Low-Complexity Variable Forgetting Factor Constant Modulus RLS-based Algorithm for Blind Adaptive Beamforming

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***Abstract*—In this paper, we propose a novel low-complexity variable forgetting factor (VFF) mechanism to enhance the performance of recursive least squares (RLS) algorithms for adaptive blind beamforming. The beamformer is designed according to the constrained constant modulus (CCM) criterion, and the proposed algorithm operates in the generalized sidelobe canceler (GSC) structure for implementation. The proposed variable forgetting factor mechanism employs a new component updated by the time average of the constant modulus (CM) cost function, to adjust the forgetting factor. A complexity comparison is provided to show its advantages over existing methods. The study of the steady-state analysis is carried out. Simulation results which are presented for a nonstationary scenario illustrate that the proposed variable forgetting factor mechanism achieves a superior performance compared to existing algorithms.** [1]

***Index Terms*—Adaptive blind beamforming, constrained constant modulus, variable forgetting factor.**

I. Introduction

Adaptive beamforming is a promising and widely investigated technology to improve the reception of a desired signal while suppressing interference at the output of a sensor array [1], [2]. It is a common task in array signal processing with applications in wireless communications, radar, sonar, and wireless sensor networks. A significant issue that is considered in adaptive beamforming is the design criterion. The constrained minimum variance (CMV) [3] based algorithms are designed in such a way that they aim to minimize the beamformer output power while subject to the constraint that the array response should be unity in the direction of desired signal. And the constrained constant modulus (CCM) criterion is a positive measure [4] of the deviation of the beamformer output from a constant modulus condition subject to a constraint on the array response of the desired signal [5]. Compared with the CMV criterion, CCM exploits a constant modulus property of the transmitted signals, utilizes the deviation to provide more information for the parameter estimation of the constant modulus constellations, and achieves a superior performance in many applications (e.g., Code Division Multiple Access (CDMA) multiuser detection, radar, beamforming and reduced-rank technique).

Numerous adaptive filtering algorithms have been developed to implement the beamformer [6], [7]. Among existing algorithms, the RLS algorithm is considered as one of the fastest and most effective methods for adaptive implementation [8]. Xu and Tsatsanis [9] developed the CMV beamformer with the RLS adaptation. It turns out that this method exhibits an improved performance and enjoys a fast convergence rate. Some works in [10], [11] have proposed the RLS algorithms based on the CCM criterion for different applications. They have shown that the CCM-RLS algorithm outperforms the CMV-RLS algorithm.

The RLS algorithm has fast convergence and can achieve a better performance in many scenarios. However in reality, users often enter and exit the system, it is difficult or even impractical to compute a predetermined value for the forgetting factor [12]. The variable forgetting factor (VFF) technique is becoming a wise choice to overcome the drawback of the fixed forgetting factor scheme. The most common method is the gradient-based variable forgetting factor (GVFF) scheme proposed in [13], where the GVFF mechanism is designed based on the MSE criterion.

In this work, we propose a novel low-complexity VFF mechanism based on the CCM-RLS algorithm using the generalized sidelobe canceler (GSC) structure for blind beamforming. The proposed VFF mechanism employs a new component updated by the time average of the CM cost function to automatically adjust the forgetting factor. We refer to the proposed variable forgetting factor mechanism as time-averaged variable forgetting factor (TAVFF). A study of the steady-state analysis is carried out, and analytical expressions to predict the MSE are obtained. Simulation results are presented for nonstationary scenarios, showing that the proposed mechanism achieves a better convergence performance compared to the existing adaptive algorithms.

II. System Model and Design of GSC Beamformer

Let us suppose that K narrowband sources impinge on a unique linear array (ULA) of M omnidirectional sensors. The sources are assumed to be in the far field with directions of arrival (DOA) θ_k for $k = 0, 1, ..., K-1$. The ith snapshot's vector of sensor array outputs can be modeled as an M-by-1 vector

$$\mathbf{r}(i) = \mathbf{A}(\theta)\mathbf{b}(i) + \mathbf{n}(i), \quad i = 1, 2, ..., N, \tag{1}$$

where $\mathbf{A}(\theta) = [\mathbf{a}(\theta_0), ..., \mathbf{a}(\theta_{K-1})]$ is the M-by-K matrix of signal steering vectors, the M-by-1 signal steering vector for a signal impinging at angle θ_k, where $k = 0, 1, ..., K-1$ is defined as

