A Joint Localization and Synchronization Technique Using Time of Arrival at Multiple Antenna Receivers

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***Abstract*—In this work, a system scheme is proposed for tracking a radio emitting target moving in two-dimensional space. The localization is based on the use of biased time-of-arrival (TOA) measurements obtained at two asynchronous receivers, each equipped with two closely spaced antennas. By exploiting the multi-antenna configuration and using all the TOA measurements up to current time step, the relative clock bias at each receiver and the target position are jointly estimated by solving a nonlinear least-squares (NLS) problem. To this end, a novel time recursive algorithm is proposed which fully takes advantage of the problem structure to achieve computational efficiency while using orthogonal transformations to ensure numerical reliability. The Cramer-Rao lower bound (CRLB) is also derived as a lower bound on the error in estimating the position and biases. Simulation results show that the mean-squared error (MSE) of the proposed method approaches the CRLB closely.**

***Keywords*—Clock bias estimation, localization, recursive nonlinear least squares, synchronization, time of arrival.**

I. Introduction

Tracking and positioning of mobile targets in a wireless network is of interest in many applications, including vehicle navigation, indoor positioning, etc. [1], [2]. The localization methods based on time-of-arrival (TOA) and time-difference-of-arrival (TDOA) measurements have received great attention, especially in ultra-wideband (UWB) impulse radio systems due to their good timing resolution [3].

In the traditional TOA-based tracking techniques, the target node (transmitter) and all the fixed anchors (receivers), have to be precisely synchronized to yield good estimation accuracy. In practice, it is often difficult and costly to maintain the desired level of synchronization between the target and the anchors. In this case, localization can still be achieved by either hyperbolic positioning using TDOA data [1], [2], or joint synchronization and positioning using TOA data [4]. However, the TDOA-based methods require one more anchor for localization as well a reliable line-of-sight (LOS) reference node which is not guaranteed all the times [2]; therefore, TOA-based methods are usually preferred in applications.

With the broad deployment of multiple antenna transceivers in emerging wireless networks, the need for a positioning system that exploits this structure is growing [5]. Unlike the above referenced methods which only employ a single antenna per anchor, several two-dimensional (2D) localization methods have been proposed that make use of anchors equipped with closely spaced antennas. For instance, in [6], a localization scheme is proposed based on the measurements of the TOAs and the TDOA at two neighboring antennas. The TDOA is used in turn to compute an angle-of-arrival (AOA) which is combined to the TOA to locate the target. However, this method requires accurate synchronization between the transmitter and the receiver; furthermore, it is sensitive to TDOA errors. The proposed scheme in [7] makes use of two anchors, each one equipped with two receiving antennas. The AOA at each anchor is estimated based on TDOA measurements and then the target position is finally determined. This method requires no synchronization between the target and the anchors; however, the considered approximations and the sensitivity to the noise in TDOA measurements deteriorate the position estimate.

In this work, by assuming a system configuration similar to that in [7], we propose a novel asynchronous localization method which uses all previous and current TOA measurements at each antenna to jointly estimate the clock biases and the current location. Knowledge of the clock bias removes the need for synchronization in the network layer and therefore reduces system costs and complexity. Furthermore, our method requires only two anchors, as compared to the standard triangulation and the hyperbolic positioning techniques which require three and four synchronized anchors, respectively [2]. At each time step, the joint parameter estimation is achieved by solving a nonlinear-least-squares (NLS) problem iteratively. For computational and memory efficiency, the proposed method is designed in a recursive manner by fully exploiting the structure of the NLS problem, whereas the orthogonal transformations are exploited to ensure numerical reliability. The Cramer-Rao-lower-bound (CRLB) on the parameter estimate covariance is also derived analytically. Through numerical simulations, by evaluating the mean-squared error (MSE) in positioning and clock bias estimation, it is shown that the mean squared error (MSE) of our proposed algorithm approaches the CRLB closely.

The organization of the paper is as follows: In Section II, the system set up and the problem statement are given. The proposed localization algorithm and the CRLB are derived in Section III and Section IV, respectively. The simulation of the proposed method and comparison with the CRLB are presented in Section V. Finally, Section VI concludes the paper.

