IMPROVED STRATEGY FOR ADAPTIVE RANK ESTIMATION WITH SPHERICAL SUBSPACE TRACKERS

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Abstract

We present an improved adaptive rank detection algorithm for on-line estimation and tracking of the signal subspace dimension in applications of spherical subspace trackers. The proposed algorithm uses different adaptive thresholds for the rank increase (up) and decrease (down) tests as well as a special set of fast tracking eigenvalue estimates in the rank decrease test, which can be obtained at little extra cost. It is based on an original investigation of the detection performance for the up and down tests that takes into account the exponential nature of the eigenvalue update in spherical subspace trackers. Through computer experiments in multi-user detection, it is shown that with the proposed algorithm, the time required to detect a rank decrease is significantly less than with existing methods.

Keywords: Rank estimation; Subspace tracking; Multi-user detection.

1. INTRODUCTION

Subspace methods rely on the decomposition of the observation space (dimension N), into the orthogonal sum of a signal subspace (dimension $r < N$) and a noise subspace. In theory, a basis of the signal subspace is provided by the rth dominant eigenvectors of the data covariance matrix. In practice, the relevant subspace information can be obtained via eigenvalue decomposition (EVD) of a sample covariance matrix.

In applications of subspace methods to dynamic signal environments, it is necessary to update the subspace information as new data become available. To avoid the computational bottleneck of the EVD or SVD computation, on the order of $O(N^3)$ operations per iteration, several fast subspace tracking algorithms have been developed. Most of these algorithms are based on the assumption of a spherical noise subspace, i.e. the noise subspace eigenvalues are all identical [3]. As a result, they can achieve computational complexity as low as $O(Nr)$ operations per time iteration, and yet maintain a level of performance comparable to exact EVD or SVD computations (see e.g. [1]). The operation of spherical subspace trackers critically depends on the availability of effective algorithms that can estimate and track r in realtime, also called *adaptive rank estimators.*

In [4], Kavcic and Yang present a general purpose adaptive rank estimator for use with spherical subspace trackers, referred to here as the KY method. In essence, it compares the eigenvalue estimates at each iteration to an adaptively set threshold in order to detect either a rank increase of decrease. The choice of the adaptive threshold in [4] is based on a simplified analysis of the probability of false alarm and miss for the test which uses probability distributions derived for batch EVD computations (i.e. long rectangular window). Despite its simplicity, the KY method typically outperforms classical information theoretic criteria. Its main limitation is the relatively long time interval (i.e. delay) needed to detect a rank decrease.

In this paper, we present an improved adaptive rank detection algorithm for use with generic spherical subspace trackers. The new algorithm uses different adaptive thresholds for the rank increase (up) and decrease (down) tests along with fast tracking eigenvalue estimates in the rank decrease test. This approach is based on a more accurate analysis of the detection performance for the tests that takes into account the exponential windowing nature of typical eigenvalue updates. The new algorithm leads to a significant reduction of the detection time of a rank decrease under a constraint of a fixed false alarm probability. This is supported by computer experiments in multi-user detection.

2. PROBLEM FORMULATION

2.1 System Model

Consider the linear model

$$\mathbf{x}(k) = A(k)\mathbf{s}(k) + \mathbf{n}(k) \qquad (1)$$

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where $\mathbf{x}(k) \in \mathbb{C}^{N\times 1}$ denotes the data vector observed at discrete-time $k \in \mathbb{Z}$, $\mathbf{s}(k) \in \mathbb{C}^{r\times 1}$ is a zero-mean, random signal vector with $r < N$, $A(k) \in \mathbb{C}^{N\times r}$ is a deterministic but unknown signal transformation matrix, and $\mathbf{n}(k) \in \mathbb{C}^{N}$ is a zero-mean background noise vector. The processes $\{\mathbf{s}(k)\}$ and $\{\mathbf{n}(k)\}$ are assumed to be uncorrelated, with respective covariance matrices at time k given by $R_s(k) = E[\mathbf{s}(k)\mathbf{s}(k)^H] > 0$ and $R_n(k) = E[\mathbf{n}(k)\mathbf{n}(k)^H] = \sigma(k)^2 I$, where $\sigma(k)^2$ is the noise variance.

