Dual Domain Echo Cancellers for Multirate Discrete Multitone Systems

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***Abstract*—Digital echo cancellers are used in duplex digital subscriber lines (DSL) transceivers to remove the echo, which is the leakage of the transmitted signal onto the collocated receiver. For discrete multitone (DMT) modulated systems, the computational complexity of these cancellers can be reduced by using the structure present in the transmitted signal. In [1] and [2], we have proposed a novel dual domain echo canceller for symmetric rate DMT systems. In practical DSL systems, asymmetric bandwidths are used for the upstream and downstream transmissions, therefore, echo cancellers must be designed to work in multirate cases. In this paper, we examine the implementation of the previously proposed dual domain canceller in the multirate scenario, and show that by using the polyphase decomposition of the signals, a reduced complexity implementation of this canceller can be achieved.**

I. Introduction

DSL technologies are used to provide broadband access over existing copper telephone lines. In these systems, the overlapping downstream and upstream data are sent over a single pair of wires, using a hybrid circuit. An impedance mismatch in this circuitry, causes the leakage of signals from the transmitter into the collocated receiver which is known as echo. Duplexing methods, such as time division duplexing (TDD) and frequency division duplexing (FDD) can be used to separate the downstream and upstream transmission. TDD and FDD are effective in terms of avoiding the echo, but the inefficient use of the available bandwidth in the case of TDD, and the requirement of precise analog filtering in the case of FDD, make them less favourable. Therefore, design of digital echo cancellers, where adaptive filters are used to cancel the echo, is of interest in various DSL technologies.

In DSL systems using DMT modulation, the direct time domain implementation of the basic echo canceller is characterized with high computational cost; however, this problem can be resolved by exploiting the structure present in the transmitted signal. Various algorithms, utilizing this property, have been introduced in the literature, such as the circular echo synthesis (CES) method by Ho *et. al* [3], the circulant decomposition canceller (CDC) method by Ysebaert *et. al* [4] and the symmetric decomposition canceller (SDC) by Pisoni and Bonaventura [5]. These methods are mainly based on an adaptive canceller in which the echo emulation is done jointly in the time and frequency domains, resulting in reduced complexity. In some cases, specially for the CES canceller, the transmission of dummy data with reduced power on the unused tones is required to attain an acceptable convergence rate.

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In [1], we introduced the dual transform domain canceller (DTDC) for DMT systems. In this method, a pair of unitary transforms is used to map the time-domain signals into alternate domains where echo emulation and weight update can be performed more conveniently. This structure also provides a unifying representation of the previous algorithms for the echo cancellation. In [2], we showed that the DTDC algorithm can be implemented with a comparable complexity to the previous methods, while performing an exact adaptive update in the dual domain, which results in a faster convergence without requiring extra excitation on the unused tones.

In practical DSL systems, the bandwidth assigned for the downstream and the upstream are different. Therefore, in the case of equal symbol rates in both directions, more samples are received at the remote terminal (RT) where larger bandwidth is demanded than at the central office (CO). Consequently, echo cancellers need to be adjusted to function in the multirate setups. In this paper, we expand the symmetric rate DTDC algorithm to multirate systems. We discuss two approaches for implementing this algorithm: the direct approach, using decimation and interpolation of the corresponding signals, and the polyphase approach, based on the polyphase decomposition of the signals. It is shown that the latter provides a more efficient implementation in terms of complexity.

In Section II, current methods for echo cancellation in DMT systems are reviewed, both for symmetric rate and multirate scenarios. In Section III, the two approaches for implementing the DTDC algorithm for multirate systems are presented. In Section IV, the implementation of these algorithms using discrete trigonometric transformers is discussed along with computational requirements. Finally, Section V includes the simulation results comparing the convergence of the proposed multirate DTDC algorithms with existing echo cancellers. Following notations are used, the square identity matrix of size M is denoted by $\mathcal{I}_M$, and the discrete Fourier transformation of size N is denoted by $\mathcal{F}_N$. Diagonal matrix with diagonal elements given by vector $\mathbf{v}$ is denoted by diag$\{\mathbf{v}\}$. Also, a new matrix extracted from a matrix $\mathcal{U}$ using its every k^{th} columns or rows in the interval i and j starting at column or row i is

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denoted by $\mathcal{U}[i:k:j\,,\,:]$ and $\mathcal{U}[:\,,\,i:k:j]$, respectively.