II. System Description

The system under consideration consists of a mobile target node equipped with a radio transmitter and at least two fixed anchors, labeled RX_1 and RX_2, each having two closely spaced receiver antennas. The moving target, at time step k, is located at position $\mathbf{x}_k = [x_k, y_k]^T \in \mathbb{R}^2$. The known position of the j-th antenna of the i-th receiver is $\boldsymbol{p}_{ij} = [X_{ij}, Y_{ij}]^T \in$

$\mathbb{R}^2$ for $i, j = 1, 2$. The distance between the two antennas at each receiver is $a = \|\boldsymbol{p}_{11} - \boldsymbol{p}_{12}\| = \|\boldsymbol{p}_{21} - \boldsymbol{p}_{22}\|$, where $\|\cdot\|$ denotes the 2-norm operation. The distance between the target and the j-th antenna of the i-th receiver, at time step k, is denoted by $d_{ij}^{(k)} = \|\mathbf{x}_k - \boldsymbol{p}_{ij}\|$. The range difference between the two antennas of the i-th receiver is denoted by $\Delta d_i^{(k)} = d_{i1}^{(k)} - d_{i2}^{(k)}$, where $|\Delta d_i^{(k)}| \leq a$. The system configuration is described in Fig. 1 and it is practically implemented in [7].

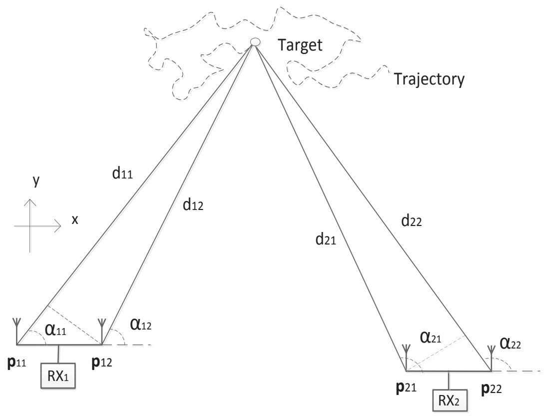


Fig. 1. The system scheme in 2D space.

At time step k, the range measurements at each receiver antenna can be obtained by multiplying the estimated TOA with the speed of wave propagation [6]. It this work, these measurements are modelled as

$$\boldsymbol{z}_k = \boldsymbol{f}(\boldsymbol{x}_k) + \boldsymbol{A}\boldsymbol{b} + \boldsymbol{n}_k, \tag{1}$$

or equivalently, in expanded vector notation,

$$\begin{bmatrix} z_{11}^{(k)} \\ z_{12}^{(k)} \\ z_{21}^{(k)} \\ z_{22}^{(k)} \end{bmatrix} = \begin{bmatrix} \|\boldsymbol{x}_k - \boldsymbol{p}_{11}\| \\ \|\boldsymbol{x}_k - \boldsymbol{p}_{12}\| \\ \|\boldsymbol{x}_k - \boldsymbol{p}_{21}\| \\ \|\boldsymbol{x}_k - \boldsymbol{p}_{22}\| \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} n_{11}^{(k)} \\ n_{12}^{(k)} \\ n_{21}^{(k)} \\ n_{22}^{(k)} \end{bmatrix}, \tag{2}$$

where $n_{ij}(k)$ is the measurement noise at the j-th antenna of the i-th receiver and b_i is the relative clock bias between the target node and the i-th anchor. For a given pair (i, j), the sequence of noise terms $n_{ij}^{(k)}$ are modeled as independent and identically distributed (iid) Gaussian random variables with zero-mean and variance σ_n^2, i.e., $n_{ij}(k) \sim \mathcal{N}(0, \sigma_n^2)$. For different (i, j) pairs, the noise terms can be considered uncorrelated with each other if the spacing between neighboring antennas of each anchor is reasonably large.

For the sake of simplicity we assume that the clock bias is constant for the localization period and the clock skew is calibrated and mitigated in the physical layer of UWB system as done in [8]. Neglecting the effect of clock skew for a short period of time has also been considered in [9]. The measurements data are stored and processed in a data fusion centre for localization. For the sake of generality, we have assumed that there is no global time reference for anchor nodes, thus the relative clock biases at each receiver are different. In this case there is no need for the anchors to communicate with each other through cables in order to be synchronized with a reference clock.