$$\mathbf{a}(\theta_k) = \left[1, e^{-2\pi j \frac{d_s}{\lambda_c}\cos\theta_k}, ..., e^{-2\pi j (M-1)\frac{d_s}{\lambda_c}\cos\theta_k}\right]^T, \tag{2}$$

where $d_s = \lambda_c/2$ is the inter-element distance of the ULA, λ_c is the wavelength and $(\cdot)^T$ denotes the transpose operation. $\mathbf{b}(i) = [b_0(i), b_1(i), ..., b_{K-1}(i)]$ is the source data, the entries of which are uncorrelated with each other, and $\mathbf{n}(i)$ is the white sensor noise, which is assumed to be a zero-mean spatially and temporally white Gaussian process.

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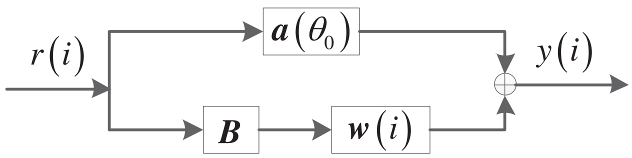

Fig. 1: Beamformer design for the GSC structure

As shown in Fig. 1, the GSC structure converts the constrained optimization problem into an unconstrained one and adopts an alternative way to design the beamformer [13]. The output of the GSC structure filter is given by

$$y(i) = (\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H \mathbf{r}(i), \tag{3}$$

where $\mathbf{w}(i) = [w_0(i), ..., w_{M-2}(i)]^T$ is the (M-1)-by-1 filter, and the matrix $\mathbf{B}$ denotes the signal blocking matrix which is orthogonal to the steering vector $\mathbf{a}(\theta_0)$, namely, $\mathbf{B}\mathbf{a}(\theta_0) = \mathbf{0}_{(M-1)\times 1}$.

The signal blocking matrix $\mathbf{B}$ can be obtained by the singular value decomposition or the QR decomposition algorithm [14]. The calculation of the weight vector based on the optimization of the CM cost function is as follows

$$\underset{\mathbf{w}(i)}{min} \quad J_{CM}(\mathbf{w}(i)) = \mathbb{E}\{[|(\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H \mathbf{r}(i)|^2 - 1]^2\}. \tag{4}$$

The objective of (4) is to minimize the array output power while maintaining the contribution from θ_0 constant. Note that $(\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H \mathbf{a}(\theta_0) = 1$.

The CCM-GSC receive filter expression that iteratively solves the problem in (4) is given by

$$\mathbf{w}(i+1) = \bar{\mathbf{Q}}^{-1}(i)\bar{\mathbf{P}}(i), \tag{5}$$

where

$$\bar{\mathbf{Q}}(i) = \mathbb{E}\left[|y(i)|^2 \mathbf{B}^H \mathbf{r}(i)\mathbf{r}^H(i)\mathbf{B}\right],$$
$$\bar{\mathbf{P}}(i) = \mathbb{E}\left[\left(\mathbf{a}^H(\theta_0) y^*(i)\mathbf{r}(i) - 1\right)^* y^*(i)\mathbf{B}^H \mathbf{r}(i)\right],$$
$$y(i) = (\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H \mathbf{r}(i).$$

III. BLIND ADAPTIVE CCM-RLS-GSC ALGORITHM AND PROBLEM STATEMENT

In this section, we describe the blind adaptive CCM-RLS-GSC algorithm for estimating the parameters of the beamformer first, and then we generalize the blind GVFF scheme for the adaptive CCM-RLS-GSC filter.

A. Adaptive CCM-RLS-GSC Beamforming

For the GSC structure depicted in Fig. 1, by employing the time-averaged estimation, we have the following cost function

$$\begin{aligned} J_{CM}(i) &= \sum_{n=1}^{i} \lambda^{i-n} \left(\left|\tilde{\mathbf{w}}^H(i)\mathbf{r}(n)\right|^2 - 1\right)^2 \\ &= \sum_{n=1}^{i} \lambda^{i-n} \left(\left|(\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H \mathbf{r}(n)\right|^2 - 1\right)^2, \end{aligned} \tag{6}$$

where λ is a forgetting factor which should be chosen as a positive constant close to 1, but less than 1, and $\tilde{\mathbf{w}}(i) = \mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i)$. By taking the gradient of (6) with respect to $\mathbf{w}^*(i)$ and equating it to zero, after further mathematical manipulations we have

$$\begin{aligned} \frac{\partial J_{CM}(i)}{\partial \mathbf{w}^*} &= \sum_{n=1}^{i} \lambda^{i-n} (\mathbf{B}^H y^*(n)\mathbf{r}(n)\mathbf{r}^H(n) y(n)\mathbf{B}\mathbf{w}(i) \\ &\quad - \mathbf{B}^H y^*(n)\mathbf{r}(n)(\mathbf{r}^H(n) y(n)\mathbf{a}(\theta_0) - 1)) = 0. \end{aligned} \tag{7}$$

