ESTIMATION OF HIGH-SPEED DATA RADIO TRANSMISSION LINE PARAMETERS

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Abstract

In this paper, we consider the distortion created by an unmatched transmission line system at the receiver of a high-speed data radio. The distortion can be modeled using a few interdependent parameters, which are to be estimated. A joint maximum likelihood (ML) estimator for the parameters is presented with the corresponding Cramér-Rao bound (CRB). A reduced complexity, approximate ML (AML) estimator is also developed. The results show that both ML and AML give reliable estimates, close to the CRB.

Keywords: *Estimation, Cable Reflections, Cramér-Rao Bound.*

1. INTRODUCTION

The presence of an impedance mismatch on the antenna-cable-radio connection of a wireless data communications system contributes to the degradation of the system's performance. In particular, this mismatch will cause reflections in the cable, as illustrated in Fig. 1. For high speed data radio receivers, these reflections correspond to intersymbol interference (ISI), effectively increasing the probability of making an error.

High reliability data radios therefore require manual tuning of the antenna-cable-radio connection, or require the presence of an automatic antenna tuning unit (ATU) to maximize system efficiency [1]. In some cases, manual tuning may not be possible and an ATU may be impractical to implement due to cost, lack of space, or other reasons. Even in the presence of an ATU or after manual tuning, a small mismatch may still be present and cause distortion. With the advance in digital signal processing (DSP) hardware, there is a genuine interest to develop a digital compensation system for the distortion created by the reflections on the cable. It is also of great interest to analyze the source of the distortion and measure its effects.

The study is performed under the context of a line-of-sight (LOS), fixed, high speed and high reliability data communications system with a long cable between the antenna and the radio transceiver. For example, the system under study could be a data communications relay station, a broadband system for remote voice, multimedia and Internet access, or a military communications system.

We propose to model the channel, defined here as the transfer function from the antenna to the receiver, using a finite set of parameters. The channel parameters are to be estimated from the received signal using the parametrized model and maximum likelihood techniques. The estimates can be used to design a compensation filter which is applied to the received signal prior to detection, or they can be used for measurement purposes. The joint maximum likelihood estimator is developed along with the Cramér-Rao bound. A reduced complexity, approximate ML estimator is also introduced. Results, comparing the MSE of the estimates for the AML and ML estimator to the CRB are presented. The results indicate that reliable estimates can be obtained from both the ML and AML estimator.

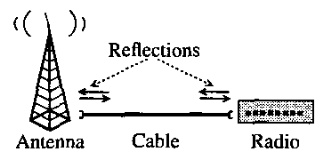

Fig. 1 Reflections from Impedance Mismatch

2. PROBLEM FORMULATION

The antenna-cable-radio connection may be represented using phasor notation by the transmission line model shown in Fig. 2, where V_S is the amplitude of the source voltage or antenna, Z_S is the antenna output impedance, γ is the cable's propagation constant, Z_0 is the cable's characteristic impedance, z is the distance from the antenna on the cable, L_c is the cable's length, Z_L is the load or receiver input impedance, and V_L is the receiver's input voltage.

Assuming an unmatched antenna-cable-radio connection, i.e. $Z_S \neq Z_0 \neq Z_L$, reflections occur at both ends of the transmission line and both forward (V^+) and backward (V^-) travelling waves are present. The voltage V_L measured at the load in steady-state is therefore given by the sum of all the travelling wave voltages measured at position $z = L_c$. Using properties of transmission lines reflections,

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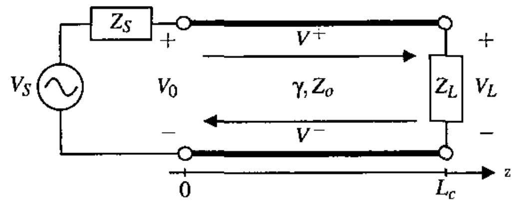

Fig. 2 Transmission Line Model

it can be shown that V_L can be expressed by

$$V_L = V_0 e^{-\gamma L_c} \frac{1+\Gamma_L}{1-\Gamma_L \Gamma_S e^{-\gamma 2L_c}}, \tag{1}$$

where Γ_S and Γ_L are the source and load reflection coefficients, respectively, and $V_0 = V_S Z_0/(Z_0+Z_S)$. Expression (1) is valid provided $|\Gamma_L \Gamma_S e^{-\gamma 2L_c}| < 1$, which is always the case by definition.

