = \frac{1}{M}\sum_{l=1}^{M}\frac{1}{N}\sum_{t=1}^{N} x_l(t-(h-1)t_0)\mathbf{X}_l^H(t) \tag{6}$$

$$\widehat{\mathbf{r}}_h = (\widehat{\mathbf{r}}'_h - \widehat{\sigma}^2 \mathbf{e}_h)^H, \quad (7)$$

where

$$\mathbf{e}_h = [\underbrace{0 \cdots 0}_{(h-1)} 1 0 \cdots 0], \quad (h = 1, 2, \cdots, D)$$

and $\widehat{\sigma}^2$ is the estimate of the noise variance.

(2) Gram-Schmidt (GS) orthogonalization and formation of temporal projection matrix $\mathbf{P}_\omega$:

From the vector $\{\widehat{\mathbf{r}}_h\}_{h=1}^D$, we can get D orthogonal vectors, $\{\mathbf{q}_k\}_{k=1}^D$ via GS orthogonalization. Let $\mathbf{Q}_\omega = [\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_D]$, then compute the temporal projection matrix $\mathbf{P}_\omega = \mathbf{I} - \mathbf{Q}_\omega \mathbf{Q}_\omega^H$, which spans the null space of $\{\mathbf{a}(\omega_k)\}_{k=1}^D$.

(3) Estimate the unknown frequencies with the temporal projection matrix $\mathbf{P}_\omega$:

The frequencies $\{\omega_k\}_{k=1}^D$ are estimated as the D largest peaks of the function $P(\omega) = (\mathbf{a}^H(\omega)\mathbf{P}_\omega \mathbf{a}(\omega))^{-1}$, searching over the frequency sector of interest.

Step 2. Direction and Mutual Coupling Estimation

(1)Estimate of direction vectors, η'_l and η_l:

$$\widehat{\eta}'_l = \frac{1}{m}\sum_{h=1}^{m} \frac{1}{N}\sum_{t=1}^{N} y_l(t-(h-1)t_0)\mathbf{Y}^H(t-(h-1)t_0) \quad (8)$$

$$\widehat{\eta}_l = (\widehat{\eta}'_l - \widehat{\sigma}^2 \mathbf{e}_l)^H, \quad (9)$$

where

$$\mathbf{e}_l = [\underbrace{0 \cdots 0}_{(l-1)} 1 0 \cdots 0] \quad (l = 1, 2, \cdots, p)$$

(2) Via Gram-Schmit orthogonalization of $\{\widehat{\eta}_l\}_{l=1}^p$, the p orthogonal vectors, $\{\zeta_l\}_{l=1}^p$ and spatial orthogonal projection matrix $\mathbf{P}_\theta = \mathbf{I} - \mathbf{Q}_\theta \mathbf{Q}_\theta^H$ are obtained, where $\mathbf{Q}_\theta$=[ζ_1 $\zeta_2 \cdots \zeta_p$]

(3) For each frequency $\{\omega_k\}_{k=1}^D$ estimated in Step 1, the directions θ_k and mutual coupling matrix $\mathbf{C}(\omega_k)$ can be estimated with following equations iteratively :

$$(\mathbf{B}^H(\boldsymbol{\theta})\mathbf{C}^H(\omega_k)\mathbf{P}_\theta \mathbf{C}(\omega_k)\mathbf{B}(\boldsymbol{\theta}))^{-1}$$

$$\widehat{\mathbf{C}}(\omega_k) = (\mathbf{G}(\omega_k)^{-1}\mathbf{w})(\mathbf{w}^H \mathbf{G}(\omega_k)^{-1}\mathbf{w})^{-1}$$

where $\mathbf{G}(\omega_k) = \sum_{k=1}^{p_k} \mathbf{B}(\theta_k)^H \mathbf{P}_\theta \mathbf{B}(\theta_k)$ and $\mathbf{w} = [1, 0, \cdots, 0]^T$.

5 Computer Simulations

A uniform circular array with 8 sensors is used for the simulation. Four equal-power narrow-band sources are located at the far field of the array with the center frequencies and directions: $S(\theta_{11}, f_1) = (30°, 8.5MHz)$, $S(\theta_{12}, f_1) = (60°, 8.5MHz)$, $S(\theta_{21}, f_2) = (90°, 6.5MHz)$, $S(\theta_{22}, f_2) = (120°, 6.5MHz)$. Additive noise is injected with SNR of 15 dB referenced to the signal source. 100 snapshots of array data are accumulated. Fig.1 shows the frequency estimation. For the 2 estimated frequencies, Fig.2 and Fig.3 show the spatial spectrum for the DOA estimations with unknown and estimated MCM, respectively. Fig.4 and Fig5 show the mean square error (MSE) of estimated DOA and frequency and their theoretical low bound. 30 Monte Carlo simulations are made for the $DOA = 30°$ and $f = 8.5MHz$