Letting $\tilde{\mathbf{r}}(n) = y^*(n)\mathbf{r}(n)$, $\mathbf{x}(n) = \mathbf{B}^H \tilde{\mathbf{r}}(n)$ and $d(n) = \mathbf{a}^H(\theta_0)\tilde{\mathbf{r}}(n) - 1$, we rewrite (7) as

$$\sum_{n=1}^{i} \lambda^{i-n} \left(\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(i) - \mathbf{x}(n) d^*(n)\right) = 0. \tag{8}$$

For the sake of concise presentation, we define $\mathbf{Q}(i) = \sum_{n=1}^{i} \lambda^{i-n}\mathbf{x}(n)\mathbf{x}^H(n)$, $\mathbf{P}(i) = \sum_{n=1}^{i} \lambda^{i-n}\mathbf{x}(n) d^*(n)$, thus we get

$$\mathbf{w}(i) = \mathbf{Q}^{-1}(i)\mathbf{P}(i), \tag{9}$$

$$\mathbf{Q}(i) = \lambda\mathbf{Q}(i-1) + \mathbf{x}(i)\mathbf{x}^H(i), \tag{10}$$

$$\mathbf{P}(i) = \lambda\mathbf{P}(i-1) + \mathbf{x}(i) d^*(i). \tag{11}$$

To avoid the matrix inversion and reduce the complexity, we employ the matrix inversion lemma [13] to update $\mathbf{Q}^{-1}(i)$ iteratively. According to the definition of $\mathbf{Q}(i)$, we can get

$$\mathbf{Q}^{-1}(i) = \lambda^{-1}\mathbf{Q}^{-1}(i-1) - \frac{\lambda^{-1}\mathbf{Q}^{-1}(i-1)\mathbf{x}\mathbf{x}^H \lambda^{-1}\mathbf{Q}^{-1}(i-1)}{1 + \lambda^{-1}\mathbf{x}^H \mathbf{Q}^{-1}(i-1)\mathbf{x}}. \tag{12}$$

Substituting (11), (12) into (9), finally we obtain

$$\mathbf{w}(i) = \mathbf{w}(i-1) + \mathbf{k}(i) e^*(i), \tag{13}$$

where

$$\mathbf{k}(i) = \frac{\mathbf{Q}^{-1}(i-1)\mathbf{x}(i)}{\lambda + \mathbf{x}^H(i)\mathbf{Q}^{-1}(i-1)\mathbf{x}(i)}, \tag{14}$$

$$e(i) = d(i) - (\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i-1))^H \mathbf{x}(i). \tag{15}$$

By using (9) - (15) with the initial values, we can implement the adaptive CCM-RLS-GSC beamforming algorithm.

B. Blind GVFF Scheme in Beamformer Design For GSC Structure

By following the gradient-based VFF mechanism with the MSE criterion [13], we can derive the blind GVFF mechanism with the GSC structure which is given as follows.

$$\begin{aligned} \lambda(i+1) = \Bigg[\lambda(i) \\ - u\frac{\partial\left(\left|(\mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i))^H \mathbf{r}(i)\right|^2 - 1\right)^2}{\partial\lambda}\Bigg]_{\lambda^-}^{\lambda^+}, \end{aligned} \tag{16}$$

where

$$\frac{\partial\left(\left|\left(\mathbf{a}\left(\theta_0\right)-\mathbf{B}\mathbf{w}\left(i\right)\right)^H\mathbf{r}\left(i\right)\right|^2-1\right)^2}{\partial\lambda}$$
$$=4\left(\left|\left(\mathbf{a}\left(\theta_0\right)-\mathbf{B}\mathbf{w}\left(i\right)\right)^H\mathbf{r}\left(i\right)\right|^2-1\right) \quad (17)$$
$$\times\left|\left(\mathbf{a}\left(\theta_0\right)-\mathbf{B}\mathbf{w}\left(i\right)\right)^H\mathbf{r}\left(i\right)\right|\mathbf{B}\frac{\partial\mathbf{w}^H\left(i\right)}{\partial\lambda}\mathbf{r}\left(i\right),$$

