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SUBSPACE-BASED LINE AND CURVE EXTRACTION FROM NOISY IMAGES *

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Abstract

In this paper, we are interested in the use of subspace tracking techniques for image analysis. More specifically, we present a new model which enables the parametric extraction of lines and curves of any shape. The lines and curves parameters, i.e. angle and offset, are tracked as the image is scanned from the top to the bottom by a sliding window. Based on this model, we propose a new Subspace-based Adaptive Line Extraction algorithm (SALE). SALE, an adaptive extent of the SLIDE algorithm, takes advantage of the latest developments in subspace tracking algorithms. SALE outputs a parametric description of detected lines and curves, by tracking their offset and angle throughout the image; its reduced complexity is directly related to the number of extracted features.

1 Introduction

Image analysis/understanding is used in various areas of research such as biomedical, robotics and astronomy imaging; its main goal is the extraction of a desired information from a set of noisy observations.

Depending on the type of application, specific models are used to extract the desired information. In astronomy for instance, the atmosphere blurs the images by spreading each pixel over a limited surrounding area and the deblurring problem is then treated as a deconvolution issue. In computer vision applications, one is mainly interested in the extraction of features such as lines and contours, or shape recognition. In image restoration, the main task consists in reducing the noise level from the image.

One notices a common point to all the above applications: the information one seeks is often localized, i.e. the information contained in a single pixel is usually related to the value of a limited number of surrounding pixels only; in other words, a pixel is correlated with its near surrounding only. Therefore, most image analysis/understanding techniques could be formulated so as to extract the desired information by performing several local analysis within the image.

Such a "localized" analysis supposes a sliding window to be moved over the image so as to extract the features locally. An accurate model is also required to link all extracted local features for a good analysis/understanding of the whole image. Fish's deblurring algorithm [1] is a perfect example of such an image analysis procedure; it uses the singular value decomposition (SVD) locally so as to independently deblur small regions of the image, by scanning the whole image with a square sliding window.

Generally, an SVD model fits well to problems in which a parameter has to be estimated through the observation of a series of data. Even though the SVD is known to be an elegant way to process separately the noise and signal components of a data series, its use in image processing applications is still limited, mostly because of its heavy computational load. In fact, the computation of the SVD of numerous small blocks of an image, e.g. as in Fish's algorithm, is still a much complex task.

Assuming a SVD-based model which enables the extraction of local features, the main idea we are going to develop in this paper is the following: under specific conditions which shall be stated later, the features of a small image region can be obtained by slightly updating the SVD-based representation of a near adjacent region. In other words, one may not have to re-compute the SVD of each image block, if it can be obtained by slightly updating the SVD of

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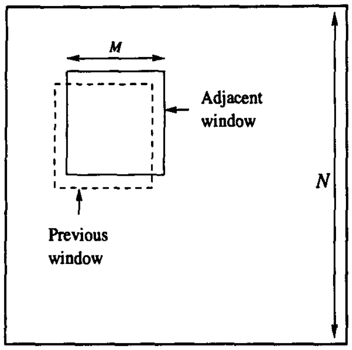

Figure 1: Traitement spatial d'une image par translation d'une fenêtre d'observation.

a previous near block. This technique which we call a "SVD-based sliding window" procedure has a lower complexity.

Intuitively, the above "sliding window" assumption along with the need to track the local features as they are extracted from different regions of an image, has a strong analogy with subspace tracking techniques. The core of subspace tracking algorithms generally consists in the updating of an orthogonal decomposition, i.e. the eigenvalue decomposition (EVD) of a correlation matrix or the SVD of a data matrix, as new data become available. For instance, subspace tracking algorithms are used to estimate and track the direction of arrival (DOA) of plane waves using antenna arrays. Therefore, they fit perfectly to image analysis applications which require a scan-like processing.

Let us assume that an image of size N has to be processed by computing the SVD of several overlapping M-sized blocks, as shown in Fig. 1. On one hand, the exact computation of the SVD of each block from scratch requires $O(M^3)$ operations; on the other hand, by using subspace tracking techniques, the SVD of any block can be obtained by slightly updating the one from the preceeding block in $O(M^2)$ operations or less. Thus, the SVD of all blocks can be obtained with fewer operations! Note that in several applications, one does not compute directly the SVD of the image pixel values, but rather the SVD of an intermediate representation, as will be shown in Sections 2 and 3. However, the advantage of the above principle remains the same.

In summary, the ability of the SVD to be used to track parameters, the low-complexity associated to local SVDs compared to global SVD, and the fact that a pixel is generally correlated with its near surrounding only, justify the need to adapt subspace tracking methods to image analysis/understanding applications.

