A REDUCED COMPLEXITY MULTI-RATE ECHO CANCELLER FOR DMT BASED DSL SYSTEMS

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Abstract

In the past years, a number of researchers have investigated the use of digital echo cancellation systems for transmission technologies based on discrete multi-tone (DMT) modulation such as used in asymmetrical digital subscriber lines (ADSL) and the likes. This paper reports on a novel set of techniques that reduces the computational complexity of the multi-rate echo canceller structure at the lower transmit rate side of the link, e.g. the remote terminal (RT) side in ADSL. In this case, the echo canceller located in the terminal processes the signals into the entire frequency bandwidth (higher rate). In fact, the echo channel estimated during the processing corresponds to the imperfect hybrid channel combined with the low pass filtering action of the analog-to-digital converter (ADC). In such a situation, we can exploit the special structure of the echo channel to reduce the computational complexity of the echo cancellation processing. Specifically, this is achieved by zeroing the high frequency coefficients of the echo channel estimate. The frequency representation of the echo allows the exploitation of certain properties of the inverse Fast Fourier Transform (IFFT) algorithm to further reduce the complexity.

Experimental simulations for an ADSL like configuration show a computational complexity reduction factor of as much as 67% without significantly affecting the residual echo level.

Keywords—*Echo; echo canceller; DMT; multi-rate; ADSL.*

1 Introduction

A telephone line is characterized by the twisted-wire-pair that links the customer's house to the central office. While the advantage of the DSL lies in the fact that every user has a dedicated link, one inconvenience is that the data transmitted to and from the customer is conveyed through the same unshielded wire. The separation of the upstream and downstream signals (coupling) is usually performed by a hardware circuit called a hybrid. Unfortunately, during the separation of the received and transmit signals, the hybrid introduces interference in the received signal. This interference is called "echo" because it consists of (several) replications of the transmit signal at much lower levels (-40dB and below). In the case where the transmit signal and receive signal occupy different frequency bands, the echo interference has little significance even though echo interference leakage exists. However, in the case where there is an overlap in the frequency bands occupied by the transmit and received signals, for instance the echo cancelling mode in ADSL, the echo interference becomes much more significant and has to be removed through a digital signal processing algorithm called an echo canceller.

A number of echo canceller structures have been studied in the literature [1–6]. Each of them can be separated into two parts: adaptation and emulation. The adaptation corresponds to the part where the echo channel estimate is adapted, while the emulation corresponds to the part where the echo interference signal is recreated and subtracted from the received signal. Echo canceller structures differ in the way the adaption and emulation are realized.

Basic echo cancellation used to be completely realized in time domain. Cioffi *et al.* [1] were one of the first to investigate specific echo cancellation for discrete multi-tone (DMT) modulation. The innovation brought by [1] consisted of executing the adaptation in the frequency domain in such a manner that it reduces computational complexity. Later on, Ho *et al.* [2] exploited completely the frequency-domain idea previously introduced and elaborated an EC that perform the emulation mostly in the frequency domain. Thus, the only time-domain processing left was to make the recreated echo signal appear periodic. This set of time-domain operations is referred to as cyclic echo synthesizer (CES). In term of complexity and regardless of the frame alignment, the structure proposed by [2] is very optimized.

The venue of the CES led Jones *et al.* [3] to investigate the optimal frame alignment between the received and transmit signals. Indeed, by choosing a specific frame alignment, the number of operations to be processed in the CES can be reduced by half when compared with zero frame alignment of [2]. Based on [3] and [2], Ysebaert *et al.* [6] took advantage of the frame alignment and created another echo canceller algorithm with the aim once again to reduce the total computational complexity on both transmitter sides. An interesting modification in this structure is the replacement of the IFFT/FFT pairs, previously required at the unaligned modem side, by a single FFT.

With respect to the multi-rate case, Ho *et al.* [2] were the first to propose two structures to mitigate the echo problem. One structure is for the low bandwidth side and another is for the high bandwidth side. Ysebaert *et al.* [6] reused the same principles in their proposed EC multi-rate version. This paper focusses on the multi-rate scheme of the low bandwidth side, typically being the remote terminal side in ADSL. It proposes simplifications made possible by the particular frequency response of the echo channel estimated. The paper also proposes a new initialization method that allows faster convergence of the echo channel estimates.