Using properties of the propagation constant γ, we can express (1) in the form of a frequency dependent transfer function. First, we let $\gamma L_c = \alpha L_c + j\omega\tau$, where α is the attenuation constant, $\tau = L_c/\mu$ is the time required for the wave to propagate from one end of the transmission line to the other, with μ being the corresponding propagation velocity. Both α and τ are assumed to be independent of the angular frequency $\omega = 2\pi f$ in the neighborhood of the carrier. We assume that the channel is affected by an automatic gain controller and a synchronization device at the receiver that both make slight estimation errors, leaving a small gain and synchronization offset represented by ϕ and ϵ respectively. The frequency dependent channel transfer function may then be given by

$$H(\omega) \triangleq \frac{V_L}{V_0} = \frac{\phi e^{-j\omega\epsilon}}{1-\psi e^{-j\omega 2\tau}}, \tag{2}$$

where $\psi = \Gamma_L \Gamma_S e^{-\alpha 2L_c}$ is a complex-valued constant, also assumed independent of ω in the carrier neighborhood.

We now develop the lowpass equivalent (baseband) representation of the communication system of interest. The lowpass received signal $r(t)$ at the input of the radio receiver can be expressed as

$$r(t) = \underbrace{s(t) * h(t)}_{u(t)} + n(t) \tag{3}$$

where $s(t)$, $h(t)$, and $n(t)$ represent the lowpass equivalent transmitted signal, channel and noise respectively, and $u(t)$ is the noiseless lowpass equivalent received signal. The lowpass equivalent transmitted signal, $s(t)$, consists of a sum of pulse shaped (possibly complex) symbols A_k:

$$s(t) = \sum_{k=-\infty}^{\infty} A_k p(t-kT), \tag{4}$$

where $p(t)$ is the pulse shaping function, assumed to be bandlimited to $|\omega| < W$. The sequence A_k represents the realization of a random process consisting of independent and identically distributed random variables taking values from the set $\{A_1, \ldots, A_Q\}$ where Q is the constellation size. We further assume that sequence A_k has ergodic properties in the mean and autocorrelation.

Using the time-domain lowpass equivalent channel impulse response, obtained using the inverse Fourier transform of (2) and (3), the received signal may be expressed as

$$r(t) = \phi \sum_{k=-\infty}^{\infty} \sum_{l=0}^{\infty} \psi^l A_k p(t-kT-2l\tau-\epsilon) + n(t), \tag{5}$$

where the lowpass noise, $n(t)$, is complex circular Gaussian [2] with zero mean and power spectral density N_o for $|\omega| < W$ and zero elsewhere.

3. ML ESTIMATOR

As indicated by (2), the channel transfer function depends on five parameters, namely ϕ, $\psi = |\psi| e^{j\measuredangle\psi}$, τ and ϵ. Using the results of the previous section, the probability model for the received signal is first derived, from which the joint ML estimator for the parameters is developed.

The joint probability density function (pdf) $f(\boldsymbol{x};\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the vector of parameters to be estimated, i.e.,

$$\boldsymbol{\theta} = [\phi, |\psi|, \measuredangle\psi, \tau, \epsilon]^T, \tag{6}$$

and $\boldsymbol{x}$ is the vector of data observations, represents a family of probability density functions parametrized by $\boldsymbol{\theta}$. Because we assume that $\boldsymbol{\theta}$ is unknown but fixed, the estimation problem belongs to the category of *point estimation*. As such, the maximum likelihood parameter is obtained by choosing among the family of pdfs the one that fits the "best" to the observed data. The parameters are then directly obtained from the selected probability density function.