and $[.]_{\lambda^-}^{\lambda^+}$ denotes the truncation to the limits of the range $[\lambda^-,\lambda^+]$, u denotes a small, positive step-size. Letting $\mathbf{Y}\left(i\right)=\frac{\partial\mathbf{w}^H(i)}{\partial\lambda}$, and recalling that $\mathbf{w}\left(i\right)=\mathbf{w}\left(i-1\right)+\mathbf{k}\left(i\right)e^*\left(i\right)$, we can obtain $\mathbf{Y}\left(i\right)$ through taking the gradient of $\mathbf{k}\left(i\right)$ with respect to $\lambda\left(i\right)$. Actually, we generate two new quantities $\frac{\partial\mathbf{k}(i)}{\partial\lambda}$ and $\frac{\partial\mathbf{Q}^{-1}(i)}{\partial\lambda}$, which can be solved by doing the differentiate operation. The CCM-RLS-GSC beamformer with the GVFF scheme is implemented by using (16), (17) and the updated equations of $\mathbf{Y}\left(i\right)$, $\frac{\partial\mathbf{k}(i)}{\partial\lambda}$, $\frac{\partial\mathbf{Q}^{-1}(i)}{\partial\lambda}$ with initial values.

IV. Proposed TAVFF Scheme

In this section, we first describe the proposed low-complexity time averaged variable forgetting factor scheme that adjusts the forgetting factor for the adaptive CCM-RLS-GSC algorithm. Based on this scheme, the computational complexity analysis for the proposed TAVFF scheme is presented.

A. Blind TAVFF Mechanism

Motivated by the variable step-size mechanism for least mean square (LMS) algorithm in [15], we denote a new component $\phi\left(i\right)$ which is updated by the instantaneous CM cost function. It can be expressed by the following time-averaged expression

$$\phi\left(i\right)=\alpha\phi\left(i-1\right)+\beta(|\left(\mathbf{a}\left(\theta_0\right)-\mathbf{B}\mathbf{w}\left(i\right)\right)^H\mathbf{r}\left(i\right)|^2-1)^2, \quad (18)$$

where $0<\alpha<1$ and $\beta>0$. Generally, α is close to 1, and β is set equal to a small value. The updated component $\phi\left(i\right)$ is a small value, and it changes rapidly as the instantaneous value of the cost function.

The proposed low-complexity VFF scheme is given by

$$\lambda\left(i\right)=\left[\frac{1}{1+\phi\left(i\right)}\right]_{\lambda^-}^{\lambda^+} \quad (19)$$

where the value of variable forgetting factor $\lambda\left(i\right)$ is close to 1. It is worth to point out that other rules have been experimented and the TAVFF scheme is a result of several attempts to devise a simple and yet effective mechanism.

Let us derive the steady-state first order and second order statistical properties for the variable forgetting factor. Using (19) and with the aid of Taylor's formula, when $i\to\infty$ we can write

$$\mathbb{E}[\lambda\left(\infty\right)]\approx 1-\mathbb{E}\left[\phi\left(\infty\right)\right]+\mathbb{E}\left[\phi^2\left(\infty\right)\right]+o\left(\mathbb{E}\left[\phi^3\left(\infty\right)\right]\right) \quad (20)$$

and

$$\mathbb{E}[\lambda^2\left(\infty\right)]\approx 1-2\mathbb{E}\left[\phi\left(\infty\right)\right]+3\mathbb{E}\left[\phi^2\left(\infty\right)\right]+o\left(\mathbb{E}\left[\phi^3\left(\infty\right)\right]\right). \quad (21)$$

Since $0<\alpha<1$, from (18) we can see $\lim_{i\to\infty}\mathbb{E}[(|\tilde{\mathbf{w}}\left(i\right)\mathbf{r}\left(i\right)|^2-1)^2]=J_{\min}+J_{ex}\left(\infty\right)$ [16], where $J_{\min}$ denotes the CCM minima and $J_{ex}\left(\infty\right)$ denotes the steady-state excess error of the CM cost function, $J_{\min}\gg J_{ex}\left(\infty\right)$ [17]. Subsequently, we obtain

$$\mathbb{E}[\phi\left(i\right)]=\frac{\beta(J_{\min}+J_{ex}(\infty))}{1-\alpha}\approx\frac{\beta J_{\min}}{1-\alpha}. \quad (22)$$