Under the above assumptions, the covariance matrix of the data vector $\mathbf{x}(k)$ takes form

$$R_x(k) = A(k)R_s(k)A(k)^H + \sigma(k)^2 I \tag{2}$$

The EVD of $R_x(k)$ is expressed as

$$R_x(k) = U(k)\Lambda(k)U(k)^H \tag{3}$$

where $\Lambda(k) = \text{diag}(\lambda_1(k), \ldots, \lambda_N(k))$ contains the eigenvalues of $R_x(k)$ in non-increasing order, i.e.

$$\lambda_1(k) \geq \ldots \geq \lambda_r(k) > \lambda_{r+1}(k) = \ldots = \lambda_N(k) = \sigma(k)^2 \tag{4}$$

and $U(k) = [\mathbf{u}_1(k), \ldots, \mathbf{u}_N(k)]$ contains the corresponding eigenvectors in orthonormal form. Introducing $U_s(k) = [\mathbf{u}_1(k), \ldots, \mathbf{u}_r(k)]$ and $U_n(k) = [\mathbf{u}_{r+1}(k), \ldots, \mathbf{u}_N(k)]$, we have that $A(k)^H U_n(k) = 0_{r\times(N-r)}$, or equivalently, $\text{span}\{A(k)\} = \text{span}\{U_s(k)\}$, where span$\{.\}$ denotes the column span of its matrix argument. We refer to the orthogonal subspaces $\text{span}(U_s(k))$ and $\text{span}(U_n(k))$ as the signal and noise subspaces, respectively. Subspace methods rest on the above formulation of $R_x(k)$.

2.2 Spherical EVD Tracking

In the application of subspace methods to dynamic signal environments, it is necessary to recompute the desired EVD estimates as new data become available. This can be achieved e.g. by first updating the sample covariance matrix and then recomputing its EVD. The following recursive estimate of $R_x(k)$ is often used:

$$\hat{R}_x(k) = \alpha\hat{R}_x(k-1) + (1-\alpha)\mathbf{x}(k)\mathbf{x}(k)^H \tag{5}$$

where $0 < \alpha < 1$ is a forgetting factor introduced to deemphasize past data. Several well-proven numerical algorithms are then available for computing the EVD of Hermitian matrix $\hat{R}_x(k)$ [2]. However, recomputing this EVD *ab initio* at each time step k using such algorithms entails a very high computational cost of $O(N^3)$ operations per time iteration.

Several fast subspace tracking algorithms have been developed that can update the desired EVD information at a much reduced cost. Of particular interest here is a class of so-called spherical subspace trackers (SST), which enforce an equality constraint on the estimated noise subspace eigenvalues (i.e. shperical noise subspace) so as to reduce the dimensionality of the EVD updating problem (e.g. [3, 1]). As a result, SST can achieve computational complexity as low as $O(Nr)$, yet maintain a level of performance comparable to an exact EVD computation.

Given the signal subspace dimension r, SST algorithms typically track $r+1$ EVD components, namely: the signal subspace eigenvalues, say $l_i(k)$ $(i = 1, \ldots, r)$; a single, average noise subspace eigenvalue, say $l_N(k)$; the signal subspace eigenvectors, say $\mathbf{v}_i(k)$ $(i = 1, \ldots, r)$, also represented by the matrix $V_S(k) = [\mathbf{v}_1(k), \ldots, \mathbf{v}_r(k)]$; and an additional noise subspace eigenvector, say $\mathbf{v}_N(k)$. The following steps are common to several SST algorithms:

Data projection:

$$\mathbf{y}_S(k) = V_S(k-1)^H\mathbf{x}(k) \tag{6}$$

$$\mathbf{x}_N(k) = \mathbf{x}(k) - V_S(k-1)\mathbf{y}_S(k) \tag{7}$$

$$y_{r+1}(k) = \|\mathbf{x}_N(k)\| \tag{8}$$

$$\mathbf{v}_N(k-1) = \mathbf{x}_N(k)/y_{r+1}(k) \tag{9}$$

Eigenvector updating:

$$[V_S(k), \mathbf{v}_N(k)] = \mathcal{T}\{[V_S(k-1), \mathbf{v}_N(k-1)]\} \tag{10}$$

where the explicit form of the transformation $\mathcal{T}$ depends on the specific SST algorithm being considered.