III. Localization Method

At time step k, to estimate $\boldsymbol{x}_k$ (and estimate $\boldsymbol{x}_l$ for $l = 1, 2, \ldots, k-1$ as well for post-processing), we use all the measurement equations in (1) up to and including step k and solve the following nonlinear-least-squares (NLS) problem

$$\min_{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k, \boldsymbol{b}} \left\| \begin{bmatrix} \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_k \end{bmatrix} - \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}_1) \\ \vdots \\ \boldsymbol{f}(\boldsymbol{x}_k) \end{bmatrix} - \begin{bmatrix} \boldsymbol{A} \\ \vdots \\ \boldsymbol{A} \end{bmatrix} \boldsymbol{b} \right\|_2. \tag{3}$$

The solution gives the maximum likelihood estimates of $\boldsymbol{x}_1$, ..., $\boldsymbol{x}_k$ and $\boldsymbol{b}$ at time step k. Instead of the above problem formulation, we could remove the clock bias by subtracting the TOA measurement equations of two neighboring antennas at each receiver in (2), and then estimate the position. However, due to the close distance between the neighboring antennas, this estimation method is ill-conditioned in practice. Moreover, as the clock biases are assumed to be constant, the measurement equations at different time steps are related. Therefore, without the exploitation of the previous measurements in estimating the current position, the information from the past is lost. Thus, the latter method would lead to an inaccurate and less robust estimate of $\boldsymbol{x}_k$ than that obtained by solving (3).

Solving the minimization problem (3) becomes computationally expensive as k grows, therefore, for computational efficiency, we develop a recursive method by using the idea given in [10].

A. Initial Estimate of the First Location Using AOA

At time step $k = 1$, we estimate the position using an AOA-based method. The AOA, at the first antenna of i-th receiver $i = 1, 2$, can be computed from the measured parameters as

$$\hat{\alpha}_{i1} = \arccos\left(\frac{a^2 - (\Delta z_i^{(1)})^2 + 2z_{i1}^{(1)}\Delta z_i^{(1)}}{2z_{i1}^{(1)}a}\right), \tag{4}$$

where the range differences $\Delta z_i^{(1)} = z_{i1}^{(1)} - z_{i2}^{(1)}$, which are related to the TDOA[1], can be stated as

$$\Delta z_i^{(1)} = \Delta d_i^{(1)} + \Delta n_i^{(1)}, \tag{5}$$

where $\Delta n_i^{(1)} = n_{i1}^{(1)} - n_{i2}^{(1)}$ and $\Delta n_i^{(1)} \sim \mathcal{N}(0, 2\sigma_n^2)$. The sensitivity of the angles to the errors in TOA (bias and noise) is small while the sensitivity to the TDOA errors is large [12].

Using the AOA at antenna locations $[X_{11}, Y_{11}]^T$ and $[X_{21}, Y_{21}]^T$, the first location $\boldsymbol{x}_1$, can be initially estimated by (see [1])

$$\tilde{\boldsymbol{x}}_1 = \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \end{bmatrix} = \boldsymbol{\Gamma}^{-1} \begin{bmatrix} Y_{11}\cos(\hat{\alpha}_{11}) - X_{11}\sin(\hat{\alpha}_{11}) \\ Y_{21}\cos(\hat{\alpha}_{21}) - X_{21}\sin(\hat{\alpha}_{21}) \end{bmatrix}, \tag{6}$$

where

$$\boldsymbol{\Gamma} = \begin{bmatrix} -\sin(\hat{\alpha}_{11}) & \cos(\hat{\alpha}_{11}) \\ -\sin(\hat{\alpha}_{21}) & \cos(\hat{\alpha}_{21}) \end{bmatrix}. \tag{7}$$

We do some more iterations based on the method explained below to improve this initial estimate.

[1]The effect of antenna delay error on TDOA measurement can be neglected for a target far enough compared to the antenna distance [11].

B. Recursive NLS Algorithm

To solve (3), suppose that at time step $k-1$, an estimate of $\boldsymbol{x}_l$, denoted by $\tilde{\boldsymbol{x}}_l$, has been obtained for $l = 1, \dots, k-1$. At time step k, we take $\tilde{\boldsymbol{x}}_k = \tilde{\boldsymbol{x}}_{k-1}$ as an initial estimate of $\boldsymbol{x}_k$ (if $k = 1$, the initial estimate of $\boldsymbol{x}_1$ has been given in (6)). By the Taylor series expansion,