Using (18), by computing the square of $\phi\left(i\right)$ we obtain $\phi^2\left(i\right)=\alpha^2\phi^2\left(i-1\right)+2\alpha\beta\phi\left(i-1\right)(|(\mathbf{a}(\theta_0)-\mathbf{B}\mathbf{w}\left(i\right))^H\mathbf{r}\left(i\right)|^2-1)^2+\beta^2(|(\mathbf{a}(\theta_0)-\mathbf{B}\mathbf{w}\left(i\right))^H\mathbf{r}\left(i\right)|^2-1)^4$. Since β^2 is quite small, $0<\alpha^2<1$, and $\phi\left(i-1\right)$ and $(\mathbf{a}(\theta_0)-\mathbf{B}\mathbf{w}(i))^H r(i)|^2-1)^2$ are uncorrelated, by taking the expectation we know that $\mathbb{E}[\phi^2\left(i\right)]$ converges and when $i\to\infty$, we obtain

$$\mathbb{E}\left[\phi^2\left(\infty\right)\right]\approx\frac{2\alpha\beta\mathbb{E}\left[\phi\left(\infty\right)\right]J_{\min}}{1-\alpha^2}. \quad (23)$$

By substituting (22) and (23) into (20) and (21), respectively, we have the steady-state statistical properties for the variable forgetting factor:

$$\mathbb{E}\left[\lambda\left(\infty\right)\right]\approx 1-\frac{(1-\alpha^2)\beta J_{\min}-2\alpha\beta^2 J_{\min}^2}{(1-\alpha)(1-\alpha^2)}, \quad (24)$$

$$\mathbb{E}\left[\lambda^2\left(\infty\right)\right]\approx 1-\frac{2(1-\alpha^2)\beta J_{\min}-6\alpha\beta^2 J_{\min}^2}{(1-\alpha)(1-\alpha^2)}. \quad (25)$$

B. Computational Complexity

We study the additional computational complexity of the proposed TAVFF mechanism and GVFF mechanism which is listed in Table I. We compute the number of additions and multiplications to compare the different parts of those two VFF mechansims. Compared to the GVFF mechanism, the proposed TAVFF mechanism reduces the computational complexity significantly. It requires only a few fixed number of operations while the GVFF mechanism has an additional complexity proportional to the number of senors M.

TABLE I: Additional Computational Complexity

	Number of operations per symbol	
mechanism	multiplications	additions
TAVFF	5	3
blind GVFF	$12M^2-12M+3$	$5M^2-8M+5$

V. Steady-State Analysis

A. Convergence of the Mean Weight Vector

In this part, we make some approximations and derive several expressions to show the convergence of the mean weight vector for the CCM-RLS-GSC beamformer with the proposed TAVFF mechanism.

Let us start from the weight vector calculation equation which is given by (13), by multiplying both sides of (13) with $\mathbf{Q}\left(i\right)$, we obtain

$$\mathbf{Q}\left(i\right)\mathbf{w}\left(i\right)=\mathbf{Q}\left(i\right)\mathbf{w}\left(i-1\right)+\mathbf{Q}\left(i\right)\mathbf{k}\left(i\right)e^*\left(i\right). \quad (26)$$

Recalling that

$$\mathbf{Q}\left(i\right)=\lambda\left(i\right)\mathbf{Q}\left(i-1\right)+\mathbf{x}\left(i\right)\mathbf{x}^H\left(i\right), \quad (27)$$

$$\mathbf{Q}(i)\,\mathbf{k}(i) = \mathbf{x}(i), \tag{28}$$

$$e(i) = d(i) - \mathbf{w}^H(i-1)\,\mathbf{x}(i). \tag{29}$$

Substituting (27), (28) and (29) into (26), we have

$$\mathbf{Q}(i)\,\mathbf{w}(i) = \lambda(i)\,\mathbf{Q}(i-1)\,\mathbf{w}(i-1) + \mathbf{x}(i)\,d^*(i). \tag{30}$$

Then, we multiply (27) with the optimal receiver $\mathbf{w}_0$ and subtract the resulting equation from (30), which yields

$$\begin{aligned}\mathbf{Q}(i)\left(\mathbf{w}(i) - \mathbf{w}_0\right) =& \lambda(i)\,\mathbf{Q}(i-1)\left(\mathbf{w}(i-1) - \mathbf{w}_0\right)\\ &+ \mathbf{x}(i)\left(d(i) - \mathbf{x}^H(i)\,\mathbf{w}_0\right).\end{aligned} \tag{31}$$