Eigenvalue updating: Most SST use exponential windowing to update the eigenvalue estimates:

$$l_i(k) = \alpha l_i(k-1) + (1-\alpha)|y_i(k)|^2 \tag{11}$$

where $y_i(k)$ denotes the ith entry of vector $\mathbf{y}_S(k)$. A slightly different update is used for the noise eigenvalue:

$$l_N(k) = \alpha l_N(k-1) + (1-\alpha)\frac{|y_{r+1}(k)|^2}{N-r} \tag{12}$$

where the factor $1/(N-r)$ reflects the presence of the noise averaging operation.

2.3 Adaptive Rank Estimation (KY method)

The operation of SST critically depends on the availability of effective algorithms for estimation and tracking of r, also called *adaptive rank estimators*. In [4], Kavcic and Yang present an adaptive rank estimator that is specially designed for use with SST. This estimator, referred to here as the KY method, makes advantageous use of the average noise subspace eigenvalue available with SST to adaptively set a threshold in a basic rank detection test. Compared to traditional information theoretic criteria, KY method offers several advantages, including: reduced complexity, ease of integration with popular SST algorithms and in many cases, superior detection performance.

The basic idea behind KY method is to keep track of the rank r and then, after a simple threshold comparison, allow the rank to change by either ± 1 or 0 at each time step. To overcome the smoothing effects of spherical averaging on the noise subspace eigenvalues in the case of a sudden rank increase, it is proposed in [4] to track an additional noise eigenvalue and eigenvector. This is achieved by running the SST algorithm with $r+1$ instead of r and slightly modifying the updating equation for $l_N(k)$. The basic test in KY method can be formulated as follows:

```
if l_r(k) < γ(k) then "rank deflation"
      incorporate l_{r+1}(k) into l_N(k)
      delete v_{r+1}(k) from V_S(k)
      r ← r − 1
elseif l_{r+1}(k) > γ(k) then "rank inflation"
      set l_{r+2}(k) = l_N(k)
      set v_{r+2}(k) = v_N(k)
      r ← r + 1
else "rank unchanged"
γ(k+1) = φ²l_N(k)
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where $\gamma(k)$ is the adaptive threshold and ϕ is the so-called thresholding factor. The reader is referred to [4] for additional details.

The main drawback of the KY algorithm is the relatively long time delay needed for the detection of a sudden rank decrease, as compared to the detection of a rank increase (see Section V).

3. DETECTION PERFORMANCE WITH EXPONENTIAL WINDOW

In [4], the performance of the above threshold comparison test is analyzed with the aim of determining an optimal value for parameter ϕ. The estimated eigenvalues are treated as random variables from which the probability of underestimation/overestimation of r are computed. This analysis is based on the following assumptions:

a) process $\mathbf{x}(k)$ in (1)-(2) is wide sense stationary, where the value of r is assumed fixed;
b) a rectangular window of length N (instead of an exponential one) is used in the computation of $\hat{R}_x(k)$.

As a consequence of a), the dynamic properties of the detection variables under a true rank increase/decrease cannot be investigated. In particular, no information is provided by the theory about the average detection time, so that no plausible explanations can be obtained for the above noted difference between rank increase and rank decrease. As a consequence of b), the role of the forgetting factor α can not be directly tied in to the optimal value of ϕ.