II. Background

In this section, we briefly review some of the methods used for echo cancellation in DMT-modulated systems. We examine these cancellers first for symmetric rate systems, and later examine their implementation in the multirate systems.

A. Symmetric rate echo cancellers

In a symmetric DMT transceiver with symbol frame size of N and cyclic prefix size of υ, the emulated echo at each symbol period is expressed by

$$\mathbf{y}_e^k = \mathcal{U}^k\,\mathbf{w}^k \tag{1}$$

where $\mathbf{w}^k$ is the weight vector of the echo canceller. For the general asynchronous case with a delay of Δ samples between the echo frames and received far-end frames, the matrix $\mathcal{U}^k$ in (1) is an $N\times N$ Toeplitz matrix consisting of elements from symbols, $\mathbf{u}^{k-1}$, $\mathbf{u}^k$ and $\mathbf{u}^{k+1}$. The first row of this matrix is $[u_\Delta^k, \cdots, u_0^k, u_{N-1}^k, \cdots, u_{N-\upsilon}^k, u_{N-1}^{k-1}, \cdots, u_{\Delta+\upsilon+1}^{k-1}]$ and its first column is $[u_\Delta^k, \cdots, u_{N-1}^k, u_{N-\upsilon}^{k+1}, \cdots, u_{N-1}^{k+1}, u_0^{k+1}, \cdots, u_{\Delta-\upsilon-1}^{k+1}]^T$, where u_i^k $(i = 0, \cdots, N-1)$ denotes the i^{th} sample of the k^{th} symbol.

Various echo cancellers for DMT systems use different decompositions of the Toeplitz matrix $\mathcal{U}^k$. For example, in [3], Ho *et al.* introduced the CES canceller, where the matrix $\mathcal{U}^k$ is decomposed as a sum of a circulant matrix and a residual component. Using this decomposition, the circulant part can be diagonalized using the Fourier transform. Therefore, the echo emulation can be done jointly in the time and frequency domains, where the complexity of the frequency-domain part is reduced to one complex multiplication per tone. Complexity is further reduced by performing the approximate weight update in the frequency domain, where a per tone LMS update is performed using the elements obtained from the diagonalization of the circulant part.

In [4], Ysebaert *et al.* proposed the CDC method to ameliorate the convergence of the CES canceller. In this approach, the Toeplitz matrix $\mathcal{U}^k$ is decomposed into a sum of a circulant matrix and a skew-circulant matrix. The latter is then transformed into a complex valued circulant matrix, and the two circulant matrices are diagonalized using the Fourier transform. Using similar approximation as in CES method, the per tone approximate weight update is performed in the frequency domain using the elements obtained from the diagonalization of the circulant components of the matrix $\mathcal{U}^k$.

In [5], Pisoni and Bonaventura proposed a symmetric decomposition canceller (SDC) where the Toeplitz matrix $\mathcal{U}^k$ is decomposed using discrete cosine and sine transforms (DCT and DST). They also derive the relationship between the SDC and CDC and use the symmetric decomposition to achieve a more efficient implementation for the latter.

In [1], we considered a general decomposition of the Toeplitz matrix $\mathcal{U}^k$, given by

$$\mathcal{U}^k = \mathcal{G}_N^H\,\mathcal{S}^k\,\mathcal{G}_N, \tag{2}$$

where the $2N\times N$ matrix $\mathcal{G}_N$ contains unitary submatrices which are constant for all symbol periods, and matrix $\mathcal{S}^k$ is calculated based on the transmitted symbols. We also showed that the general form in (2) provides a uniform representation of the decompositions used in the echo cancellers discussed earlier. Based on this general decomposition, a novel dual transform domain canceller (DTDC) was proposed for the symmetric rate DMT systems.