The information available at the receiver for the estimation consists of the received signal in (5), over an observation period $T_o \gg T$:

$$r(t) = u(t) + n(t), \quad 0 \le t \le T_o. \tag{7}$$

To simplify, $r(t)$ is represented using a discrete (and ideally finite) set of related observations. Using the Fourier series representation of $r(t)$, we obtain

$$R_m \triangleq \frac{1}{T_o} \int_0^{T_o} r(t) e^{-j\omega_m t} dt, \tag{8}$$

where $\omega_m = \frac{2\pi m}{T_o}$, $m \in \mathbb{Z}$. The Fourier series representation is complete so it is possible to recover the original observation signal $r(t)$ from the set of R_m. In other words, R_m incorporates all the information contained in $r(t)$.

Since the noise term $n(t)$ is complex circular Gaussian, by linearity $r(t)$ and the set of R_m are also complex circular Gaussian random variables. Let the mean of the Fourier series coefficients be $U_m \triangleq E[R_m]$, then if $WT_o \gg 2\pi$, it can be shown that the Fourier series coefficients corresponding to different "frequencies" are uncorrelated (e.g. [3]), namely:

$$E[(R_m - U_m)(R_p - U_p)^*] \simeq \frac{N_o}{T_o}\delta[m-p] \quad (9)$$

$$E[(R_m - U_m)(R_p - U_p)] = 0. \quad (10)$$

Furthermore, since $p(t)$ is bandlimited to W, we have

$$U_m \simeq 0 \qquad \text{for } |l| > M, \quad M = \left\lceil \frac{WT_o}{2\pi} \right\rceil, \quad (11)$$

where $\lceil a \rceil$ is the largest integer less than or equal to a. Therefore, if we let $\boldsymbol{x} = [R_{-M}, \ldots, R_M]^T$ be the vector of data observations and $\boldsymbol{y} = [U_{-M}, \ldots, U_M]^T$ be the corresponding mean vector, then the probability density function $f(\boldsymbol{x}; \boldsymbol{\theta})$ can be expressed as

$$f(\boldsymbol{x};\boldsymbol{\theta}) = \frac{1}{C}\exp\left\{-(\boldsymbol{x}-\boldsymbol{y})^H \frac{T_o}{N_o}(\boldsymbol{x}-\boldsymbol{y})\right\}, \quad (12)$$

where $C = (\sqrt{\pi}N_o/T_o)^{2M+1}$.

To get the joint ML estimator, the logarithm of (12) is taken to obtain the log likelihood function $\ell(\boldsymbol{x};\boldsymbol{\theta})$:

$$\ell(\boldsymbol{x};\boldsymbol{\theta}) \triangleq -\log(C) - \frac{T_o}{N_o}(\boldsymbol{x}-\boldsymbol{y})^H(\boldsymbol{x}-\boldsymbol{y}). \quad (13)$$

Since C is a constant incorporating terms independent of the parameters, the term $\log(C)$ may be ignored. Therefore, the log likelihood equation becomes:

$$\ell(\boldsymbol{x};\boldsymbol{\theta}) = -\frac{T_o}{N_o}\sum_{m=-M}^{M}|R_m - U_m|^2 \quad (14)$$

The joint maximum likelihood estimate $\hat{\boldsymbol{\theta}}_{\text{ML}}$ is obtained by maximizing (14) with respect to the parameter $\boldsymbol{\theta}$, i.e.:

$$\hat{\boldsymbol{\theta}}_{\text{ML}} = \arg\max_{\boldsymbol{\theta}} \ell(\boldsymbol{x};\boldsymbol{\theta}) \quad (15)$$

The joint ML estimator can be implemented for example using a multi-dimensional search algorithm.