In the following, we relax the above assumptions and analyze the detection performance of basic tests for the rank increase and rank decrease situations. In both cases, we attempt to determine the value of α that minimizes the detection time under a constraint of a fixed probability of false alarm.

3.1 Dynamic Signal Model

We consider a simplified dynamic model of the data vector $\mathbf{x}(k)$ in which the signal subspace dimension r may or not change by ± 1 at time $k=0$. Specifically, we model $\mathbf{x}(k)$ in terms of a Karhunen Loeve expansion

$$\mathbf{x}(k) = \sum_{l=1}^{N} c_l(k)\,\mathbf{u}_l \tag{13}$$

where $\mathbf{u}_l \equiv \mathbf{u}_l(k)$ are constant, orthonormalized eigenvectors, and $c_l(k)$ are uncorrelated random coefficients, modelled as zero-mean, complex circular Gaussian random variables with variances

$$E[\,|c_l(k)|^2\,] = \lambda_l(k) \tag{14}$$

We assume that all the eigenvalue parameters $\lambda_l(k)$ are constant over time (i.e. $\lambda_l \equiv \lambda_l(k)$), except for $\lambda_r(k)$ and $\lambda_{r+1}(k)$ which will be allowed to change only at time $k=0$. The exact nature of the change depends on specific hypotheses, to be presented below.

We also assume an idealized SST model in which the eigenvector estimates are error free, that is:

$$V_S(k) \equiv U_S, \quad \mathbf{v}_N(k) = \mathbf{u}_{r+1} \tag{15}$$

This is motivated by experimental observations suggesting that the rank detection performance is not very sensitive to small errors in the eigenvector estimates.

3.2 Exponentially Weighted χ^2

To simplify the analysis of the detection performance of the rank increase/decrease tests, we find it convenient to first introduce an exponentially weighted chi-square distribution, as follows.

Let z_k be a sequence of i.i.d. chi-square random variables with n degrees of freedom, i.e. $z_k \sim \chi^2(n)$. Let the sequence η_k be defined recursively through

$$\eta_k = \alpha\eta_{k-1} + \frac{(1-\alpha)}{n} z_k \tag{16}$$

We refer to z_k as an exponentially weighted chi-square random variable with parameters α and n. It is possible to derive expressions for the moments of η_k by making use of (16). For instance, we note that $E(\eta_k) = 1$.

We define the cumulative distribution function (CDF) of η_k as

$$F(x;\alpha,n) \triangleq \text{Prob}(\eta_k \leq x), \quad x \in \mathbb{R} \tag{17}$$

Apparently, a close form expression for the above CDF does not exist, although various schemes are available to device analytical approximations. In this work, we resort to Monte Carlo simulation of the defining recursion (16) to evaluate the CDF $F(x;\alpha,n)$ for different values of the parameters α and n.

3.3 Detection of a Rank Increase

A rank increase is modelled by the following behavior of the process eigenvalues: for all k, $\lambda_1 \geq \ldots \geq \lambda_r \geq \sigma^2 + S$ and $\lambda_{r+2} = \ldots = \lambda_N = \sigma^2$; for $k < 0$, $\lambda_{r+1}(k) = \sigma^2$ while for $k \geq 0$, two possibilities:

$$\begin{aligned} H_0^u &: \lambda_{r+1}(k) = \sigma^2 \\ H_1^u &: \lambda_{r+1}(k) = \sigma^2 + S \end{aligned}$$

where H_0^u corresponds to no change in the rank r, H_1^u corresponds to a rank increase by 1, and $S > 0$ represents the additional power in mode $r+1$ under H_1^u.