The organization of the paper is as follows. The next section, section II, describes the optimal echo canceller for symmetric and asymmetric rates in terms of mathematical equations. Afterward, section III presents the innovations brought by this paper. Finally, section IV discusses the computational complexity reduction these innovations allow in relation with the simulation results.

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2 Echo cancellation in DMT

2.1 Symmetrical rate structure

A DMT based DSL communication system necessarily contains a certain number of tones; for the purpose of our explanations we refer to this number as $N/2$. Thus the number of samples required to represent one DMT symbol is N. We model the echo interference signal as a linear convolution between the near-end transmit signal and the finite impulse response (FIR) of the echo channel. To translate the model into mathematical equations, we represent the time-domain samples of the near-end transmit symbol by u_j^i, where i is the symbol index and j the sample index. We can use these elements to form a Toeplitz matrix Γ^i composed of three different transmit symbols. The first row is defined by the vector $[u_{\Delta+1}^i \dots u_1^i u_N^i \dots u_{N-L+1}^i u_N^{i-1} \dots u_{\Delta+L+2}^{i-1}]$ and the first column by $[u_{\Delta+1}^i \dots u_N^i u_{N-L+1}^{i+1} \dots u_N^{i+1} \dots u_1^{i+1} \dots u_{\Delta-L}^{i+1}]$. In the previous matrix definition, L is the length of the prefix used in the system and Δ is the delay in samples between the far-end and near-end signals. Thus we can define the echo interference signal for symbol i by

$$y^i = \Gamma^i h. \tag{1}$$

In this equation h is a vector representing the echo channel; it combines together the imperfect separation of the near-end and far-end signal in the hybrid as well as the imperfection of the front-end filters. The length of the echo channel is $M < N$; however, to allow matrices multiplications, h is zero padded to length N. Given that the echo canceller algorithm produces echo channel FIR estimates, denoted by the vector w^i, we can define the frequency residual echo signal as

$$E^i = \mathcal{F}(y^i - \Gamma^i w^i) \tag{2}$$

where $\mathcal{F}$ represents the N-points DFT-matrix.

In [2], the authors introduce a NxN circulant matrix C with the first column equals to $[u_{\Delta+1}^i \dots u_N^i u_1^i \dots u_\Delta^i]$. Based on this matrix, authors in [6] built another more effective circulant matrix named $\tilde{C}$ where the first column is constructed by choosing the first Nb elements of the first row of C, concatenated with the $N - Nb$ first elements of the first column of C. More precisely the first column of $\tilde{C}$ equals

$$\begin{aligned}\tilde{C}_1 = [&u_{\Delta+1}^i \dots u_{N-N_b+\Delta}^i \; u_{N-N_b+L+\Delta+1}^{i-1} \dots u_N^{i-1} \\ &u_{N-L+1}^i \dots u_N^i \; u_1^i \dots u_\Delta^i].\end{aligned} \tag{3}$$

Using this matrix as a calculus artifact we obtain the following frequency residual echo signal

$$\begin{aligned}E^i &= \mathcal{F}(y^i - \Gamma^i w^i) \\ &= \mathcal{F}(y^i - (\Gamma^i - \tilde{C} + \tilde{C})w^i) \qquad (4) \\ &= \mathcal{F}(y^i - \tilde{\chi}^i w^i) - \mathrm{diag}(\tilde{U}^i)W^i \qquad (5)\end{aligned}$$

with $\tilde{\chi}^i = \Gamma^i - \tilde{C}$ and $\tilde{U}^i = \mathcal{F}\tilde{C}_1$. In the last equation, the time-domain operations consist in CES and the operations realized in the frequency-domain are performed by the frequency-domain echo canceller (FEC). The minimal computational complexity is achieved for a particular combination of Δ and N_b. For more information regarding the proper choice of Δ and N_b see [6].

