DECENTRALIZED COLLABORATIVE UPLINK BEAMFORMING WITH ROBUSTNESS AGAINST CHANNEL MISMATCHES

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ABSTRACT

We present a distributed algorithm for collaborative uplink transmit beamforming that provides robustness against uncertainties in the channel state information. Our algorithm is derived using the available information about the second-order statistics of the channel and the possibly erroneous channel state. It can be applied to both line-of-sight propagation and flat fading channels. The beamforming coefficients of each terminal are computed locally using the available information about its channel and a single parameter that is broadcasted from the base station to all the cooperating terminals.

1. INTRODUCTION

Collaborative beamforming has been recently proposed to improve the performance of wireless communication systems by exploiting the spatial characteristics of the channel [1]. The main idea behind collaborative beamforming is to consider groups of nearby terminals as a virtual antenna array forming a spatial beam in the direction of the target base station. Collaborative beamforming techniques differ from their classical counterparts due to their distributed nature since the array elements are distributed among different terminals. Thus, only limited amount of information can be shared between the cooperating terminals with possible errors and delays.

Many algorithms have been recently proposed to provide robustness against array manifold errors for line-of-sight (LOS) propagation environments [2], [3], [4]. Robust transmit beamforming for fading channels was also considered in [5] and [6] with robustness against mismatches in the channel state and covariance matrix, respectively. However, all the algorithms presented in [2]–[6] were derived based on the assumption that the array elements are located within a single processing unit. Hence, they are not suitable for collaborative transmission scenarios where the beamforming coefficients have to be locally computed by the cooperating terminals.

In [7], we have presented a collaborative beamforming algorithm with robustness against channel mismatches. The beamforming coefficients are computed by the base station using the uplink measurements and fed back to the cooperating terminals. In this paper, we modify our algorithm in [7] to allow for the computation of the beamforming vector for each terminal locally with minimum feedback from the base station. First, we review the unified signal model for both LOS and flat fading channels initially introduced in [7]. Our signal model divides the available channel information into two parts: a perfectly known part that corresponds to the second-order statistics of the channel or the local array manifolds of the cooperating terminals, and a possibly erroneous estimate of the *channel realization driving vector* that captures the randomness of the channel and is assumed to belong to a predefined uncertainty set. We formulate the beamforming problem as minimizing the total transmitted power while preserving the received signal at the base station for all the channel vectors within the uncertainty set. We provide a closed-form solution for the optimal weight vector using the method of Lagrange multipliers. This solution allows the beamforming coefficients to be computed by each terminal using its local channel information and the Lagrange multiplier which is computed at the base station and broadcasted to all the cooperating terminals. Hence, our algorithm is well-suited to collaborative transmission scenarios where a limited amount of information has to be shared among the terminals. We also show that our algorithm is equivalent to an optimum power allocation strategy among the eigen beams of the channels of the cooperating terminals based on the strength of each eigen beam and the uncertainty in their channel estimates.

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2. SIGNAL MODEL

We consider the uplink of a narrowband wireless communication system where M terminals are collaboratively transmitting a common signal to the same base station. The mth terminal is equipped with a k_m-element antenna array. The received baseband signal at the base station at the ith time instant can be written as

$$y(i) = \sum_{m=1}^{M} \boldsymbol{w}_m^H \boldsymbol{h}_m s(i) + w(i) = \boldsymbol{w}^H \boldsymbol{h} s(i) + v(i) \qquad (1)$$

where $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively, $\boldsymbol{h}_m$ is the $k_m \times 1$ vector containing the channel coefficients from the mth terminal to the base station, $\boldsymbol{w}_m$ is the $k_m \times 1$ beamforming vector of the mth terminal, and $v(i)$ is white Gaussian noise with zero mean and variance σ_v^2. The $K \times 1$ stacked channel vector is $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_M^T]^T$ and the corresponding stacked beamforming vector is $\boldsymbol{w} = \left[\boldsymbol{w}_1^T, \ldots, \boldsymbol{w}_M^T\right]^T$ where $K = \sum_{m=1}^{M} k_m$.