In this paper, we address the issue of using subspace updating techniques for the detection of lines and curves in an image. We start our argument by modifying a model initially proposed by Aghajan [2]; in this model, straight lines are processed as "plane waves", and array processing techniques are used to compute their angle and offset. We propose an adaptive model in which the image is analyzed by sliding a rectangular window from the top to the bottom. As the image is scanned, the local SVDs are continuously updated and the curves parameters can be tracked. Based on this model, we propose a new Subspace-based Adaptive Line Extraction algorithm (S.A.L.E), which detects lines of any shape, i.e. curves. Moreover, this model enables the parametric extraction of curves, i.e. their angle and offset versus each row number. Therefore, SALE might be suitable to a class of manufacturing and military applications.

The paper is organized as follows: In Section 2, we review Aghajan's model and introduce an adaptive model for curve extraction. In section 3, we present the SALE algorithm. Section 4 presents experimental results obtained from synthetic and real satellite images. Concluding remarks are provided in Section 5.

2 Model for parametric line and curve extraction

The extraction of lines and curves fron an image is a well-known issue for which, there already exist a wide variety of solutions; the list of related applications is a long one which shall not be stated here. However, we would like to mention that in this paper, we are interested in the *parametric* extraction of lines and curves, i.e. their exact location on the 2-D plane of the image. Thus, we would like to develop a method which does not output a processed image containing the extracted features, but a parametric description of those curves and lines. For instance, in the case of a straight line, its parametric extraction consists in the estimation of its offset and its angle only. Such a method is performed by the SLIDE algorithm which is biefly explained below.

2.1 Straight lines extraction: the SLIDE algorithm

SLIDE stands for Subspace-based LIne DEtection [2]. It is an alternative solution to the Radon Trans-

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form [3] which enables the parametric extraction of straight lines. Proposed by Aghajan et. al, it is based on techniques used in the area of array processing. The underlying model of SLIDE is illustrated on Fig. 2.

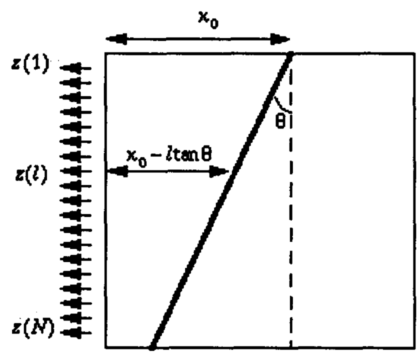

Figure 2: Model for straight line parameters estimation using the SLIDE algorithm.

Let us assume that hypothetical sensors are located along the left-side of the image. If the l^{th} row of the image contains r non-zero pixels located at columns q_i, $i = 1 \cdots r$, the "signal" $z(l)$ of the l^{th} sensor is

$$z(l) = \sum_{i=1}^{r} e^{-j\mu q_i} \quad (1)$$

where μ is a constant parameter. If the r black pixels are due to r straight lines, each with an offset x_{0i} and an angle θ_i, the l^{th} sensor's output is

$$z(l) = \sum_{i=1}^{r} a_l(\theta_i)s_i + n(l) \quad l = 1, \cdots, N \quad (2)$$

where $a_l(\theta_i) = e^{j\mu(l-1)\tan\theta_i}$ and $s_i = e^{-j\mu x_{0i}}$. This scenario is effectively encoding the line angles as frequencies of cisoidals components in the measurements $[z(1), \cdots, z(N)]$; thus, θ_i can be estimated using harmonic retrieval techniques based on the covariance matrix estimation.

SLIDE estimates the angles θ_i in four steps:

- perform a spatial smoothing on the data by defining a series of "measurement vectors", i.e. consider a series of overlapping vectors

$$\mathbf{z}(l) = [z(l), \cdots, z(l + M - 1)]^T \quad (3)$$

- compute the EVD of the associated sample covariance matrix

$$R_{\mathbf{zz}} = \frac{1}{N - M + 1} \sum_{l=1}^{N-M+1} \mathbf{z}(l)\mathbf{z}(l)^H = \sum_{j=1}^{M} \lambda_j \mathbf{v}_j \mathbf{v}_j^H; \quad (4)$$

- from the eigenvalue spectrum $\{\lambda_j\}$, estimate the number r of straight lines with the minimum description length (MDL) criterium [4];

- extract the angles θ_i with the ESPRIT algorithm [5], using the signal eigenvectors, i.e. the signal subspace basis $V_S = [\mathbf{v}_1, \cdots, \mathbf{v}_r]$.