$$\boldsymbol{f}(\boldsymbol{x}_l) \approx \boldsymbol{f}(\tilde{\boldsymbol{x}}_l) + \boldsymbol{J}(\tilde{\boldsymbol{x}}_l)(\boldsymbol{x}_l - \tilde{\boldsymbol{x}}_l), \tag{8}$$

where the Jacobian matrix

$$\boldsymbol{J}(\tilde{\boldsymbol{x}}_l) = \begin{bmatrix} (\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{11})^T / \|\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{11}\|_2 \\ (\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{12})^T / \|\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{12}\|_2 \\ (\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{21})^T / \|\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{21}\|_2 \\ (\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{22})^T / \|\tilde{\boldsymbol{x}}_l - \boldsymbol{p}_{22}\|_2 \end{bmatrix} \in \mathbb{R}^{4\times 2}. \tag{9}$$

Note that each row of the Jacobian matrix is a unit vector which points from an anchor's antenna to the target and it will change little if the position of the target does not change much. Using the right hand side of (8) to replace $\boldsymbol{f}(\boldsymbol{x}_l)$ in (3) we obtain the linear LS problem:

$$\min_{\boldsymbol{x}_1,\dots,\boldsymbol{x}_k,\boldsymbol{b}} \left\| \begin{bmatrix} \boldsymbol{z}_1 - \boldsymbol{f}(\tilde{\boldsymbol{x}}_1) \\ \vdots \\ \boldsymbol{z}_k - \boldsymbol{f}(\tilde{\boldsymbol{x}}_k) \end{bmatrix} - \begin{bmatrix} \boldsymbol{J}(\tilde{\boldsymbol{x}}_1)(\boldsymbol{x}_1 - \tilde{\boldsymbol{x}}_1) \\ \vdots \\ \boldsymbol{J}(\tilde{\boldsymbol{x}}_k)(\boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k) \end{bmatrix} - \begin{bmatrix} \boldsymbol{A} \\ \vdots \\ \boldsymbol{A} \end{bmatrix} \boldsymbol{b} \right\|, \tag{10}$$

Let the QR factorization of $\boldsymbol{J}(\tilde{\boldsymbol{x}}_l)$, be

$$\boldsymbol{Q}_l^T \boldsymbol{J}(\tilde{\boldsymbol{x}}_l) = \begin{bmatrix} \boldsymbol{R}_l \\ \boldsymbol{0} \end{bmatrix}, \tag{11}$$

where $\boldsymbol{Q}_l = [\underset{2}{\boldsymbol{U}_l}, \underset{2}{\boldsymbol{V}_l}] \in \mathbb{R}^{4\times 4}$ is orthogonal and $\boldsymbol{R}_l \in \mathbb{R}^{2\times 2}$ is upper triangular. The QR factorization in this work is achieved by the Householder transformation, which is efficient and numerically stable [13]. Furthermore,

$$\boldsymbol{U}_l^T \boldsymbol{A} \equiv \boldsymbol{F}_l, \quad \boldsymbol{V}_l^T \boldsymbol{A} \equiv \boldsymbol{G}_l, \tag{12}$$

$$\boldsymbol{U}_l^T (\boldsymbol{z}_l - \boldsymbol{f}(\tilde{\boldsymbol{x}}_l)) \equiv \boldsymbol{r}_l, \quad \boldsymbol{V}_l^T (\boldsymbol{z}_l - \boldsymbol{f}(\tilde{\boldsymbol{x}}_l)) \equiv \boldsymbol{s}_l. \tag{13}$$

Then transforming the linear LS problem (10) by these orthogonal matrices $\boldsymbol{Q}_l$ and reordering gives

$$\min_{\boldsymbol{x}_1,\dots,\boldsymbol{x}_k,\boldsymbol{b}} \left\| \begin{bmatrix} \boldsymbol{r}_1 \\ \vdots \\ \boldsymbol{r}_k \\ \boldsymbol{s}_1 \\ \vdots \\ \boldsymbol{s}_k \end{bmatrix} - \left[\begin{array}{ccc|c} \boldsymbol{R}_1 & & & \boldsymbol{F}_1 \\ & \ddots & & \vdots \\ & & \boldsymbol{R}_k & \boldsymbol{F}_k \\ \hline & & & \boldsymbol{G}_1 \\ & & & \vdots \\ & & & \boldsymbol{G}_k \end{array}\right] \begin{bmatrix} \boldsymbol{x}_1 - \tilde{\boldsymbol{x}}_1 \\ \vdots \\ \boldsymbol{x}_k - \tilde{\boldsymbol{x}}_k \\ \hline \boldsymbol{b} \end{bmatrix} \right\|. \tag{14}$$