In this canceller, the unitary submatrices of $\mathcal{G}_N$ are used to transform the time domain signals and filter weights, providing a more convenient domain to perform the echo emulation and adaptive weight update. In this algorithm, the transformed emulated echo is

$$\mathbf{Y}_e^k = \mathcal{G}_N\mathbf{y}_e^k = \Phi^k\boldsymbol{\omega}^k. \tag{3}$$

where Φ^k consists of the transformed input samples, *i.e.*, $\Phi^k = \mathcal{G}_N\,\mathcal{G}_N^H\,\mathcal{S}^k$, and $\boldsymbol{\omega}^k = \mathcal{G}_N\,\mathbf{w}^k$ is the mapped weight vector in the dual domain. As shown in [1], the error signal for the proposed DTDC is calculated by

$$\mathbf{E}^k = \mathbf{Y}^k - \Phi^k\boldsymbol{\omega}^k \tag{4}$$

where $\mathbf{Y}^k = \mathcal{G}_N\,\mathbf{y}^k$ is the transformed received signal. Using the LMS algorithm, the echo weights are updated by

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu\,(\Phi^k)^H\,\mathbf{E}^k. \tag{5}$$

The above equation offers a complete and exact adaptation for the transform domain canceller. In existing echo cancellers, the dual transformation is partially used in the emulation part but avoided in the weight update part, in order to reduce the computational cost. As shown in [2], the adaptive weight update (5) in the transform domain is equivalent to

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \,\mu\,(\mathcal{S}^k)^H\mathbf{E}^k. \tag{6}$$

Therefore, for the decompositions in which the submatrices of $\mathcal{S}^k$ are diagonal or at most tridiagonal, the echo canceller weights can be updated using no approximation and with low computational complexity.

B. Multi rate echo cancellers

In practical DSL systems, the bandwidth assigned for the downstream and the upstream are different. Therefore, in the case of equal symbol rates in both directions, more samples are received at the RT where larger bandwidth is demanded than at the CO. Hence, echo cancellers need to be adjusted to function in a multirate configuration.

At the RT transceiver, the transmitted signal bandwidth is κ times smaller than that of the received signal. Hence, because of the non-ideal reconstruction filter at the digital-to-analog convertor (DAC), the higher frequencies of the transmitted signal can leak into the received signal. This aliasing is modelled by the interpolation of the transmitted signal before the convolution, which in the time domain corresponds to padding $\kappa - 1$ zeros between adjacent samples. At the CO transceiver, the transmitted signal bandwidth is κ times larger than that of the received signal and aliasing occurs because of the non-ideal anti-aliasing filter at the analog-to-digital convertor (ADC). To consider the effect of the aliasing in the echo canceller, the emulated echo is decimated which corresponds to downsampling by factor κ, in the time domain.

In [3], the CES echo canceller is implemented for the multirate case directly using the interpolation and decimation in the time and frequency domains. This means that the matrix $\mathcal{U}^k$ in (1) is interpolated and decimated, at the RT and CO sides, respectively. For the frequency-domain operations, the transmitted symbol is replicated, at the RT, and the emulated echo is blocked and added together, at the CO side. It should be noted that the echo channel is always modelled at the higher data rate. In [4], it is shown that a similar approach as in [3] is possible for implementing the CDC multirate echo canceller. Furthermore, by using the polyphase components of the time domain signals and the weight vector, the computational complexity of the canceller can be reduced. In this approach the transformations at the higher rate, needed for mulirate CDC, is replaced by multiple lower rate transformations, which is more efficient to implement.