2.1. Line-of-Sight Propagation Environment

The channel vector of the mth terminal can be written as

$$\boldsymbol{h}_m = e^{-j2\pi f_0 T_m(\theta_m)} \boldsymbol{a}_m(\theta_m) \qquad (2)$$

where $\boldsymbol{a}_m(\theta_m) = [1, e^{-j2\pi f_0 \tau_{m,2}(\theta_m)}, \ldots, e^{-j2\pi f_0 \tau_{m,k_m}(\theta_m)}]^T$, f_0 is the carrier frequency, $\tau_{m,i}(\theta_m)$ is the propagation delay of the signal transmitted from the ith antenna of the mth terminal towards the base station, located in the direction θ_m, relative to that transmitted from the first antenna of the mth terminal, and $T_m(\theta_m)$ is the propagation delay from the first antenna of the mth terminal relative to virtual antenna located at a common reference point. We can write the stacked channel vector as

$$\boldsymbol{h} = \boldsymbol{V}\boldsymbol{n} \tag{3}$$

where $\boldsymbol{n} = [e^{-j2\pi f_0 T_1(\theta_1)}, \ldots, e^{-j2\pi f_0 T_M(\theta_M)}]^T$ is the so-called channel realization driving vector, the $K \times M$ matrix

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{a}_1(\theta_1) & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{a}_2(\theta_2) & \boldsymbol{0} & \vdots \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{a}_M(\theta_M) \end{bmatrix}, \tag{4}$$

and $\boldsymbol{0}$ is column vector of zeros with appropriate dimension. Hence, the channel vector can be decomposed into the product of a matrix $\boldsymbol{V}$ that contains the local array manifold vectors of the M terminals and a vector $\boldsymbol{n}$ containing their phase offsets. Note that the uncertainty in the relative location and/or the synchronization error of the mth terminal can be modeled as an error in the propagation delay T_m. Thus, we can model the actual channel vector as

$$\boldsymbol{h} = \boldsymbol{V}(\hat{\boldsymbol{n}} + \boldsymbol{\Delta}) \tag{5}$$

where $\hat{\boldsymbol{n}} = [e^{-j2\pi f_0 \hat{T}_1}, \ldots, e^{-j2\pi f_0 \hat{T}_M}]^T$ is the estimate of $\boldsymbol{n}$ and $\{\hat{T}_m\}$ are the presumed delay offsets.

2.2. Flat Fading Propagation Environment

In the case of multipath flat fading channels, the channel vector of the mth terminal can be written as $\boldsymbol{h}_m = \boldsymbol{R}_m^{\frac{1}{2}}\boldsymbol{n}_m$ [8], where $\boldsymbol{R}_m$ is the covariance matrix of the channel vector of the mth terminal, and $\boldsymbol{n}_m$ is a $k_m \times 1$ vector of independent zero mean, unit variance, complex Gaussian random variables. Note that we have assumed that the channel vector of each terminal is independent of that of the other terminals, i.e., the terminals are well-separated in space. Hence, we can write the stacked channel vector as $\boldsymbol{h} = \boldsymbol{V}\boldsymbol{n}$ where the $K \times K$ matrix $\boldsymbol{V}$ is given by

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{R}_1^{\frac{1}{2}} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_2^{\frac{1}{2}} & \boldsymbol{0} & \vdots \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{R}_M^{\frac{1}{2}} \end{bmatrix} \tag{6}$$

and $\boldsymbol{n} = [\boldsymbol{n}_1^T, \ldots, \boldsymbol{n}_M^T]^T$ is the $K \times 1$ channel realization driving vector. We can assume that the channel is quasi-stationary, i.e., the second-order statistics of the channel are almost constant within a certain stationarity time interval [8]. Hence, we can model the actual stacked channel vector $\boldsymbol{h}$ by the same model as that in Eq. (5) where $\hat{\boldsymbol{n}} = [\hat{\boldsymbol{n}}_1^T, \ldots, \hat{\boldsymbol{n}}_M^T]^T$ and $\boldsymbol{\Delta}$ is the corresponding error vector.

3. ROBUST TRANSMIT BEAMFORMING

If the cooperating terminals have perfect knowledge of the channel vector, the optimum beamformer that maximizes the transmission efficiency, i.e., the ratio between the received signal-to-noise ratio (SNR) at the base station and the transmitted power, is given by $\boldsymbol{w} = \alpha\boldsymbol{V}\boldsymbol{n}$ where α is a scalar that determines the transmitted power. However, at the transmission instant each terminal has a possibly erroneous estimate $\hat{\boldsymbol{n}}$ of the channel realization driving vector which might degrade the received SNR at the base station.