4. APPROXIMATE ML

The joint ML estimator presented above requires a high-dimensional search for the maximum of the log likelihood function in (14). This represents a computationaly intensive and possibly slow operation. The reduced complexity, approximate ML estimator that we propose is based on the hypothesis that some of the parameters are not as tightly coupled as others. In particular, we make the assumption that since ϵ and ϕ are only indirect consequences of the transmission line reflections, they may be estimated separately from ψ and τ. The estimation of the five parameters is thus performed through the iteration of two independent estimation procedures. From (14), we define the first cost function at iteration step number $i+1$ as

$$J_{a,i+1} = \sum_{m=-M}^{M} |R_m - \phi e^{-j\omega_m \epsilon} \underbrace{\frac{S_m}{1-\psi_i e^{-j\omega_m 2\tau_i}}}_{K_m(i)}|^2 \quad (16)$$

where ψ_i and τ_i are considered constant and S_m is the Fourier series coefficients of the pulse shaped transmitted sequence $s(t)$. Minimization of $J_{a,i+1}$ with respect to ϵ can be realized by maximizing the time-correlation of R_m with $K_m(i)$ and is therefore independent of the gain ϕ, which may be obtained directly as

$$\phi_{i+1} = \frac{\sum_{m=-M}^{M} \text{Re}\{R_m e^{j\omega_m \epsilon_{i+1}} K_m^*(i)\}}{\sum_{m=-M}^{M} |K_m(i)|^2}. \quad (17)$$

Once the estimates for ϕ_{i+1} and ϵ_{i+1} are available, they are used for the estimation of ψ_{i+1} and τ_{i+1}, through the minimization of the second cost function

$$J_{b,i+1} = \sum_{m=-M}^{M} \left| R_m - \frac{\phi_{i+1} e^{-j\omega_m \epsilon_{i+1}} S_m}{1-\psi e^{-j\omega_m 2\tau}} \right|^2, \quad (18)$$

with respect to ψ and τ, considering ϕ_{i+1} and ϵ_{i+1} constant. The two estimations steps are iterated until convergence or until a pre-determined maximum number of iterations is reached.

5. CRAMÉR-RAO BOUND

The Cramér-Rao bound gives a lower limit on the variance of any unbiased estimate, such that [4]:

$$\text{Var}[\hat{\theta}_i] \geq [\boldsymbol{J}^{-1}]_{ii}, \quad (19)$$

where $\boldsymbol{J}$ is the Fisher information matrix with entry (i,j)

$$J_{i,j} = -E\left[\frac{\partial^2 \ell(\boldsymbol{x};\boldsymbol{\theta})}{\partial\theta_j \partial\theta_i}\right]. \quad (20)$$

Through simple differentiation and using the fact that $E[R_m] = U_m$, we obtain the (i,j) entry of the Fisher information matrix as

$$J_{i,j} = \frac{2T_o}{N_o}\sum_{m=-M}^{M} \text{Re}\left[\left(\frac{\partial U_m}{\partial\theta_j}\right)\left(\frac{\partial U_m}{\partial\theta_i}\right)^*\right]. \quad (21)$$

It can be shown that for long observation time, i.e. $T_o \gg T$, the Fourier series coefficients of the received mean can be expressed approximately as

$$U_m \simeq \frac{1}{T_o} H(\omega_m) P(\omega_m) D(\omega_m), \quad (22)$$

where $D(\omega)$ is the discrete Fourier transform (DFT) of the sequence of transmitted symbols A_k associated to the observation interval and $P(\omega_m)$ represents the Fourier transform of the pulse shape function $p(t)$ evaluated at ω_m. Without loss of generality, let $T_o = NT$. Then $k \in \{0, 1, \ldots, N-1\}$, where N is the number of symbols in the observation interval. Using the ergodic property of the sequence of A_k, the magnitude square of $D(\omega_m)$ can be reduced to

$$|D(\omega_m)|^2 = \sum_{n=0}^{N-1}\sum_{l=0}^{N-1} A_n A_l^* e^{-j\omega_m(n-l)T} \simeq NP_A, \quad (23)$$

where P_A is the constellation average power. We also observe that only $H(\omega_m)$ is dependent on the parameters in (22). It is convenient to arrange the partial derivative of $H(\omega_m)$ with respect to each parameter in a vector such that