By denoting the weight error vector $\boldsymbol{\varepsilon}(i) = \mathbf{w}(i) - \mathbf{w}_0$ and the optimum error $e_0(i) = d(i) - \mathbf{w}_{_0}^H(i)\,\mathbf{x}(i)$, then we have

$$\mathbf{Q}(i)\,\boldsymbol{\varepsilon}(i) = \lambda(i)\,\mathbf{Q}(i-1)\,\boldsymbol{\varepsilon}(i-1) + \mathbf{x}(i)\,e_0^*(i). \tag{32}$$

By taking the expectation and due to the fact that, when $i \to \infty$, we have $\mathbb{E}\left[\mathbf{x}(i)\,e_0^*(i)\right] = \mathbf{0}$, we can arrive the following expression

$$\mathbb{E}\left[\mathbf{Q}(i)\,\boldsymbol{\varepsilon}(i)\right] = \mathbb{E}\left[\lambda(i)\,\mathbf{Q}(i-1)\,\boldsymbol{\varepsilon}(i-1)\right]. \tag{33}$$

When $i \to \infty$, we assume that $\mathbf{Q}(i)$ and $\boldsymbol{\varepsilon}(i)$ are uncorrelated, since $0 < \mathbb{E}\left[\lambda(i)\right] < 1$, we obtain

$$\mathbb{E}\left[\boldsymbol{\varepsilon}(i)\right] = 0. \tag{34}$$

We can see that the expected weight error converges to zero as $i \to \infty$.

B. Convergence of MSE

Without loss of generality, we assume that user 1 is the desired user, when $i \to \infty$, the steady-state MSE is given by

$$\begin{aligned}\lim_{i\to\infty}\xi_{mse}(i) &= \lim_{i\to\infty}\mathbb{E}\left[\left|b_0(i) - \tilde{\mathbf{w}}^H(i)\,\mathbf{r}(i)\right|^2\right]\\ &= \lim_{i\to\infty}\mathbb{E}[1 - 2b_0(i)\,\tilde{\mathbf{w}}^H(i)\,\mathbf{r}(i)\\ &+ \tilde{\mathbf{w}}^H(i)\,\mathbf{r}(i)\,\mathbf{r}^H(i)\,\tilde{\mathbf{w}}(i)]\\ &= \lim_{i\to\infty}\mathbb{E}[1 - 2b_0(i)\sum_{k=0}^{K-1} b_k(i)\\ &+ \left(\tilde{\mathbf{w}}_0 + \tilde{\varepsilon}(i)\right)^H\mathbf{R}(i)\left(\tilde{\mathbf{w}}_0 + \tilde{\varepsilon}(i)\right)]\\ &= -1 + \mathbb{E}\left[\tilde{\mathbf{w}}_{_0}^H\mathbf{R}(i)\,\tilde{\mathbf{w}}_0\right]\\ &+ \lim_{i\to\infty}\mathbb{E}\left[tr\left[\mathbf{R}(i)\,\tilde{\varepsilon}(i)\,\tilde{\varepsilon}^H(i)\right]\right],\end{aligned} \tag{35}$$

where $\tilde{\mathbf{w}}(i) = \mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}(i)$, $\tilde{\mathbf{w}}_0(i) = \mathbf{a}(\theta_0) - \mathbf{B}\mathbf{w}_0(i)$ and $\tilde{\varepsilon}(i) = \tilde{\mathbf{w}}(i) - \tilde{\mathbf{w}}_0$. Using (32), we obtain the following difference equation for the coefficient error vector

$$\boldsymbol{\varepsilon}(i) \approx \lambda(i)\,\boldsymbol{\varepsilon}(i-1) + \mathbf{Q}^{-1}(i)\,\mathbf{x}(i)\,e_0^*(i). \tag{36}$$

By multiplying $-\mathbf{B}$ on both sides of (36), we have

$$\tilde{\varepsilon}(i) \approx \lambda(i)\,\tilde{\varepsilon}(i-1) - \mathbf{B}\mathbf{Q}^{-1}(i)\,\mathbf{x}(i)\,e_0^*(i). \tag{37}$$