In the context of SST, a basic rank increase test is provided by

$$l_N(k) \underset{H_0}{\overset{H_1}{\gtrless}} \phi^u \sigma^2 \tag{18}$$

where $\phi^u > 1$ is a user selected multiplicative threshold factor. We first derive an expression for the probability of rejecting H_0 when it is indeed in force. This probability, simply called the probability of false alarm (FA), can be defined as

$$P_{FA} = \int_{\phi^u\sigma^2}^{\infty} f_{l_N(k)}(x)dx \tag{19}$$

where $f_{l_N(k)}(x)$ denotes the probability density function (PDF) of $l_N(k)$ under H_0.

To evaluate P_{FA}, in (19), we first note that under the assumed dynamic model and hypothesis H_0^u, random variable $|y_{r+1}(k)|^2$ in (12) can be expressed as $\sigma^2 z_0(k)/2$ where $z_0(k)$ is chi-square with $2(N-r)$ degrees of freedom. From there, it can be verified that

$$P_{FA} = 1 - F(\phi^u;\alpha,2(N-r)) \tag{20}$$

Next, we investigate the detection time under H_1^u. Due to the rank increase at time $k=0$, the power associated to the $(r+1)$th eigenvector is now greater than the noise level. Specifically, for $k \geq 0$

$$|y_{r+1}(k)|^2 = \frac{\sigma^2}{2}[(1+SNR)z_1(k) + z_2(k)] \tag{21}$$

where $z_1(k) \sim \chi^2(2)$ and $z_2(k) \sim \chi^2(2(N-r-1))$ and where we introduce the *signal-to-noise ratio*

$$\text{SNR} = \frac{S}{\sigma^2} \tag{22}$$

In this work, a suitable measure of the detection time under H_1^u is defined as the time required by the expected value of $l_N(k)$ in (12) to raise from σ^2 at time $k=-1$ (prior to rank increase) to the threshold level $\phi^u\sigma^2$ of the rank increase test. Let

$$\mu_N(k) \triangleq E[l_N(k)] \tag{23}$$

Combining (21) and (12), the following recursion may be obtained for $k \geq 0$

$$\mu_N(k) = \alpha\mu_N(k-1) + \sigma^2(1-\alpha)\left(1 + \frac{SNR}{N-r}\right) \tag{24}$$

with initial condition $\mu_N(-1) = \sigma^2$. Provided $\phi^u < 1 + \text{SNR}/(N-r)$, $\mu_N(k)$ will reach the threshold level $\phi^u\sigma^2$ at time

$$T^u = \frac{\ln(1 - \frac{(N-r)(\phi^u-1)}{\text{SNR}})}{\ln\alpha} - 1 \tag{25}$$

One possible approach to optimize the performance of the rank increase test is to select the value of the forgetting factor α that minimizes the detection time T^u under a constraint of a fixed probability of false alarm P_{FA}. Note that for given values of $N-r$ and SNR, the detection time T^u is a function of α and ϕ^u. Also note that under a constraint of a fixed P_{FA} in (19), ϕ^u becomes a function of the α. Thus, we define

$$T_{min}^u = \arg\min_{\alpha} T^u(\alpha,\phi^u) \quad \text{s.t.} \quad P_{FA} = \text{cte} \tag{26}$$

A typical plot of T_{min}^u versus SNR for $N-r=5$, and $P_{FA} = 0.01$ is shown in Fig. 1. Plots of T^u (25) versus SNR for fixed values of α is also included.

3.4 Detection of a Rank Decrease

A rank decrease is modelled by the following behavior of the process eigenvalues: for all k, $\lambda_1 \geq \ldots \geq \lambda_{r-1} \geq \sigma^2 + S$ and $\lambda_{r+1} = \ldots = \lambda_N = \sigma^2$; for $k < 0$, $\lambda_r(k) = \sigma^2 + S$ while for $k \geq 0$, two possibilities:

$$\begin{aligned} H_0^d &: \lambda_r(k) = \sigma^2 + S \\ H_1^d &: \lambda_r(k) = \sigma^2 \end{aligned}$$

where H_0^d corresponds to no change in the rank r, H_1^d corresponds to a rank decrease by 1.