III. Multi rate dual transform domain canceller

In this section, we present the expansion of the DTDC algorithm in [1], [2] for the multirate DSL systems. Two implementations are presented for the cancellers both at the RT and the CO transceivers: the direct approach and the polyphase decomposition approach.

A. Decimated DTDC

1) Direct approach decimated DTDC: As discussed earlier, at the CO transceiver, the emulated echo needs to be decimated, which is expressed in the time domain by

$$\mathbf{y}_e^k = \mathbf{D}\,\mathcal{U}^k\,\mathbf{w}^k. \tag{7}$$

In (7), $\mathcal{U}^k$ and $\mathbf{w}^k$ are the Toeplitz data matrix and the time-domain weight vector, respectively at the higher data rate, *i.e.*, the matrix $\mathcal{U}^k$ has the dimension $\kappa N \times \kappa N$ and the vector $\mathbf{w}^k$ has the length κN, and the $N \times \kappa N$ matrix $\mathbf{D} = \mathcal{I}_{\kappa N}[1 : \kappa : \kappa N\ , :]$ performs the downsampling. Hence, the emulated echo in the dual domain is given by

$$\mathbf{Y}_e^k = \mathcal{G}_N\,\mathbf{D}\,\mathcal{G}_{\kappa N}^H\,\mathcal{S}^k\,\mathcal{G}_{\kappa N} = \mathbf{w}^k\Phi_{\text{dec}}^k\,\boldsymbol{\omega}^k. \tag{8}$$

In (8), the data matrix $\mathcal{U}^k$ is replaced by its decomposition performed at higher rate, *i.e.*, $\mathcal{U}^k = \mathcal{G}_{\kappa N}^H\mathcal{S}^k\mathcal{G}_{\kappa N}$, while the mapping into the dual domain, done after the downsampling, is performed at the lower rate. It should be noted that for clarity a subscript is used for denoting the size of the mapping matrix $\mathcal{G}$, where the $2N \times N$ matrix is denoted by $\mathcal{G}_N$, and the $2\kappa N \times \kappa N$ matrix is denoted by $\mathcal{G}_{\kappa N}$. As a result, in (8), the matrix Φ_{dec}^k has the dimension $2N \times 2\kappa N$, and the transformed weight vector $\boldsymbol{\omega}^k = \mathcal{G}_{\kappa N}\mathbf{w}^k$. Using the decimated emulated echo, the error signal can be calculated from (4).

Since the weights are updated at the higher rate, the error signal needs to be upsampled in the time domain $\mathbf{e}_{\text{int}}^k$ and transformed into the dual domain $\mathbf{E}_{\text{int}}^k = \mathcal{G}_{\kappa N}\mathbf{e}_{\text{int}}^k$. Using the error vector $\mathbf{e}_{\text{int}}^k$, the weights are updated by

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu\,(\mathcal{S}^k)^H\,\mathbf{E}_{\text{int}}^k. \tag{9}$$

As it can be seen, in this implementation, the decomposition of the matrix $\mathcal{U}^k$ and the mapping of error signal are done at the higher data rate. In the next section, we show that these calculations can be done at the lower rate after proper adjustments, which results in reduced complexity.

2) Polyphase approach decimated DTDC: We next present a more efficient implementation of the decimated DTDC, using the polyphase decomposition approach as in [4]. The decimated emulated echo, in (7), can be rewritten in terms of polyphase components of the data matrix and the weight vector, *i.e.*,

$$\mathbf{y}_e^k = \sum_{l=1}^{\kappa}\mathcal{U}_l^k\,\mathbf{w}_l^k \tag{10}$$

where the $N \times N$ polyphase matrices of $\mathcal{U}^k$ are $\mathcal{U}_l^k = \mathcal{U}^k[1 : \kappa : \kappa N\ , l : \kappa : \kappa N]$, and the length N polyphase vectors of $\mathbf{w}^k$ are $\mathbf{w}_l^k = \mathbf{w}^k[l : \kappa : \kappa N]$. It can be easily verified that the polyphase matrices $\mathcal{U}_l^k$ are Toeplitz matrices, therefore, each one can be decomposed as in (2). In this case the decomposition $\mathcal{U}_l^k = \mathcal{G}_N^H\mathcal{S}_l^k\mathcal{G}_N$ is done at the lower data rate. Using these decompositions, (10) can be rewritten as