To find a new estimate of $\boldsymbol{x}_l$ for $l = 1, \dots, k$ based on (14), we need to solve the following problem first:

$$\hat{\boldsymbol{b}} = \arg\min_{\mathbf{b}} \left\| \begin{bmatrix} \boldsymbol{s}_1 \\ \vdots \\ \boldsymbol{s}_k \end{bmatrix} - \begin{bmatrix} \boldsymbol{G}_1 \\ \vdots \\ \boldsymbol{G}_k \end{bmatrix} \boldsymbol{b} \right\|. \tag{15}$$

The computational complexity in solving the above LS problem grows linearly with k [13]. Thus, to make our algorithm efficient, we solve (15) in a recursive way. Suppose that by the end of time step $k-1$ we have done the following orthogonal transformations:

$$\boldsymbol{W}_{k-1}^T \begin{bmatrix} \boldsymbol{G}_1 \\ \vdots \\ \boldsymbol{G}_{k-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_{k-1} \\ \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{W}_{k-1}^T \begin{bmatrix} \boldsymbol{s}_1 \\ \vdots \\ \boldsymbol{s}_{k-1} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{s}}_{k-1} \\ \bar{\boldsymbol{s}}_{k-1} \end{bmatrix}, \tag{16}$$

where $\boldsymbol{W}_{k-1}$ is orthogonal, $\boldsymbol{T}_{k-1} \in \mathbb{R}^{2\times 2}$ is nonsingular upper triangular and $\hat{\boldsymbol{s}}_{k-1} \in \mathbb{R}^2$. Then at time step k, we compute the $\boldsymbol{R}_k$, $\boldsymbol{r}_k$, $\boldsymbol{s}_k$, $\boldsymbol{F}_k$, and $\boldsymbol{G}_k$ in (11)–(13). Then we compute the following orthogonal transformations as

$$\widetilde{\boldsymbol{W}}_k^T \begin{bmatrix} \boldsymbol{T}_{k-1} \\ \boldsymbol{G}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_k \\ \boldsymbol{0} \end{bmatrix}, \widetilde{\boldsymbol{W}}_k^T \begin{bmatrix} \hat{\boldsymbol{s}}_{k-1} \\ \boldsymbol{s}_k \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{s}}_k \\ \check{\boldsymbol{s}}_k \end{bmatrix}, \bar{\boldsymbol{s}}_k \equiv \begin{bmatrix} \bar{\boldsymbol{s}}_{k-1} \\ \check{\boldsymbol{s}}_k \end{bmatrix}, \tag{17}$$

where $\widetilde{\boldsymbol{W}}_k$ is orthogonal, $\boldsymbol{T}_k \in \mathbb{R}^{2\times 2}$ is non-singular upper triangular and $\hat{\boldsymbol{s}}_k \in \mathbb{R}^2$. Note that we only compute (17), while (16) is just stated for understanding (17) and it is never computed explicitly. Thus, from (17), the solution of $\hat{\boldsymbol{b}}$ to the LS problem (15) should satisfy the reduced 2×2 upper triangular linear system

$$\boldsymbol{T}_k \hat{\boldsymbol{b}} = \hat{\boldsymbol{s}}_k. \tag{18}$$

Then, from the top part of (14) we can obtain the estimate of position $\boldsymbol{x}_l$ based on the measurements upto time epoch k, i.e., $\boldsymbol{x}_{l|k}$, by solving the upper triangular linear systems

$$\boldsymbol{R}_l(\boldsymbol{x}_{l|k} - \tilde{\boldsymbol{x}}_l) = \boldsymbol{r}_l - \boldsymbol{F}_l \hat{\boldsymbol{b}}, \quad l = 1, \dots, k. \tag{19}$$

Since the application considered here is a real-time one (i.e., we are not interested in $\boldsymbol{x}_{l|k}$ for $l = 1, \dots k-1$) and the computation power and the memory may be limited, we would solve the upper triangular linear system in (19) only for $l = k$.

Since the original LS problem (3) is nonlinear, some iterations may be needed for more accurate estimates. The algorithm is summarized in Algorithm 1, in which ϵ is a given tolerance and $K_{\max}$ is an assigned maximum allowable number of iterations.