We consider the following test for rank decrease:

$$l_r(k) \underset{H_1}{\overset{H_0}{\gtrless}} \phi^d(S+\sigma^2) \tag{27}$$

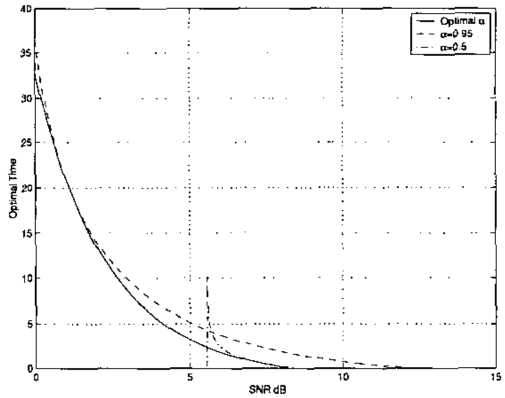

Fig. 1 Optimum detection time versus SNR for rank increase test.

where $\phi^d < 1$ is a selected multiplicative threshold factor. We define the probability of a false alarm (i.e. false rank change detection) as the probability of rejecting H_0^d when it is indeed in force. This probability can be defined as

$$P_{FA} = \int_{-\infty}^{\phi^d(S+\sigma^2)} f_{l_r(k)}(x)\,dx \tag{28}$$

where $f_{l_r(k)}(x)$ denotes the PDF of $l_r(k)$ under H_0^d. Proceeding as in the previous subsection, it can be shown that

$$P_{FA}^d = F(\phi^d; \alpha, 2) \tag{29}$$

where the function $F()$ is defined in (17).

Due to the rank decrease at time $k = 0$ under H_1^d, the average power of the (r)th signal component is now reduced to the noise floor. Specifically, for $k \geq 0$

$$|y_r(k)|^2 = \frac{\sigma^2}{2} z_3(k) \tag{30}$$

where $z_3(k) \sim \chi^2(2)$. We define the detection time under H_1^d as the time required by the expected value of $l_r(k)$ to decay from $S + \sigma^2$ at time $k = -1$ (prior to rank decrease) to the threshold level $\phi^d(S + \sigma^2)$ of the rank decrease test.

Proceeding as in Section 3.4, the following formula may be obtained for the detection time:

$$T^d = \frac{\ln(\phi_d(1 + \frac{1}{\text{SNR}}) - \frac{1}{\text{SNR}})}{\ln \alpha} - 1 \tag{31}$$

As in (26), we consider the minimization of the detection time T^d with respect to α under a constraint of fixed P_{FA}. Specifically, we define

$$T_{min}^d = \arg\min_{\alpha} T^d(\alpha, \phi^d) \quad \text{s.t.} \quad P_{FA} = \text{cte} \tag{32}$$

A typical plot of T_{min}^d versus SNR for $P_{FA} = 0.01$ is illustrated in Fig. 2. The Figure also includes plots of T^d (31) versus SNR for fixed values of α.

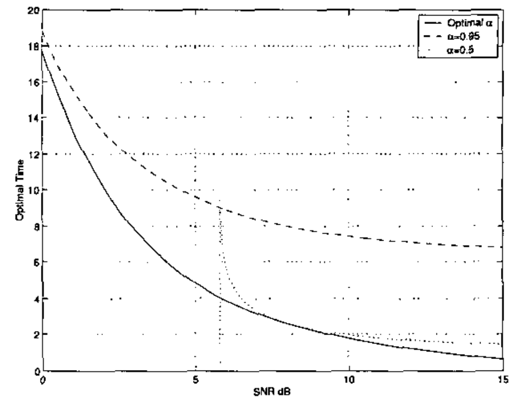

Fig. 2 Optimum detection time versus SNR for rank decrease test.

4. THE NEW ALGORITHM

Above theory points to a fundamental limitation of the KY algorithm. Indeed, the latter uses the same combination of threshold γ and forgetting factor α for both the rank increase and decrease tests. However, according to the theory in Section 3, different combinations of forgetting factors and thresholds are needed to achieve the optimum detection time in the rank increase and decrease tests. Thus, KY algorithm does not have the intrinsical ability to optimize the performance of both tests simultaneously. In fact, the situation gets worst at high SNR since the gap between the optimal values of the thresholds can be shown to increase with SNR.

To overcome this limitation, we need to modify the basic rank detection algorithm of KY to allow the use of different combinations of forgetting factors and thresholds for the rank increase and decrease tests. This can be achieved by using a combination of the tests in (18) and (27). Ideally, one would like to implement these tests using the optimal values of (α, ϕ^u) and (α, ϕ^d) prescribed by (26) and (32). However, this is not practically feasible since these optimal values depend on the SNR parameter in (22). Also, the exact relationships between the thresholds parameters ϕ^u, ϕ^d and α in practice differ from those derived in Section 3 on the basis of the simplified signal model (13)-(15).