$$\mathbf{y}_e^k = \sum_{l=1}^{\kappa}\mathcal{G}_N^H\,\mathcal{S}_l^k\,\mathcal{G}_N\,\mathbf{w}_l^k. \tag{11}$$

Therefore, the transformed emulated echo is given by

$$\mathbf{Y}_e^k = \sum_{l=1}^{\kappa}\Phi_l^k\boldsymbol{\omega}_l^k, \tag{12}$$

where $\Phi_l^k = \mathcal{G}_N\mathcal{G}_N^H\mathcal{S}_l^k$ and the transformed weight vectors $\boldsymbol{\omega}_l^k = \mathcal{G}_N\mathbf{w}_l^k$, which can be calculated using only the transformations at the lower data rate, unlike the direct approach.

The error signal, calculated from (4), can be used directly to adaptively update the polyphase weight vectors $\boldsymbol{\omega}_l^k$, *i.e.*,

$$\boldsymbol{\omega}_l^{k+1} = \boldsymbol{\omega}_l^k + \mu\,(\mathcal{S}_l^k)^H\mathbf{E}^k \quad \text{for } l = 1, \cdots, \kappa. \tag{13}$$

As shown above, the polyphase decomposition can be used to replace the transformation at the higher data rate with multiple lower rate transformations, as shown later, requires less computations.

B. Interpolated DTDC

1) Direct approach interpolated DTDC: As discussed earlier, at the RT transceiver, the transmitted signal needs to be interpolated before the echo emulation, as in

$$\mathbf{y}_e^k = \mathcal{U}_{\text{int}}^k\,\mathbf{w}^k \tag{14}$$

where the $\kappa N \times \kappa N$ matrix $\mathcal{U}_{\text{int}}^k$ contains the upsampled transmitted samples, and vector $\mathbf{w}^k$ is the weight vector at the higher data rate. The interpolated data matrix $\mathcal{U}_{\text{int}}^k$ is then decomposed at the higher rate, *i.e.*, $\mathcal{U}_{\text{int}}^k = \mathcal{G}_{\kappa N}^H\mathcal{S}_{\text{int}}^k\mathcal{G}_{\kappa N}$. Based on this decomposition, the transformed emulated echo is given by

$$\mathbf{Y}_e^k = \mathcal{G}_{\kappa N}\,\mathcal{G}_{\kappa N}^H\,\mathcal{S}_{\text{int}}^k\,\mathcal{G}_{\kappa N}\,\mathbf{w}^k = \Phi_{\text{int}}^k\,\boldsymbol{\omega}^k, \tag{15}$$

where the $2\kappa N \times 2\kappa N$ matrix Φ^k_{int} is calculated based on the interpolated matrix $\mathcal{U}^k_{\text{int}}$. Thus, the error signal is calculated from (4), and can be used to update the weights:

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu\,(\mathcal{S}^k_{\text{int}})^H\,\mathbf{E}^k. \tag{16}$$

Similar to the decimated DTDC, the polyphase decomposition can be used to reduce the computational complexity of this algorithm, as discussed in the following.

2) Polyphase approach interpolated DTDC: At the RT transceiver, the emulated echo, in (14), can be expressed in terms of the matrix $\mathcal{U}^k$ which contains the input data before the upsampling, *i.e.*,

$$\mathbf{y}^k_e = \sum_{l=1}^{\kappa} \mathbf{D}_{\kappa N,l}\,\mathcal{U}^k\,\mathbf{w}^k_l, \tag{17}$$

where the polyphase vectors of $\mathbf{w}^k$ are $\mathbf{w}^k_l = \mathbf{w}^k[l:\kappa:\kappa N]$. In (17), the $\kappa N \times N$ matrices $\mathbf{D}_{\kappa N,l} = \mathcal{I}_{\kappa N}[:,\, l:\kappa:\kappa N]$ are used to upsample the emulated echo properly.