Algorithm 1 Recursive NLS localization

1: Compute $\tilde{\boldsymbol{x}}_1$ using the AOA-based method
2: **for** $k = 1, 2, \dots, N$ **do**
3:   **if** $k > 1$ **then**
4:     Take $\tilde{\boldsymbol{x}}_k = \tilde{\boldsymbol{x}}_{k-1}$
5:   **end if**
6:   Compute (11), (12) and (13) for $l = k$
7:   **if** $k = 1$ **then**
8:     Compute (16) with k replaced by 2.
9:   **else**
10:     Compute (17)
11:   **end if**
12:   Solve (18) to find $\hat{\boldsymbol{b}}$
13:   Solve (19) for $l = k$ to obtain $\boldsymbol{x}_{k|k}$
14:   **for** Iter $= 1, 2, \dots, K_{\max}$ **do**
15:     **if** $\|\tilde{\boldsymbol{x}}_k - \boldsymbol{x}_{k|k}\| \le \epsilon$ **then**
16:       Stop the iteration
17:     **end if**
18:     Repeat steps 6—13
19:   **end for**
20: **end for**

The computational analysis for each equation in our algorithm at each time step is given in Table 1. All the orthogonal transformations, including the QR factorizations are performed by the Householder transformation. Note that the total cost of the computations involved in (17), (18) (with $l = k$) at time step k is much less than that in computing (15).

At each time step, the maximum computational cost of our algorithm is the total number of flops and square roots (SQRT) operation shown in Table 1, multiplied by $K_{\max}$. As the nonlinearity in (8) is moderate, $K_{\max}$ can be chosen to be small, say 5.

TABLE I. COMPUTATIONAL COST OF THE NLS ALGORITHM

Equation	(9)	(11)	(12),(13)	(17),(18)	(19)	Total
Flops	24	34	88	56	14	216
SQRT	4	2	0	2	0	8

IV. CRAMER-RAO LOWER BOUND

The CRLB provides a lower bound on the achievable estimation error variance of all unbiased estimators. Herein, we derive the CRLB in estimating the parameter vector $\boldsymbol{\theta}^{(k)} = [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \cdots, \boldsymbol{x}_k^T, \boldsymbol{b}^T]$, based on the measurements $\boldsymbol{z}_l$ for $l = 1, \ldots, k$. Given the clock biases and the true positions of the target, the joint probability density function of the range measurements $z_{ij}^{(l)}$ is

$$\varphi^{(k)}(z_{ij}^{(l)}, l = 1 \ldots, k; i, j = 1, 2 \,|\, \boldsymbol{x}_1, \cdots, \boldsymbol{x}_k, \boldsymbol{b}) = \frac{1}{\sqrt{(2\pi\sigma_n^2)^{4k}}} e^{-\left(\sum_{l=1}^{k}\sum_{i,j=1}^{2} \frac{(z_{ij}^{(l)} - \|\boldsymbol{x}_l - \boldsymbol{p}_{ij}\| - b_i)^2}{2\sigma_n^2}\right)}, \tag{20}$$

where the above is justified due to the independence of the measurements. According to [14], the (i, j) element of the Fisher-information-matrix (FIM) $\boldsymbol{I}^{(k)} \in \mathbb{R}^{(2k+2)\times(2k+2)}$ at time step k is

$$I_{ij}^{(k)} = -\mathrm{E}\Big[\frac{\partial^2 \ln \varphi^{(k)}}{\partial\theta_i \partial\theta_j}\Big], \tag{21}$$

where θ_i is the i-th scalar element of the parameter vector $\boldsymbol{\theta}^{(k)}$. It is easy to show that $\boldsymbol{I}^{(k)}$ has the following form

$$\boldsymbol{I}^{(k)} = \begin{bmatrix} \boldsymbol{I}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{I}_{1,b} \\ \boldsymbol{0} & \boldsymbol{I}_2 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0} & \boldsymbol{I}_{k-1,b} \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{I}_k & \boldsymbol{I}_{k,b} \\ \boldsymbol{I}_{b,1} & \cdots & \boldsymbol{I}_{b,k-1} & \boldsymbol{I}_{b,k} & \boldsymbol{B}_k \end{bmatrix}, \tag{22}$$