The results in Fig. 1 suggest that the detection time for the rank increase test is not particularly sensitive to the value of α. Accordingly, for this test, we propose the use of a fixed value of α in (11)-(12), independent of the SNR parameter. This value of α is the same as that used by the underlying SST algorithm; typically, it will be close to 1. The value of ϕ^u is then adjusted to maintain the P_{FA} below a satisfactory level.

For the rank decrease test, it can be seen from Fig. 2 that the situation is different. Indeed, the detection time is more sensitive to the value of the forgetting factor α, and the use of a large (constant) value of α is not sufficient to get a detection time curve that is close to the optimal one. To this end, and particularly at high SNR, we need to use a much smaller value of α. For the rank decrease test, we therefore propose to use an additional eigenvalue estimates $l'_i(k)$, computed via exponential averaging as in (11), but using a smaller forgetting factor, say $\beta < \alpha$:

$$l'_i(k) = \beta l'_i(k-1) + (1-\beta)|y_i(k)|^2 \qquad (33)$$

As in the rank increase test, the value of ϕ_d is adjusted to maintain P_{FA} below a satisfactory level.

We can now formulate the proposed algorithm:

if $l'_r(k) < \gamma_d(k)$ then "rank deflation"
    incorporate $l'_r(k)$ into $l_N(k)$
    delete $\mathbf{v}_r(k)$ from $V_S(k)$
    $r \leftarrow r-1$
elseif $l_N(k) > \gamma_u(k)$ then "rank inflation"
    set $l_{r+1}(k) = l_N(k)$
    set $\mathbf{v}_{r+1}(k) = \mathbf{v}_N(k)$
    $r \leftarrow r+1$
else "rank unchanged"
$\gamma_d(k+1) = \phi^d l_r(k-t_d)$
$\gamma_u(k+1) = \phi^u l_N(k-t_u)$

where t_u and t_d are typical values of the detection time for rank increase and decrease. We note that with the above approach, it is not necessary to track an additional eigenvector as in the KY method. Only extra eigenvalues need to be computed.

5. EXPERIMENTS

Computer simulations have been conducted to perform the rank estimation for the subspace multiuser detection algorithm in the synchronous CDMA systems [5]. In synchronous CDMA systems, the rank of the signal subspace r is equal to the number of active users. We set the processing gain $N = 20$, which is equal to the dimension of the correlation matrix of the received signal, SNR=20dB, $\alpha = 0.95$, and $\beta = 0.5$. We change the number of active users in the system, both up and down, and apply both the KY and the proposed rank estimation algorithm to detect the change of the rank. The simulation results in Fig. 3 show that the proposed algorithm requires much less time iterations than KY to detect a rank decrease.

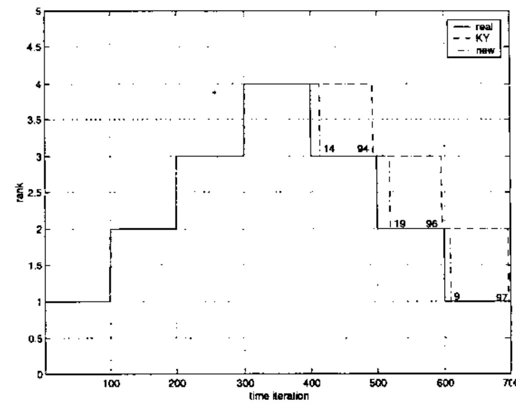

Fig. 3 Adaptive rank estimation for multi-user detection in synchronous CDMA system.

6. CONCLUSION

We have presented an improved adaptive rank detection algorithm for on-line estimation and tracking of the signal subspace dimension in applications of spherical subspace trackers. The proposed algorithm uses different adaptive thresholds for the rank increase and decrease tests as well as a special set of fast tracking eigenvalue estimates in the rank decrease test. Computer experiments in multi-user detection shown that with the proposed algorithm, the time required to detect a rank decrease is significantly reduced.

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