Let us define the ellipsoidal uncertainty set $\mathcal{A}$ as

$$\mathcal{A} = \{\tilde{\boldsymbol{n}} = \hat{\boldsymbol{n}} + \boldsymbol{D}\boldsymbol{u} \mid \|\boldsymbol{u}\| \leq 1\} \tag{7}$$

where the diagonal matrix $\boldsymbol{D}$ determines the lengths of the axes of the hyper-ellipsoid. In the case of LOS propagation, the $M \times M$ matrix $\boldsymbol{D} = \mathrm{diag}\{\varepsilon_1, \ldots, \varepsilon_M\}$, whereas, for fading channels the $K \times K$ matrix $\boldsymbol{D} = \mathrm{diag}\{[\varepsilon_1 \boldsymbol{1}_{k_1}^T, \ldots, \varepsilon_M \boldsymbol{1}_{k_M}^T]^T\}$ where $\boldsymbol{1}_k$ is the $k \times 1$ vector containing all ones. We formulate our robust beamforming problem as minimizing the total transmitted power from all the terminals while providing high gain for all the vectors in $\mathcal{A}$ [4], i.e.,

$$\min_{w} \boldsymbol{w}^H\boldsymbol{w} \quad \text{s.t.} \quad \left|\boldsymbol{w}^H\boldsymbol{V}\tilde{\boldsymbol{n}}\right| \geq 1 \quad \forall \tilde{\boldsymbol{n}} \in \mathcal{A}. \tag{8}$$

The constraint in the above optimization problem will be satisfied for all $\tilde{\boldsymbol{n}} \in \mathcal{A}$ if it is satisfied for the worst-case (mismatched) channel vector in $\mathcal{A}$. Thus, we can write (8) as

$$\min_{w} \boldsymbol{w}^H\boldsymbol{w} \quad \text{s.t.} \quad \min_{\tilde{\boldsymbol{n}} \in \mathcal{A}} \left|\boldsymbol{w}^H\boldsymbol{V}\tilde{\boldsymbol{n}}\right| \geq 1. \tag{9}$$

The minimum of $\left|\boldsymbol{w}^H\boldsymbol{V}(\hat{\boldsymbol{n}} + \boldsymbol{D}\boldsymbol{u})\right|$ over the set $\mathcal{A}$ can be found by observing that

$$\left|\boldsymbol{w}^H\boldsymbol{V}(\hat{\boldsymbol{n}} + \boldsymbol{D}\boldsymbol{u})\right| \geq \left|\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}\right| - \left|\boldsymbol{w}^H\boldsymbol{V}\boldsymbol{D}\boldsymbol{u}\right| \tag{10}$$

$$\geq \left|\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}\right| - \left\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\right\| \tag{11}$$

where (10) and (11) were derived using the triangle and Cauchy-Schwarz inequalities, respectively. The worst-case error that satisfies (10) and (11) with equality is given by

$$\boldsymbol{u} = -e^{j\phi}\frac{\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}}{\left\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\right\|} \quad \text{where } \phi = \arg\left\{\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}\right\}. \tag{12}$$

Substituting with (11) in (9), and phase-rotating the vector $\boldsymbol{w}$ so that $\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}$ is real, we can formulate the robust beamforming problem as the following second-order cone program

$$\min_{w} \ \boldsymbol{w}^H\boldsymbol{w} \quad \text{s.t.} \quad \mathrm{Im}\left\{\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}\right\} = 0$$

$$\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}} - \left\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\right\| \geq 1. \tag{13}$$

We will proceed to derive a closed-form solution of the optimization problem in (13). First, we note that the second constraint of (13) has to be satisfied with equality by the optimal weight vector, or else, we can always scale down the solution to further minimize the cost function while still satisfying the constraints. Also, the first constraint in (13) is now redundant as it is implied by the second one when it is satisfied with equality. Therefore, we can write (13) as

$$\min_{w} \boldsymbol{w}^H\boldsymbol{w} \quad \text{s.t.} \quad \boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}} - \left\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\right\| = 1. \tag{14}$$