Therefore, the matrix $\mathcal{U}^k$ can now be decomposed at the lower data rate, *i.e.*, $\mathcal{U}^k = \mathcal{G}^H_N \mathcal{S}^k \mathcal{G}_N$. Consequently, the mapped emulated echo in the dual domain is given by

$$\mathbf{Y}^k_e = \sum_{l=1}^{\kappa} \mathcal{G}_{\kappa N}\,\mathbf{D}_{\kappa N,l}\,\mathcal{G}^H_N\,\mathcal{S}^k\,\mathcal{G}_N\,\mathbf{w}^k_l \tag{18}$$

$$= \sum_{l=1}^{\kappa} \Phi^k_l \boldsymbol{\omega}^k_l, \tag{19}$$

where $2\kappa N \times 2N$ matrices $\Phi^k_l = \mathcal{G}_{\kappa N}\,\mathbf{D}_{\kappa N,l}\,\mathcal{G}^H_N\,\mathcal{S}^k$, and the transformed weight vectors $\boldsymbol{\omega}^k_l = \mathcal{G}_N \mathbf{w}^k_l$. The error signal at the high rate can be calculated from (4), however to achieve lower complexity, it is preferable to have polyphase error vectors $\mathbf{E}^k_l$. The $\mathbf{E}^k_l = \mathcal{G}_N \mathbf{e}^k_l$, are the dual domain transformation of the polyphase vecrots of the error in the time-domain $\mathbf{e}^k_l$. The resulting error vectors can now be used to update the polyphase weight vectors $\boldsymbol{\omega}^k_l$, given by

$$\boldsymbol{\omega}^{k+1}_l = \boldsymbol{\omega}^k_l + \mu\,(\mathcal{S}^k)^H \mathbf{E}^k_l \quad \text{for } l = 1, \cdots, \kappa. \tag{20}$$

In this implementation, the decomposition of the matrix $\mathcal{U}^k$ is done at lower rate; however, the formation of the Φ^k_l matrices and the mapping of the error signal still requires the transformation at the higher data rate. In the next section, we show that in the cases where the error is calculated in the time domain, this method can be implemented more efficiently.

IV. Multirate dual trigonometric canceller

The DTDC structure is defined in terms of a general decomposition of the matrix $\mathcal{U}^k$ and therefore, for its implementation, a specific decomposition must be used. In the symmetric decomposition, where the data matrix is decomposed using the DCT and the DST [5], the resulting submatrices of $\mathcal{S}^k$ are either diagonal or have non-zero elements only on the super-diagonal or on the sub-diagonal [1]. This characteristics makes this decomposition a suitable candidate for implementing the DTDC structure. In [1], we refer to this implementation of