The offsets x_{0i} are later extracted by dechirping the original sequence $\{z(l)\}$ into a new sequence $\{w(l)\}$ which this time encodes the offsets as frequencies of cisoids (see [2] for details). The above four steps are then repeated to estimate the offsets.

2.2 A new model for curve extraction

The SLIDE algorithm certainly lies on an innovative concept. However, it suffers from a number of difficulties which are mainly due to the model it is based on. For instance, it cannot deal with the curvature of a line. Also, it is not formulated to localize small extent lines.

We now propose a new model in which the curvature of a line can be tracked and updated, as the sensors are iteratively taken into account, from the top to the bottom of the image. To do this, one needs only to gather a sufficient number of measurements, i.e. a sufficient number of vectors, say M, which encode the line parameters for a specific region of the image.

Let us suppose a $N \times N$ image with a single curve which crosses the image, as shown in Fig. 3. In the case of curves, the parameters (angle and offset) are expected to vary with respect to the row index l. A quick look at the measurement vector $\mathbf{z}(l)$ in (3) shows that it corresponds to a $M \times N$ rectangular slice of the image, as shown in Fig. 3. There is here a strong analogy between the *space*-series of the image measurement vectors $\mathbf{z}(l)$ and the time-series used in the tracking of frequencies drifts of noisy sinusoids. Thus, assuming a sliding window which scans the image from the top to the bottom, one can use parameter tracking techniques based on time series measurements to track the variations of θ_i and x_{0i}, i.e. estimate $\theta_i(l)$ and $x_{0i}(l)$ for $l = 1 \cdots N$.

Note that a single measurement vector $\mathbf{z}(l)$ cannot be used to estimate $\theta_i(l)$, even though it covers an

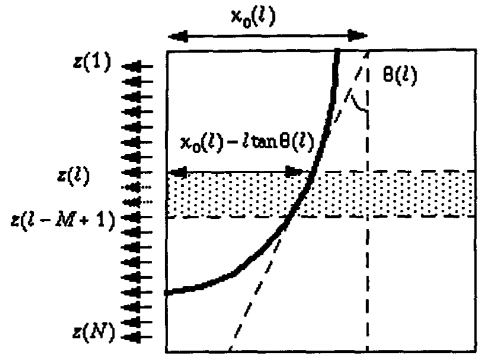

Figure 3: Adaptive model for curve extraction.

area corresponding to M rows (or sensors). This is due to the fact that subspace-based methods require the computation of either a data matrix (in which successive measurements are gathered) or a covariance matrix (which is generally approximated by a sample covariance matrix, still based on a sufficient number of successive measurements). Even though an eigenstructure-based estimation might be a little complex, it has however the advantage to enable the tracking of parameters as new measurements are added to the set of available measurements.

By analogy with adaptive array processing techniques, we propose the following model. Let us define the data matrix

$$Z(l) = \begin{bmatrix} \sqrt{\beta} Z(l-1) \\ \mathbf{z}(l)^H \end{bmatrix}; \qquad (5)$$

it is an exponential windowing of the data in which the measurements are added one after another, and in which the impact of new measurements is affected by the forgetting factor $\beta < 1$. Compute the SVD of $Z(l)$, i.e. find $U(l)$, $\Sigma(l)$ and $V(l)$ such that $Z(l) = U(l)\Sigma(l)V(l)^H$. $V(l) = [\mathbf{v}_1(l), \cdots, \mathbf{v}_M(l)]$ is a $M \times M$ unitary matrix, $U(l)$ is a $l \times M$ matrix with orthonormal columns, and $\Sigma(l) = \mathrm{diag}(\sigma_1(l), \cdots, \sigma_M(l))$ with $\sigma_1(l) \geq \sigma_2(l) \geq \cdots \geq \sigma_M(l)$.

Once the SVD of $Z(l)$ is computed, one may use the MDL criterium or any other technique to determine the number of curves $r(l)$ which cross the l^{th} row; $r(l)$ is equal to the effective rank of $\Sigma(l)$. Also, the signal subspace basis vectors $V_S(l) = [\mathbf{v}_1(l), \cdots, \mathbf{v}_{r(l)}(l)]$ are used by a projection technique such as ESPRIT [5] or root-MUSIC [7] to extract the angles $\theta_i(l)$. Note that the angles $\theta_i(l)$ are estimated at the l^{th} row only.

Regarding the offsets, each measurement vector $\mathbf{z}(l)$ is dechirped into $\mathbf{w}(l)$ which is inserted into an exponentially weighted "offset" data matrix $W(l)$, defined recursively as $Z(l)$ in (5). The offsets $x_{0i}(l)$ can then be extracted similarly to the angles.