where the sub-matrices are

$$\boldsymbol{B}_k = \mathrm{diag}\left(\sum_{l=1}^{k}\frac{2}{\sigma_n^2}, \sum_{l=1}^{k}\frac{2}{\sigma_n^2}\right),$$

$$\boldsymbol{I}_l = \frac{1}{\sigma_n^2}\begin{bmatrix} \sum_{i,j=1}^{2} \frac{(x_l - X_{ij})^2}{\|\boldsymbol{x}_l - \boldsymbol{p}_{ij}\|^2} & \sum_{i,j=1}^{2} \frac{(x_l - X_{ij})(y_l - Y_{ij})}{\|\boldsymbol{x}_l - \boldsymbol{p}_{ij}\|^2} \\ \sum_{i,j=1}^{2} \frac{(x_l - X_{ij})(y_l - Y_{ij})}{\|\boldsymbol{x}_l - \boldsymbol{p}_{ij}\|^2} & \sum_{i,j=1}^{2} \frac{(y_l - Y_{ij})^2}{\|\boldsymbol{x}_l - \boldsymbol{p}_{ij}\|^2} \end{bmatrix},$$

$$\boldsymbol{I}_{l,b} = \frac{1}{\sigma_n^2}\begin{bmatrix} \sum_{j=1}^{2} \frac{(x_l - X_{1j})}{\|\boldsymbol{x}_l - \boldsymbol{p}_{1j}\|} & \sum_{j=1}^{2} \frac{(x_l - X_{2j})}{\|\boldsymbol{x}_l - \boldsymbol{p}_{2j}\|} \\ \sum_{j=1}^{2} \frac{(y_l - Y_{1j})}{\|\boldsymbol{x}_l - \boldsymbol{p}_{1j}\|} & \sum_{j=1}^{2} \frac{(y_l - Y_{2j})}{\|\boldsymbol{x}_l - \boldsymbol{p}_{2j}\|} \end{bmatrix}. \tag{23}$$

The minimum covariance matrix in determining the unknown vector parameter $\boldsymbol{\theta}^{(k)}$ is given by $(\boldsymbol{I}^{(k)})^{-1}$ [14]. The mean-squared-error (MSE) of any unbiased estimator in determining the position at k-th time step is limited by the CRLB in estimating $\boldsymbol{x}_k$ where

$$\mathrm{CRLB}(\boldsymbol{x}_k) = (\boldsymbol{I}^{(k)})^{-1}_{2k-1,2k-1} + (\boldsymbol{I}^{(k)})^{-1}_{2k,2k}, \tag{24}$$

where $(\boldsymbol{I}^{(k)})^{-1}_{m,m}$ is the (m, m) element of $(\boldsymbol{I}^{(k)})^{-1}$. The MSE of any unbiased estimator in estimating the i-th receiver clock bias at time step k is higher than the CRLB in estimating b_i where

$$\mathrm{CRLB}(b_i) = (\boldsymbol{I}^{(k)})^{-1}_{2k+i,2k+i}, \quad i = 1, 2. \tag{25}$$

V. NUMERICAL RESULTS

In the simulation set-up we consider a square 2D area of 200×200 units and the system scheme described in Fig. 1. We assume that all the units in the plane, the ranges, biases, and the standard deviation of noises are in meters. The four antennas are located at $\boldsymbol{p}_{11} = [-51, -100]^T$, $\boldsymbol{p}_{12} = [-49, -100]^T$, $\boldsymbol{p}_{21} = [49, -100]^T$, and $\boldsymbol{p}_{22} = [51, -100]^T$, respectively. The initial point of the target is set to be $[0, 50]^T$ and the target trajectory is according to a random walk model, with iid zero-mean-Gaussian increments with standard deviation equal to 0.5m/s (see in [1, p122]). The number of samples is N=3000 and the sample interval is set to be $0.5s$ so that the consecutive range measurements are not mixed and there would be enough time for iterations. In order to simulate data in (1) for each time sample, the exact range between the target and each antenna is computed and then disturbed with the biases $b_1 = 5$ and $b_2 = -5$ and n_{ij} with standard deviation $\sigma_n = 10^{-2}$.