Then we can obtain

$$\begin{aligned}\boldsymbol{\Theta}(i) &= \mathbb{E}\left[\tilde{\varepsilon}(i)\,\tilde{\varepsilon}^H(i)\right]\\ &= \mathbb{E}\left[\lambda^2(i)\right]\boldsymbol{\Theta}(i-1)\\ &- \mathbb{E}\left[\lambda(i)\,\tilde{\varepsilon}^H(i-1)\right]\mathbb{E}\left[\mathbf{B}\mathbf{Q}^{-1}(i)\,\mathbf{x}(i)\,e_0^*(i)\right]\\ &- \mathbb{E}\left[\lambda(i)\,\tilde{\varepsilon}(i-1)\right]\mathbb{E}\left[e_0(i)\,\mathbf{x}^H(i)\,\mathbf{Q}^{-1}(i)\,\mathbf{B}^H\right]\\ &+ \mathbb{E}\left[\mathbf{B}\mathbf{Q}^{-1}(i)\,\mathbf{x}(i)\,e_0^*(i)\,e_0(i)\,\mathbf{x}^H(i)\,\mathbf{Q}^{-1}(i)\,\mathbf{B}^H\right].\end{aligned} \tag{38}$$

When $i \to \infty$, we can assume that $e_0(i)\,\mathbf{x}^H(i)$, $\mathbf{Q}^{-1}(i)$, $\mathbf{B}$ are uncorrelated, recalling that $\mathbb{E}\left[\mathbf{x}(i)\,e_0^*(i)\right] = \mathbf{0}$, we have

$$\begin{aligned}\boldsymbol{\Theta}(i) &= \mathbb{E}\left[\lambda^2(i)\right]\boldsymbol{\Theta}(i-1)\\ &+ \mathbb{E}\left[\sigma_0^2\mathbf{B}\mathbf{Q}^{-1}(i)\,\mathbf{x}(i)\,\mathbf{x}^H(i)\,\mathbf{Q}^{-1}(i)\,\mathbf{B}^H\right]\\ &= \mathbb{E}\left[\lambda^2(i)\right]\boldsymbol{\Theta}(i-1)\\ &+ \sigma_0^2\mathbf{B}\mathbb{E}\left[\mathbf{Q}^{-1}(i)\right]\mathbb{E}\left[\mathbf{x}(i)\,\mathbf{x}^H(i)\right]\mathbb{E}\left[\mathbf{Q}^{-1}(i)\right]\mathbf{B}^H,\end{aligned} \tag{39}$$

where $\sigma_0^2 = \mathbb{E}[e_0^*(i)\,e_0(i)]$. By solving (39) with the recursion, we obtain

$$\begin{aligned}\boldsymbol{\Theta}(i) &\approx \mathbb{E}^i\left[\lambda^2(\infty)\right]\boldsymbol{\Theta}(0)\\ &+ \frac{\left(1 - \mathbb{E}^i\left[\lambda^2(\infty)\right]\right)\sigma_0^2\left(1 - \mathbb{E}\left[\lambda(\infty)\right]\right)^2}{\left(1 - \mathbb{E}\left[\lambda^2(\infty)\right]\right)\mathbb{E}\left[\left|y_0(i)\right|^2\right]}\mathbf{B}\left(\mathbf{B}^H\mathbf{R}\mathbf{B}\right)^{-1}\mathbf{B}^H\\ &\approx \frac{\sigma_0^2\left(1 - \mathbb{E}\left[\lambda(\infty)\right]\right)^2}{\left(1 - \mathbb{E}\left[\lambda^2(\infty)\right]\right)\mathbb{E}\left[\left|y_0(i)\right|^2\right]}\mathbf{B}\left(\mathbf{B}^H\mathbf{R}\mathbf{B}\right)^{-1}\mathbf{B}^H,\end{aligned} \tag{40}$$

where $y_0(i) = \tilde{\mathbf{w}}_0^H(i)\,\mathbf{r}(i)$, $\mathbf{R} = \mathbb{E}\left[\mathbf{r}(i)\,\mathbf{r}^H(i)\right]$. Finally, we have the MSE at the steady-state which is given by

$$\begin{aligned}\lim_{i\to\infty}\xi_{mse}(i) &\approx -1 + \mathbb{E}\left[\tilde{\mathbf{w}}_{_0}^H\mathbf{R}(i)\,\tilde{\mathbf{w}}_0\right]\\ &+ \lim_{i\to\infty}\mathbb{E}\left[tr\left[\mathbf{R}(i)\,\boldsymbol{\Theta}(i)\right]\right].\end{aligned} \tag{41}$$

VI. SIMULATIONS

In this section, we investigate the effectiveness of the proposed TAVFF mechanism with the blind adaptive CCM-RLS-GSC beamformer through simulations. We have conducted several experiments to assess the convergence performance of the proposed algorithm and compared it with the blind adaptive CCM-RLS-GSC beamformer with the GVFF mechanism, the blind adaptive CCM-RLS-GSC beamformer with the fixed forgetting factor mechanism and the CMV-RLS-GSC beamformer with the fixed forgetting factor mechanism.