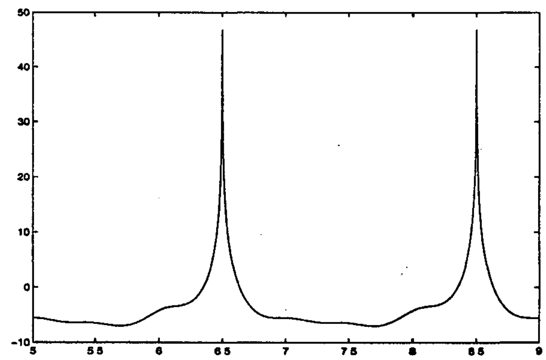

Figure 1. Frequency Estimation

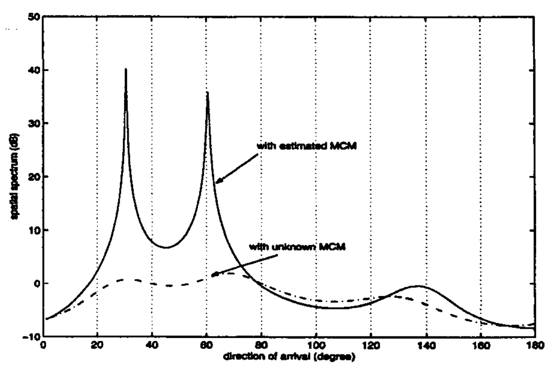

Figure 2. Spatial spectrum at the estimated frequency $f = 8.5MHz$.

6 Conclusions

In this paper a new algorithm based on separable dimension subspace method is proposed for joint estimation of signal frequencies, DOAs and the mutual coupling parameters of antenna array. The presented algorithm has been test by computer simulation studies and has been found to perform satisfactorily.

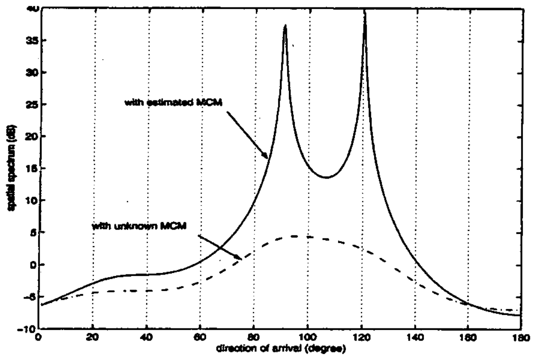

Figure 3. Spatial spectrum at the estimated frequency $f = 6.5MHz$.

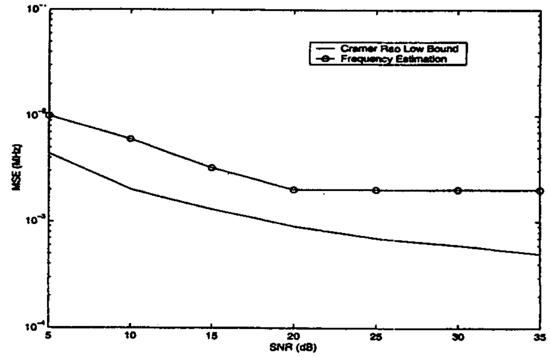

Figure 5. MSE and CRLB for the frequency estimation, $f = 8.5MHz$

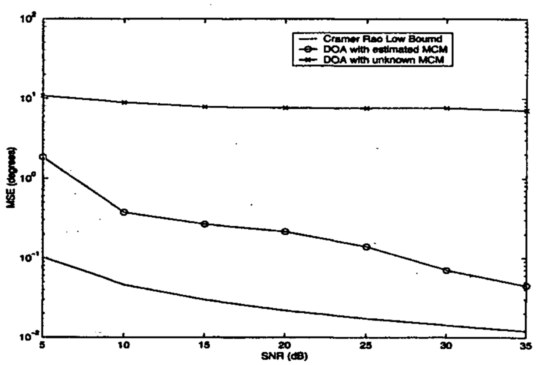

Figure 4. MSE and CRLB for the DOA estimation,$DOA = 30°$

7 Appendix: Cramer Rao Low Bound

In this appendix, we derived Cramer Rao Low Bound for the proposed separable subspace method.

The likelihood function for the data set is given by

$$L(\mathbf{X}(1), \mathbf{X}(2), \cdots, \mathbf{X}(N)/\mathbf{\Phi}) = \prod_{j=1}^{N} \frac{1}{\pi^M \|\mathbf{R}\|} \exp(-\mathbf{X}(j)^H \mathbf{R}^{-1} \mathbf{X}(j)) \quad (10)$$

where $\mathbf{X}(j) = \sum_{k=1}^{D} \mathbf{C}(\omega_k)\mathbf{B}(\boldsymbol{\theta}_k) \bigotimes \mathbf{a}(\omega_k)\mathbf{S}_k(j) + \mathbf{N}(j)$, $\mathbf{\Phi} = (\boldsymbol{\theta}, \boldsymbol{\omega}, \mathbf{C})$, and matrix $\mathbf{R}$ is

$$\mathbf{R} = E\left[\mathbf{X}\mathbf{X}^H\right] = \sum_{k=1}^{D} \mathbf{C}(\omega_k)\mathbf{B}(\boldsymbol{\theta}_k) \otimes \mathbf{a}(\omega_k)\mathbf{R}_{sk} \quad (11)$$

$$(\mathbf{C}(\omega_k)\mathbf{B}(\boldsymbol{\theta}_k) \otimes \mathbf{a}(\omega_k))^H + \sigma_{nk}^2 \mathbf{I}$$