After generating the data, the estimation process starts as described in Section III. At each time step, the recursive NLS localization algorithm described in Section III-B is performed iteratively with $\epsilon = 0.05$ and $K_{\max} = 5$. For the same trajectory of the mobile node, our algorithm is simulated for M Monte-Carlo (MC) trials, where $M = 5000$. The root-mean-square error (RMSE) for the position estimate at time step k is computed as

$$e_k^x = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\|\boldsymbol{x}_{k|k}(m) - \boldsymbol{x}_k\|^2}, \tag{26}$$

and shown in a logarithm scale in Fig. 2, where $\boldsymbol{x}_{k|k}(m)$ is the estimate of $\boldsymbol{x}_k$ in the m-th MC run. The minimum achievable standard deviation of the error $\sigma_{min} = \sqrt{\text{CRLB}(\boldsymbol{x}_k)}$ in location estimation, is computed for the considered trajectory and plotted in Fig. 2. Note that the RMSE of our method is close to the bound because in theory our position estimate at each time step is the maximum likelihood estimate. The performance of our method in estimating the bias b_i is also evaluated by computing the RMSE after M runs

$$e_k^{(i)} = \sqrt{\frac{1}{M}\sum_{m=1}^{M} \|\hat{b}_{k,i}(m) - b_i\|^2}, \quad (27)$$

where $\hat{b}_{k,i}(m)$ is the estimate of the relative bias at *i*-th anchor for *m*-th MC run. The comparison of the computed RMSE for b_1 and b_2 with the corresponding lower bound, $\sigma_{min} = \sqrt{\text{CRLB}(b_i)}$, is given in Fig. 3. Again we observe that RMSE of each clock bias is very close to the corresponding lower bound.

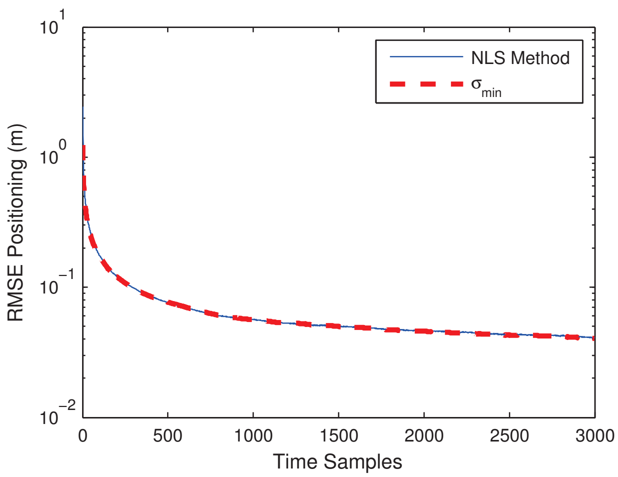
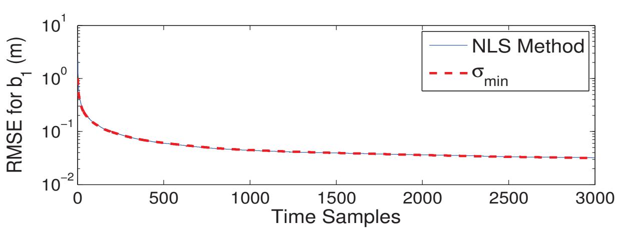
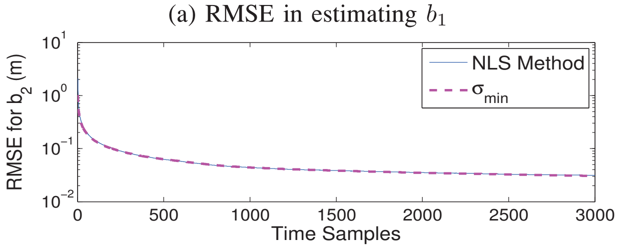


Fig. 2. The RMSE of our method and the comparison with the CRLB.



(a) RMSE in estimating b_1



(b) RMSE in estimating b_2

Fig. 3. RMSE and CRLB in estimating the biases.

VI. Conclusion and Future Work

The proposed recursive NLS method in this paper requires no anchor synchronization, unlike the conventional triangulation and hyperbolic methods. It was illustrated that the computational cost of our algorithm at each time step was low. The simulation of our method indicated that the RMSE of the position estimates was very close to the square root of the CRLB. For off-line tracking scenarios, we can easily modify our algorithm to make the position estimates in early time steps as accurate as the position estimates in late time steps by updating all the previous position estimates. In the future, we would extend this work to a joint localization and synchronization framework with unknown clock skew parameters.

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