Following the guidelines of [4], we can find a closed-form solution to (14) using the method of Lagrange multipliers and by imposing the additional constraint $\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}} - 1 \geq 0$. Since $\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}}$ is real-valued, we can write (14) as

$$\min_{w} \boldsymbol{w}^H\boldsymbol{w} \quad \text{s.t.} \quad \left\|\boldsymbol{D}\boldsymbol{V}^H\boldsymbol{w}\right\|^2 - \left|\boldsymbol{w}^H\boldsymbol{V}\hat{\boldsymbol{n}} - 1\right|^2 = 0. \tag{15}$$

The Lagrangian associated with (15) is given by

$$L(\boldsymbol{w}, \lambda) = \boldsymbol{w}^H(\boldsymbol{I}_K + \lambda\boldsymbol{Q})\boldsymbol{w} + \lambda\hat{\boldsymbol{h}}^H\boldsymbol{w} + \lambda\boldsymbol{w}^H\hat{\boldsymbol{h}} - \lambda \tag{16}$$

where $\boldsymbol{I}_K$ denotes the $K \times K$ identity matrix, the $K \times 1$ vector $\hat{\boldsymbol{h}} = \boldsymbol{V}\hat{\boldsymbol{n}}$, and the $K \times K$ Hermitian matrix $\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{D}^2\boldsymbol{V}^H - \hat{\boldsymbol{h}}\hat{\boldsymbol{h}}^H$. By equating the complex gradient of (16) to zero, we can write the optimal solution of (15) as

$$\boldsymbol{w} = -\lambda(\boldsymbol{I}_K + \lambda\boldsymbol{Q})^{-1}\hat{\boldsymbol{h}} \tag{17}$$

where the optimal value of the Lagrange multiplier λ satisfies the constraint in (15), i.e.,

$$0 = \lambda^2 \hat{\boldsymbol{h}}^H \left(\boldsymbol{I}_K + \lambda \boldsymbol{Q}\right)^{-1} \boldsymbol{Q} \left(\boldsymbol{I}_K + \lambda \boldsymbol{Q}\right)^{-1} \hat{\boldsymbol{h}} - 2\lambda \hat{\boldsymbol{h}}^H \left(\boldsymbol{I}_K + \lambda \boldsymbol{Q}\right)^{-1} \hat{\boldsymbol{h}} - 1 \quad (18)$$

We define the eigen decomposition of $\boldsymbol{Q}$ as $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{U}^H$ where $\boldsymbol{\Gamma}$ is the diagonal $K \times K$ matrix containing the eigenvalues of the matrix $\boldsymbol{Q}$ arranged in non-increasing order and $\boldsymbol{U}$ is the $K \times K$ matrix containing the corresponding eigenvectors. If we define $\boldsymbol{c} = \boldsymbol{U}^H \hat{\boldsymbol{h}}$, we can write the solution of (18) as the root of the function

$$f(\lambda) = \lambda^2 \sum_{i=1}^{K} \frac{|c_i|^2 \gamma_i}{\left(1 + \lambda\gamma_i\right)^2} - 2\lambda \sum_{i=1}^{K} \frac{|c_i|^2}{\left(1 + \lambda\gamma_i\right)} - 1 \quad (19)$$

where γ_i is the ith eigenvalue of the matrix $\boldsymbol{Q}$, and c_i is the ith entry of the vector $\boldsymbol{c}$.

The value of λ can be evaluated by solving for all the roots of (19) and selecting the root that yields the minimum value of the cost function in (15) while satisfying the additional constraint $\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} - 1 \geq 0$. However, it was shown in [4] that the additional constraint is satisfied for all values of λ greater than a threshold $\lambda_{\min}$, and that there exists only one root of (19) that satisfies $\lambda > \lambda_{\min}$ where

$$\lambda_{\min} = \frac{-1 - |c_K| \left(\gamma_K + |c_K|^2\right)^{-\frac{1}{2}}}{\gamma_K}, \quad (20)$$

γ_K is the single negative eigenvalue of the matrix $\boldsymbol{Q}$ and c_K is the corresponding entry of the vector $\boldsymbol{c}$. Therefore, Newton-Raphson method can be used to solve for the value of the optimal λ in (18), where the iterations are initialized with $\lambda_{\min}$.