3 S.A.L.E : A Subspace-based Adaptive Line Extraction algorithm

3.1 Algorithm description

Based on the model described is Section 2, we propose a new algorithm for the parametric extraction of lines and curves.

SALE tracks the curves' angle and offset by scanning the image from the top to the bottom. For each row index l, the SVD of the data matrix $Z(l)$ is not recomputed, but updated using the latest subspace updating techniques. For this purpose, SALE uses our recent Noise-Average Cross-terms SVD algorithm (NA-CSVD). NA-CSVD tracks the signal subspace only, i.e. $V_S(l) = [\mathbf{v}_1(l), \cdots, \mathbf{v}_{r(l)}(r(l))]$.

Regarding the tracking of the number of curves $r(l)$ at each row l, SALE uses NA-CSVD along with a rank estimator for spherical subspace trackers (RSST), introduced by Kavcic et. al [6]. The performance of the couple NA-CSVD/RSST has already been demonstrated [8].

Regarding the offsets $x_{0i}(l)$, they require a parallel tracking procedure, since their computation is based on a different measurement series. Indeed, once the angles $\theta_i(l)$ have been computed at l^{th} row, the corresponding measurement vector $\mathbf{z}(l)$ is dechirped according to each of the angles $\theta_i(l)$; this leads to $r(l)$ dechirped series of vectors $\mathbf{w}_i(l)$, each encoding only one offset $x_{0i}(l)$. Once again, SALE uses the NA-CSVD algorithm to track these offsets separately.

The total scan of the image consists then in N updates, one for each row of the image. To track the angles $\theta_i(l)$, NA-CSVD has the advantage to update V_S from $l-1$ to l with only $O(Mr(l))$ operations. In the case of the offsets, a rank tracking technique is not required, since each series $\mathbf{w}_i(l)$ represents only a single offset; therefore, the complexity of the updating of the $r(l)$ offsets is also $O(Mr(l))$. Thus, if the image is crossed by $\bar{r}$ curves from the top to the bottom, i.e. assuming that $r(l) = \bar{r}$ for all l, the complexity of the SVD of the whole $N \times N$ image is not $O(N^3)$ but reduced to $O(2\bar{r}MN)$. One notices that the complexity of SALE is directly related to not only the image size N, but also to the number of curves $r(l)$ to be extracted at each row.

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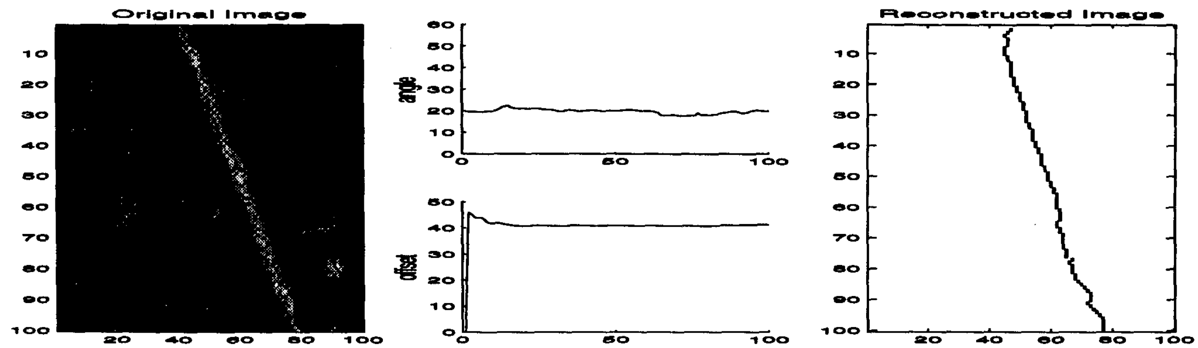

Figure 4: Road network parametric extraction using SALE algorithm

Let us define the *complexity reduction indicator* τ_{SALE} of SALE as the ratio of its complexity over the complexity of an algorithm which would re-compute the SVD of each $M \times N$ rectangular slice, i.e. $O(NM^3)$; we have

$$\tau_{SALE} = \frac{2\bar{r}}{M^2} \tag{6}$$

τ_{SALE} express also the limit of $\bar{r}$ over which SALE might be more complex. For instance, with a sliding window of $M = 10$ sensors wide, there must be at least $\bar{r} = 50$ detected curves to get $\tau_{SALE} > 1$.

3.2 Further remarks on SALE

As for array processing algorithms, several implementation issues must be addressed so as to guarantee the good performance of SALE.

Initialization: the initial subspaces $V_S(0)$ and $\Sigma(0)$ are obtained by computing the exact SVD of the $M \times N$ upper-slice of the image.