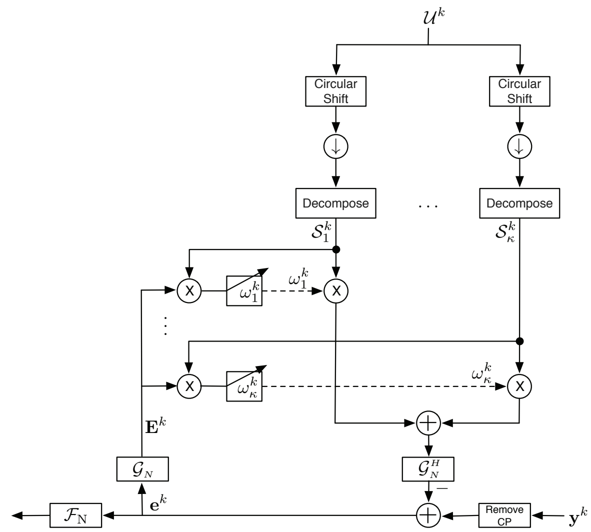

Fig. 1. Decimated DTC using polyphase approach, at CO transceiver

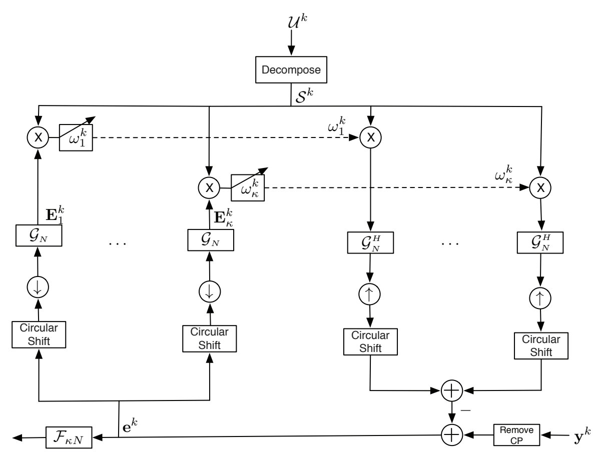

Fig. 2. Interpolated DTC using polyphase approach, at RT transceiver

DTDC, using symmetric decomposition as the dual trigonometric canceller (DTC). In this canceller, the mapping matrix $\mathcal{G}_N$ performs the DCT and DST. In addition, the error signal is calculated in the time domain and mapped into the dual domain to be used for updating the weights.

Utilizing the polyphase approach and the symmetric decomposition, the DTDC can be implemented very efficiently for multirate systems. In Fig. 1, The decimated DTC structure using polyphase approach is depicted. In this implementation, the emulated echo is calculated using (11), and the polyphase weight vectors $\boldsymbol{\omega}^k_l$ are updated by (13). In Fig. 2, the interpolated DTC structure using polyphase approach is illustrated. Since in this implementation the error signal is calculated in the time domain, the higher rate transformation $\mathcal{G}_{\kappa N}$ in (18), for forming the Φ^k_l matrices, is avoided. In addition, the polyphase vectors of the error signal $\mathbf{e}^k_l$ are formed easily in the time domain, and the mapping required for the calculation of vectors $\mathbf{E}^k_l$ is done at the lower rate.

TABLE I
COMPLEXITY COMPARISON BETWEEN THE POLYPHASE AND THE DIRECT APPROACHES FOR THE DTC ALGORITHM

Application	Direct approach	Polyphase approach
DTC - CO	$3\kappa N \log_2 \kappa N + 6\kappa N + 2$	$(\kappa N + 2N) \log_2 N + 6\kappa N + 2\kappa$
DTC - RT	$2\kappa N \log_2 \kappa N + N \log_2 N + 8\kappa N - 2N + 2$	$(2\kappa N + N) \log_2 N + 8\kappa N - 2N + 2$

In Table I, the computational complexity of the proposed direct and polyphase approaches for both decimated and interpolated DTC structures are calculated. The complexity is expressed as the number of real multiplications at the symbol rate. For the DCT and DST transformations, required for the mappings and for the decomposition of the matrix $\mathcal{U}^k$, the split-radix FFT algorithm is used. This comparison shows that by using the polyphase approach, the transformations at the higher data rate can be replaced by multiple transformations at the lower rate, which reduces the complexity considerably.