Given the optimal value of the Lagrange multiplier, the complex-valued robust beamforming weight vector can be written as

$$\boldsymbol{w} = -\lambda \left(\boldsymbol{I}_K + \lambda \boldsymbol{V}\left(\boldsymbol{D}^2 - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H\right)\boldsymbol{V}^H\right)^{-1} \boldsymbol{V}\hat{\boldsymbol{n}}. \quad (21)$$

We will proceed to further simplify (21) and show how it can be computed locally by each terminal. Using the matrix inversion lemma, we can write (21) as

$$\boldsymbol{w} = -\boldsymbol{V}\left(\lambda \boldsymbol{I} - \left((\boldsymbol{D}^2 - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H)^{-1} + \lambda \boldsymbol{V}^H\boldsymbol{V}\right)^{-1}\boldsymbol{V}^H\boldsymbol{V}\right)\hat{\boldsymbol{n}}. \quad (22)$$

For the case of LOS propagation, $\boldsymbol{V}^H\boldsymbol{V} = \boldsymbol{\Upsilon}$ where $\boldsymbol{\Upsilon} = \mathrm{diag}\{k_1, \ldots, k_m\}$, and hence, using the matrix inversion lemma we can further simplify (22) to

$$\begin{aligned} \boldsymbol{w} &= -\lambda \boldsymbol{V}\left(\boldsymbol{I}_M - \left(\frac{1}{\lambda}\boldsymbol{\Upsilon}^{-1}(\boldsymbol{D}^2 - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^H)^{-1} + \boldsymbol{I}_M\right)^{-1}\right)\hat{\boldsymbol{n}} \\ &= \frac{\boldsymbol{V}\boldsymbol{\Upsilon}^{-1}\boldsymbol{T}^{-1}\hat{\boldsymbol{n}}}{\hat{\boldsymbol{n}}^H\boldsymbol{T}^{-1}\hat{\boldsymbol{n}} - 1} \end{aligned} \quad (23)$$

where $\boldsymbol{T} = \boldsymbol{D}^2 + \frac{1}{\lambda}\boldsymbol{\Upsilon}^{-1}$. Since changing the norm of the beamforming vector affects only the transmitted power and not the transmission efficiency, we can drop the denominator of (23) and use the equivalent beamforming vector

$$\boldsymbol{w} \equiv \boldsymbol{V}\left(\boldsymbol{D}^2\boldsymbol{\Upsilon} + \frac{1}{\lambda}\boldsymbol{I}_M\right)^{-1}\hat{\boldsymbol{n}}. \quad (24)$$

Hence, the beamforming vector for the mth terminal can be obtained by weighting its classical weight vector estimate $e^{-j2\pi f T_m}\boldsymbol{a}_m(\theta_m)$ by $\left(\varepsilon_m^2 k_m + \frac{1}{\lambda}\right)^{-1}$. This is an optimal power allocation strategy for each terminal based on the uncertainty in its phase offset.

In the case of fading channels, we can write the robust beamformer weight vector in (22) as

$$\boldsymbol{w} = \frac{1}{\beta}\boldsymbol{V}\left(\boldsymbol{D}^2\boldsymbol{R} + \frac{1}{\lambda}\boldsymbol{I}_K\right)^{-1}\hat{\boldsymbol{n}} \quad (25)$$

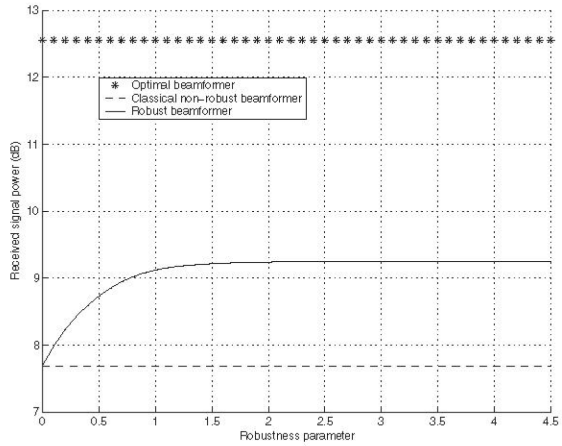

Fig. 1. Average received signal power versus ε.