We assume that the DOA of the desired user is known by the receiver. In each experiment, a total of N=1000 runs are carried out to obtain the curves. In all simulations, the source power including the desired user and interferers are the same, the noise is zero mean spatially and temporally white Gaussian noise and the input SNR is 15dB. Simulations are performed by an ULA containing $M = 16$ sensor elements with half-wavelength inter-element spacing. The BPSK scheme is employed to modulate the signals.

Fig. 2 shows the signal to interference plus noise ratio (SINR) performance of the desired user versus the number of snapshots in a nonstationary scenario for the proposed TAVFF scheme, the GVFF scheme and the conventional fixed forgetting factor schemes. In this experiment, the system starts with $K = 5$ users including one desired user. After 1000 snapshots, two more interferers enter the system. This change reduces the output SINR suddenly and degrades the performance of all the algorithms. It is evident that the output SINR of our proposed mechanism is superior and converges much faster than the other mechanisms.

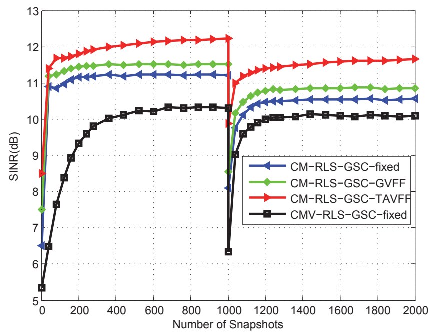

Fig. 2: SINR performance in a nonstationary environment

In the next experiment as shown in Fig. 3, it illustrates the beampatterns of the array of the existing and proposed algorithms. The DOA of the desired user is $\theta_0 = 50°$. There are four interferers with the same power as the source ($\theta_1 = 20°, \theta_2 = 40°, \theta_3 = 60°, \theta_4 = 70°$). From Fig. 3, the mainlobe beams of the proposed algorithms direct at the direction of the desired user. The proposed mechanism have nulls at the directions of arrival of the interferers, which forms the nulls deeper than that of the existing mechanism.

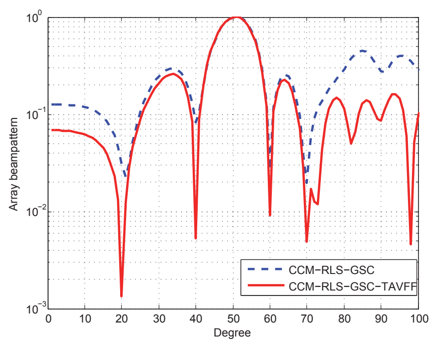

Fig. 3: Array beampattern versus degree for the proposed and existing algorithms

In the last experiment, Fig. 4 compares the steady-state MSE performance of the proposed CCM-RLS-GSC algorithm with the TAVFF mechanism between the theoretical analysis and the simulation results. It can be seen that the proposed algorithm converge quickly and reach the steady-state which is in a good match with the analytical result. The simulation and analysis results agree well with each other.

VII. Conclusion

In this paper, we developed a low-complexity variable forgetting factor scheme employing a new component updated by the time average of the CM cost function for blind adaptive beamforming with a GSC structure. A complexity comparison was given for illustrating the advantage of the proposed algorithms over the existing GVFF scheme. Furthermore, we investigated the convergence of the proposed scheme, and derived expressions to predict the steady-state MSE of the adaptive CCM-RLS-GSC algorithm with the TAVFF mechanism. The simulation results verified the analytical results and showed that the proposed scheme significantly outperforms the existing methods in terms of convergence and steady-state performance at a low complexity.

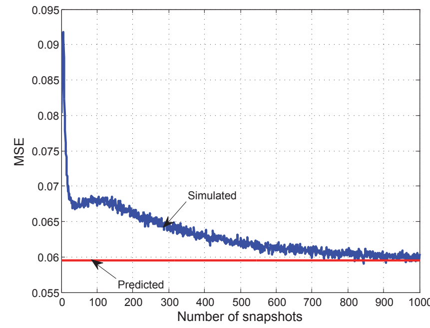

Fig. 4: Analytical MSE versus simulated performance for the proposed CCM-RLS-GSC algorithm with the TAVFF mechanism

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