V. SIMULATION RESULTS

In this section, we use simulation experiments to evaluate the convergence behaviour of the proposed multirate dual transform domain echo canceller and the existing echo cancellers. In the simulations, an ADSL system over the carrier serving area (CSA) loop $\#4$ is used. DMT modulation is employed where, tones 7-31 and 33-255 are allocated for the upstream and downstream, respectively; where each tone transmits a 4-QAM signal constellation. The downstream and upstream signal are transmitted at -40 dBm/Hz, and the external additive noise is white Gaussian noise at -140 dBm/Hz. The transmit block length in the upstream and downstream is 64 and 512, respectively and the corresponding cyclic prefix length is 5 and 40, respectively, *i.e.*, $N = 64$ and $\kappa = 8$. The true echo channel transfer function contains 512 samples at 2.2 MHz, including the effect of the hybrid and the transmitter and receiver filters. The echo channel is estimated by an FIR filter having 300 taps, and the weights are initiated with all zero values. In all the simulations, the step sizes are chosen in a way that all of the compared algorithms have a similar slope in the first part of the curve, *i.e.*, having same initial rate of convergence.

In Fig. 3, the echo cancellation methods are compared at the RT transceiver, without the transmission of dummy data on the unused tones as required in [3]. The CES canceller has the worst performance because of the lack of excitation on the unused tones. The DTC method achieves the same performance as the normalized LMS (NLMS) method, but with much reduced complexity [2]. Similar results are achieved at the CO transceiver as shown in Fig. 4.

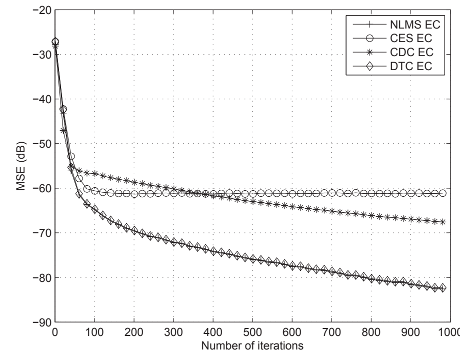

Fig. 3. Convergence behaviour of various echo cancellers at the RT transceiver

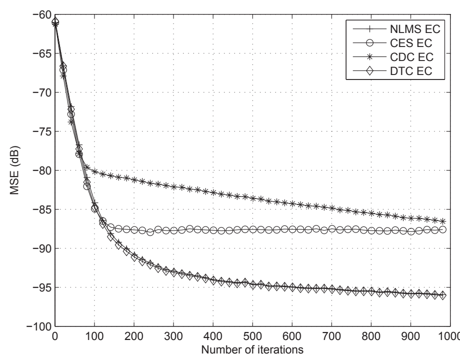

Fig. 4. Convergence behaviour of various echo cancellers at the CO transceiver

VI. CONCLUSION

In this paper, we have expanded the dual transform domain canceller proposed for symmetric rates in [1], [2] for practical multirate DSL systems. Two approaches are discussed: direct approach and the polyphase approach. It is shown that the cancellers using the former approach have lower computational complexity, since the transformations at the higher data rate are replaced by multiple lower data rate transformations. In addition, the proposed method have the same convergence behaviour as the LMS algorithm.

REFERENCES

[1] N. Ehtiati and B. Champagne, "Dual transform domain echo canceller for discrete multitone systems," in *IEEE Global Telecommunications Conf.*, Honolulu, HI, 2009.

[2] N. Ehtiati and B. Champagne, "Comparative evaluation of the dual transform domain echo canceller for DMT-based systems," in *IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, Dallas, TX, 2010.

[3] M. Ho, J. M. Cioffi, and J. A. C. Bingham, "Discrete multitone echo cancelation," *IEEE Trans. Commun.*, vol. 44, no. 7, pp. 817–825, July 1996.

[4] G. Ysebaert, F. Pisoni, M. Bonaventura, R. Hug, and M. Moonen, "Echo cancellation in DMT-receivers: Circulant decomposition canceler," *IEEE Trans. Signal Process.*, vol. 52, no. 9, pp. 2612–2624, Sept. 2004.

[5] F. Pisoni and M. Bonaventura, "A multi-carrier echo canceller based on symmetric decomposition," in *Proc. IEEE Workshop on Signal Process. Syst. Design and Implementation*, 2005, pp. 233–238.