where $\beta = \hat{\boldsymbol{n}}^H\left(\boldsymbol{D}^2 + \frac{1}{\lambda}\boldsymbol{R}^{-1}\right)^{-1}\hat{\boldsymbol{n}} - 1$ and the $M \times M$ block-diagonal matrix $\boldsymbol{R} = \boldsymbol{V}^H\boldsymbol{V}$. Therefore, normalizing the transmitted power such that the constant $\beta = 1$, the mth terminal beamforming vector is given by

$$\boldsymbol{w}_m \equiv \boldsymbol{R}_m^{\frac{1}{2}}\left(\varepsilon_m^2\boldsymbol{R}_m + \frac{1}{\lambda}\boldsymbol{I}_{k_m}\right)^{-1}\hat{\boldsymbol{n}}_m. \quad (26)$$

Let the eigen decomposition of $\boldsymbol{R}_m$ be $\boldsymbol{R}_m = \boldsymbol{E}_m\Sigma_m\boldsymbol{E}_m^H$. Therefore, we can write (26) as

$$\boldsymbol{w}_m \equiv \sum_{k=1}^{k_m} \frac{\lambda\sqrt{\sigma_{m,k}}\,\boldsymbol{e}_{m,k}^H\hat{\boldsymbol{n}}_m}{1 + \lambda\varepsilon_m^2\sigma_{m,k}}\boldsymbol{e}_{m,k} \quad (27)$$

where $\boldsymbol{e}_{m,k}$ is the kth eigen vector of $\boldsymbol{R}_m$ and $\sigma_{m,k}$ is the associated eigen value. Thus, our robust beamforming technique is an optimum power allocation strategy along the eigen beams of the channel covariance matrix based on the uncertainty in the channel estimate and the power associated with each eigen mode. When the channel is perfectly known, i.e., $\varepsilon_m = 0$, the beamforming gain increases as the eigen beam power increases. However, when there is any uncertainty in the channel estimate, the beamforming gain does not necessarily increase as the eigen beam power increases, i.e., the beamforming gain decreases if $\sigma_{m,k}$ increases beyond $1/(\lambda\varepsilon_m^2)$.

Therefore, for both LOS propagation and fading environments, the cooperating terminals can start transmission using their classical non-robust beamforming vectors. With the base station feeding back the parameter λ computed using the uplink measurements, each terminal can compute its robust beamforming vector based on the knowledge of its channel realization vector estimate and covariance matrix (local array manifold) only. This process can be repeated to track any changes in the operating environment.

4. NUMERICAL SIMULATIONS

Simulation 1: Line-of-sight propagation environment

We consider the uplink of a wireless communication system with $M = 5$ cooperating terminals. Each terminal is equipped with an antenna array of $k_1 = 4$, $k_2 = 3$, $k_3 = 2$, $k_4 = 4$, and $k_5 = 5$ elements with half-wavelength spacing. The antenna arrays of the first, third, and fourth terminals are located parallel to the X-axis with the center of the arrays presumed to be at $[10.75\lambda, 5\lambda]$, $[15.25\lambda, 0]$, and $[12.75\lambda, -3\lambda]$, respectively. The arrays of the second and fifth terminals are located parallel to the Y-axis with the center of the arrays presumed to be at $[15\lambda, 6.5\lambda]$ and $[18\lambda, \lambda]$, respectively. The actual location of the mth terminal is displaced along the X- and

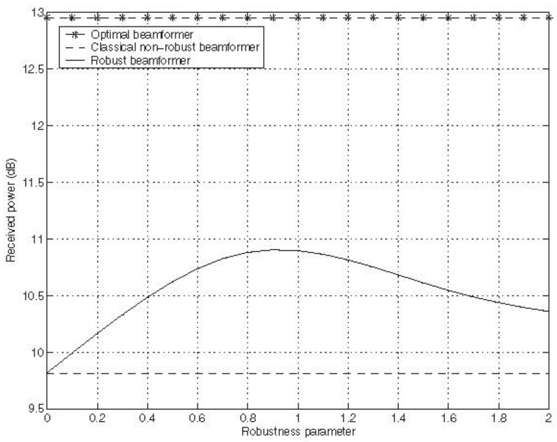

Fig. 2. Average received signal power versus ε.