$$\boldsymbol{\nabla H} = \left[\frac{\partial H(\omega_m)}{\partial \theta_1}, \ldots, \frac{\partial H(\omega_m)}{\partial \theta_5}\right]^H, \quad (24)$$

where the subscript 1 to 5 corresponds to the parameter indices in (6). It is then easy to substitute (22) in (21) and use (23) and (24) to obtain the Fisher information matrix:

$$\boldsymbol{J} = \frac{2NP_A}{T_o N_o} \sum_{m=-M}^{M} |P(\omega_m)|^2 \mathrm{Re}\left[\boldsymbol{\nabla H}\ \boldsymbol{\nabla H}^H\right]. \quad (25)$$

6. RESULTS

The estimation algorithms were implemented and tested using computer simulations. For each experiment, a training sequence of $N = 1000$ QAM-32 symbols was transmitted through the channel, modeled as an IIR filter. The parameters were estimated from the received signal, sampled at half the symbol rate ($T/2$). The mean square error (MSE) of the estimates was then computed and averaged over two hundred experiments to obtain reliable statistics. The actual parameters were chosen to appropriately reflect a realistic physical situation. Since the length of the cable is usually approximately or exactly known, the propagation time τ is initialized to the proper value in the algorithms. The other values are all initialized to zero with the exception of the gain, ϕ, which is initialized to the value it would take if no reflection were present i.e., $\phi_0 = 1$.

A quick observation of Fig. 3 and Fig. 4 indicates that the ML and AML estimators give similar MSE, relatively close to the CRB for all cases. Indeed, the MSE of the AML estimator is always within approximately 0.3dB from that of the ML estimator.

It can be observed from Fig. 3 on the left graph that the average MSE for the AML estimators is approximately 5.9dB and 3.4dB from the CRB for $\measuredangle\psi$ and $|\psi|$ respectively. The time delay and sychronization offset MSE are closer to the CRB; 0.84dB for τ and -0.81dB for ϵ. The latter value, which is lower than the CRB, may be explained by simulation artifacts that could be coming from the optimization procedure. The MSE for ϕ shown on the right graph in Fig. 4 is approximately 5.1dB from the CRB, showing similar estimation performance to ψ.

7. CONCLUSION

In this paper, a new method for estimating transmission line parameters for high-speed data radio was proposed. The method may be used for measurements or in conjunction with a compensation filter to reduce the effects of impedance mismatch on the radio link performance. The ML and AML methods presented have a MSE close to the corresponding CRB.

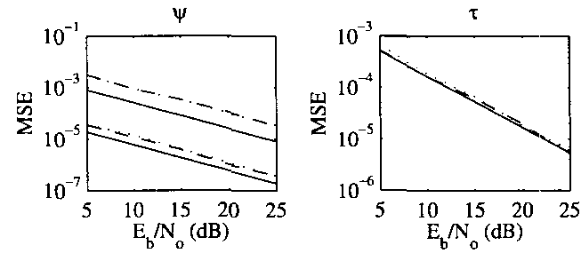

Fig. 3 MSE for ML (dashed line) and AML (dotted line) estimators with CRB (solid line). The left graph shows the results for $\measuredangle\psi$ (top curves) and $|\psi|$ (bottom curves). The right graph shows the corresponding curves for τ.

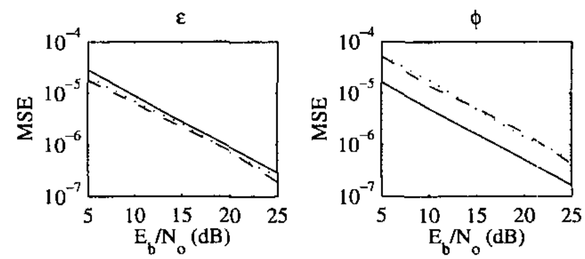

Fig. 4 MSE for ML and AML estimators with CRB; the left graph shows the results for ϵ and the right graph shows the corresponding curves for ϕ.

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