Assuming all sources have signal-noise rate $P = \mathbf{R}_{sk}/\sigma_n^2$, then the Cramer Rao low bound is

$$CRLB(\Phi) = [J_{mn}]^{-1} \quad (12)$$

where the $m, n - th$ element of the Fisher information matrix are

$$J_{mn} = N \cdot tr\{\mathbf{R}^{-1} \cdot \partial\mathbf{R}/\partial\Phi_m \cdot \mathbf{R}^{-1} \cdot \partial\mathbf{R}/\partial\Phi_n\} \quad (13)$$

7.1 DOA terms

$$\begin{aligned} J_{\theta\theta} &= 2N \cdot \Re(tr(\dot{\mathbf{A}}_\theta P \mathbf{A}^H \mathbf{R}^{-1} \mathbf{A} P \dot{\mathbf{A}}_\theta^H \mathbf{R}^{-1}) \\ &+ tr(\dot{\mathbf{A}}_\theta P \mathbf{A}^H \mathbf{R}^{-1} \dot{\mathbf{A}}_\theta P \mathbf{A}^H \mathbf{R}^{-1})) \end{aligned} \quad (14)$$

where $\mathbf{A} = \mathbf{C}\ \mathbf{B}(\boldsymbol{\theta}) \otimes \mathbf{a}(\omega)$ and $\dot{\mathbf{A}}_\theta = \mathbf{C}\ (\partial\mathbf{B}(\boldsymbol{\theta})/\partial\boldsymbol{\theta}) \otimes \mathbf{a}(\omega) = \mathbf{C}\ \dot{\mathbf{B}}(\boldsymbol{\theta}) \otimes \mathbf{a}(\omega)$.

7.2 Frequency ω terms

$$\begin{aligned} J_{\omega\omega} &= 2N \cdot \Re(tr(\dot{\mathbf{A}}_\omega P \mathbf{A}^H \mathbf{R}^{-1} \mathbf{A} P \dot{\mathbf{A}}_\omega^H \mathbf{R}^{-1}) \\ &+ tr(\dot{\mathbf{A}}_\omega P \mathbf{A}^H \mathbf{R}^{-1} \dot{\mathbf{A}}_\omega P \mathbf{A}^H \mathbf{R}^{-1})) \end{aligned} \quad (15)$$

where

$$\begin{aligned} \dot{\mathbf{A}}_\omega &= \mathbf{C}(\partial\mathbf{B}(\boldsymbol{\theta})/(\partial\omega) \otimes \mathbf{a}(\omega) + \mathbf{C}\mathbf{B}(\boldsymbol{\theta}) \otimes (\partial\mathbf{a}(\omega)/\partial\omega) \\ &= \mathbf{C}\dot{\mathbf{B}}_\omega \otimes \mathbf{a} + \mathbf{C}\mathbf{B} \otimes \dot{\mathbf{a}}_\omega \end{aligned}$$

7.3 Mutual coupling terms

Suppose circulant mutual coupling matrix with only a single coupling coefficient given by

$$C_{12} = \mu e^{j\xi} \quad (16)$$

where μ and ξ are magnitude and phase of mutual coupling coefficient. Then, we have

$$\begin{aligned} J_{\mu\mu} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\mu^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1})) \\ J_{\xi\xi} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\xi P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\xi^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\xi P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\xi P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{17}$$

where $\dot{\mathbf{A}}_\mu = (\partial\mathbf{C}/\partial\mu)\cdot\mathbf{B}\otimes\mathbf{a}$ and $\dot{\mathbf{A}}_\xi = (\partial\mathbf{C}/\partial\xi)\cdot\mathbf{B}\otimes\mathbf{a}$

7.4 Cross terms

The cross terms are derived as

$$\begin{aligned} J_{\theta\omega} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\theta P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\omega^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\theta P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\omega P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{18}$$

$$\begin{aligned} J_{\theta\mu} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\theta P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\mu^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\theta P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{19}$$

$$\begin{aligned} J_{\theta\xi} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\theta P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\xi^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\theta P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\xi P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{20}$$

$$\begin{aligned} J_{\omega\mu} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\omega P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\mu^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\omega P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{21}$$

$$\begin{aligned} J_{\omega\xi} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\omega P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\xi^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\omega P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\xi P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{22}$$

$$\begin{aligned} J_{\mu\xi} &= 2N \cdot \Re\,(tr\,(\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}P\dot{\mathbf{A}}_\xi^H\mathbf{R}^{-1}) \\ &+ \ tr\,(\dot{\mathbf{A}}_\mu P\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\xi P\mathbf{A}^H\mathbf{R}^{-1})) \end{aligned} \tag{23}$$

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