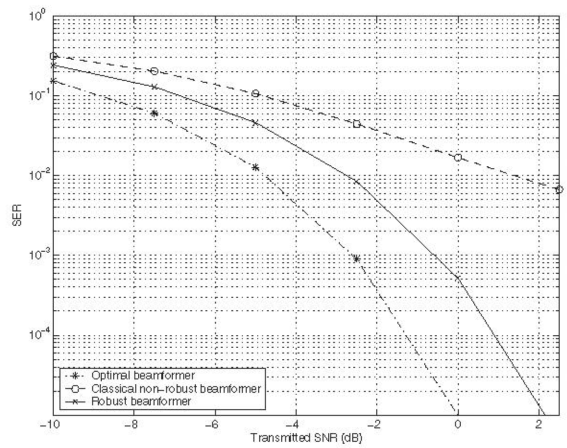

Fig. 3. Average SER versus transmitted SNR.

Y-axes from its nominal location by independent random displacements that are uniformly distributed between $[-0.5\lambda\delta_m, 0.5\lambda\delta_m]$ where $\delta_1 = 0.2$, $\delta_2 = 4$, $\delta_3 = 2$, $\delta_4 = 1$, and $\delta_5 = 0.2$. The uncertainty set $\mathcal{A}$ is formed using the values $\{\varepsilon_m = \varepsilon\delta_m\}$. The desired base station is located along $\{\theta_m = 0°\}_{m=1}^{5}$, where θ_m is measured relative to the X-axis, and the wave propagation model is planar. Simulation results are averaged over 10^4 Monte Carlo runs.

Fig. 1 shows the average received power by the desired base station using our robust beamformer versus different choices of the parameter ε that correspond to different sizes of the robustness set $\mathcal{A}$. It also shows the average received power obtained using the classical weight vector $\boldsymbol{w} = \alpha\boldsymbol{V}\hat{\boldsymbol{n}}$, and the maximum received power using the optimal beamformer, i.e., with perfect knowledge of the channel. Note that all the beamforming vectors are normalized to have unit norm. We can clearly see the performance improvements (more than 1 dB gain in SNR) achieved by our beamformer compared to the classical one. Moreover, it is not very sensitive to the exact size of the uncertainty set and performs well over a wide range of ε.

Simulation 2: Flat fading environment

We consider the same collaborative transmission scenario described in the previous simulation. The propagation environment for each of the 5 terminals is modeled as a Ricean flat-fading channel with Ricean K-factor equal to 0.1, 0.2, 0.2, 0.3, and 0.5 and LOS arrival angles 10°, 20°, 30°, 40°, and 50° for the 1st to 5th user, respectively. The scattered component of the received signal due to each of the 5 terminals has a Laplacian power-angle-profile with mean angle of arrival 90°, 150°, 270°, 40°, and 180° and angular spread 10°, 3°, 6°, 5°, and 2° for the 1st to 5th user, respectively. We generate 100 independent channel realizations. For each channel realization, the estimate of the channel realization driving vector of the mth terminal is obtained as

$$\hat{\boldsymbol{n}}_m = \boldsymbol{n}_m + \frac{\delta_m \|\boldsymbol{n}_m\|}{\|\boldsymbol{\Delta}_m\|}\boldsymbol{\Delta}_m \tag{28}$$

where $\boldsymbol{\Delta}_m$ is a standard circular Gaussian vector with independent components, and δ_m is the relative magnitude of the error in the channel vector estimate. The values of δ_m are given by 0.4, 2, 4, 4, and 0.2 for $m = 1$ to 5, respectively. The set $\mathcal{A}$ is formed using $\{\varepsilon_m = \varepsilon\delta_m\}$. Simulation results are averaged over 50 independent realizations of $\{\boldsymbol{\Delta}_m\}$ and 100 independent channel realizations.

Fig. 2 shows the average received signal power at the base station versus different values of the parameter ε. All the beamforming vectors have been normalized to have unit norm. From this figure, we can clearly see that the proposed robust beamforming technique can improve the received signal power by more than 1 dB compared to the classical non-robust beamformer. We can also notice that the received signal power does not degrade considerably over a wide range of the size of the robustness set. Fig. 3 shows the average symbol error rate (SER) versus the transmitted signal power for different beamformers for the QPSK constellation. For our robust beamformer, we have selected the value of ε that yields the highest received SNR. We can clearly see from Fig. 3 that the power gain offered by our beamforming technique is translated into a corresponding gain in the average SER.

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