Angles and offsets matching: each offset data series $\mathbf{w}_i(l)$ is obtained after the measurement $\mathbf{z}(l)$ is dechirped using the appropriate angle $\theta_i(l)$; since the subspaces are not "signed", we ensure this match to be done by continuously sorting the offsets and angles in separate tables.

scan direction: scanning the image from the top to the bottom provides a good estimates of vertical line, while scanning from left to-right provides a good description of horizontal ines. Thus, depending on the image to process, SALE can be performed twice in two orthogonal directions. Note that additional software is required to combine the two results.

pre and post-processing: One may optionally pre-process the image by thresholding the pixels to remove the background, or by enhancing the curves using a gradient mask. By providing a list of parameters $(\theta_i(l), x_{0i}(l))$ instead of an output image, SALE makes post-processing tasks such as outliers removal easier, since they are performed on a 1-D list of data and require no 2-D search algorithms.

4 Experimental results

In this Section, we test the SALE algorithm in various simulations and experiments. Fig. 4 illustrates a parametrized road extraction from a very noisy satellite image containing a single distorted line; its angle (around 20°) and offset (around 40 pixels) are tracked as the line crosses the image from the top to the bottom. The parameters provided by SALE are then used to reconstruct the detected road. In Fig. 5, the SALE algorithm is compared to SLIDE and the Radon Transform. SLIDE cannot provide any information regarding the extent of the detected lines, as well as the Radon transform whose peaks indicates only the angles and offsets, but not the extent of the detected lines. One notices that SALE has the advantage to provide a more precise parametric description of small extent lines.

5 Conclusion

In this paper, we have proposed a new model for the tracking of curve parameters in an image. By encoding the angle and offset of the curves, we use well-known subspace tracking techniques for the parameters tracking. Based on this model, we have proposed a parametric line and curve extraction algorithm, SALE, which extracts the curve parameters

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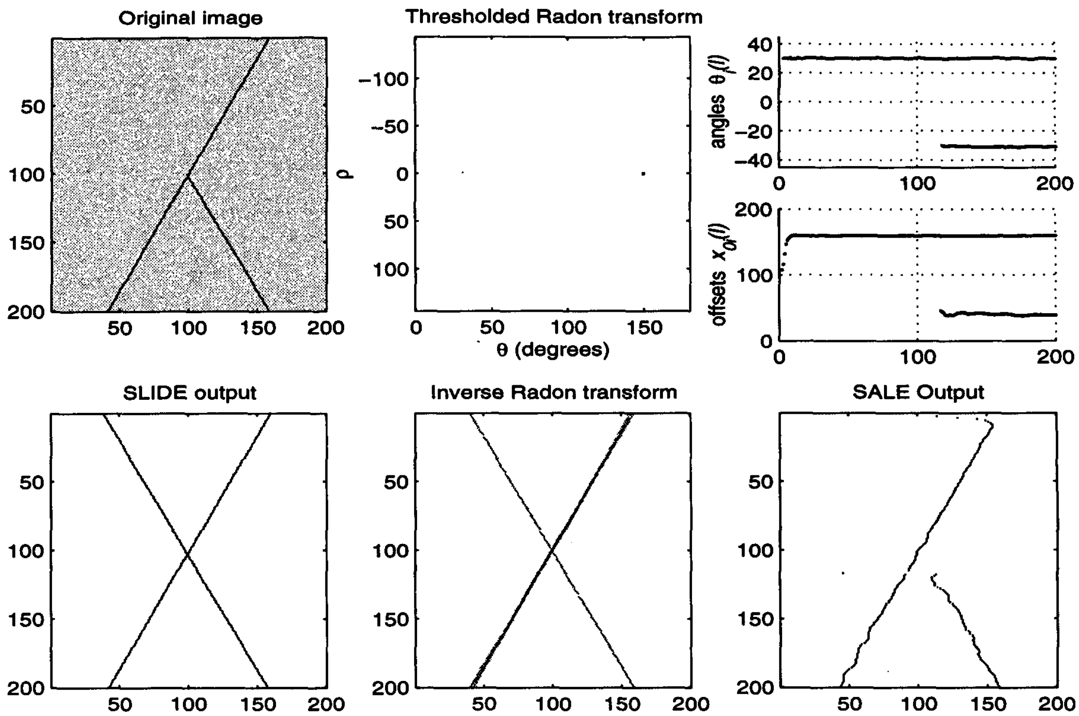

Figure 5: Lines extraction from a noisy image, using SALE, SLIDE and the Radon transform

adaptively for each row of the image. SALE outputs only the necessary curves description, i.e. their angles and offsets. SALE is a less complex solution which has the advantage to provide the parametric localization of small extent curves.

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