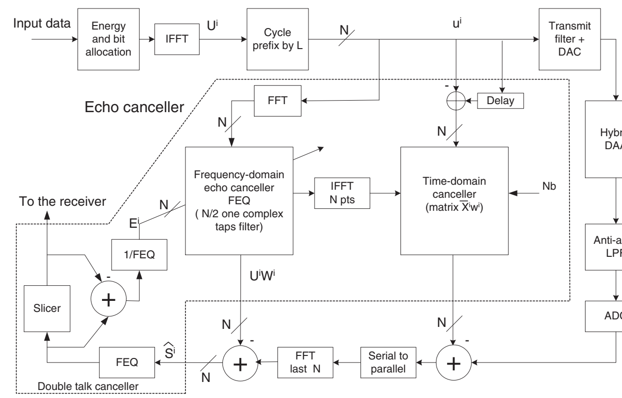

Figure 1: Symmetric-rate ADSL echo canceller at RT

The frequency echo channel estimates (W^{i+1}) are updated by LMS equation :

$$W^{i+1} = W^i + \mu \, \mathrm{diag}(\tilde{U}^i)^* E^i \tag{6}$$

where * denotes complex conjugation and μ the step size. Figure 1 illustrates the optimal symmetric rate echo canceller system. The update of the echo channel estimate, defined by the previous equation, as well as the formation of the echo signal estimate ($\tilde{U^i W^i}$)) are performed by the FEC. The time-domain canceller operates in the right middle box, it creates the signal corresponding to the CES ($\tilde{\chi}^i w^i$). The residual echo signal E^i is obtained by subtracting the slicer output from the "corrected" far-end signal $\hat{S}^i$(i.e. once the echo signal estimate has been removed). Here, FEQ means frequency equalizer; it is required in order to correct the attenuation of the far-end signal due to the propagation channel.

2.2 Multi-rate structure

The multi-rate aspect needs to be taken into account when designing DMT echo canceller. The first specifications in regards of the two multi-rate EC structures (one for each transmission side) have been established in [2] and had remained applicable for the improvement brought later on in [6]. In this paper we will focus on the lower bandwidth transmission side, i.e. the remote terminal side in ADSL. The main characteristic of such modem is that the transmit bandwidth size is k times smaller than the received bandwidth size. Thus, in order to have the same rate for the input signals of the previously defined echo canceller, the number of samples (in frequency-domain as well as in time-domain) from the transmit signal needs to match that of the receive signal. In [2,6], the authors not only solved the rate matching problem but also annihilate the effect of the imperfect digital to analog converter (DAC).

The processing involved in a DAC consists of interpolating the input signal and passing it into an analog filter. Obviously the filtering operation is not ideal and frequency leakage occurs

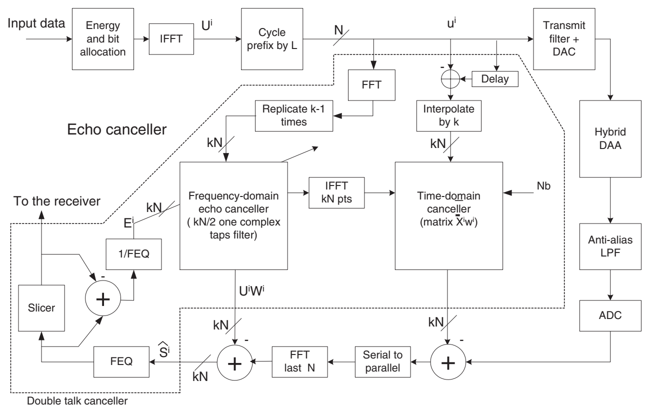

Figure 2: Multi-rate ADSL echo canceller at RT

in the resulting analog signal. Thus, it is possible to interpolate the input signal just as the DAC does, by replicating $k-1$ times the frequency component of the transmit signal, and by inserting $k-1$ zeros between successive time-domain samples. By doing so, the echo channel estimated by the EC corresponds in fact to the imperfect hybrid channel combined with the low-pass filter channel of the DAC. The structure of the EC is illustrated in Figure 2. In this system, there are N samples/symbol in the transmit signals therefore, the received signal contains kN samples/symbol.

3 New structure

The aim of this paper is to benefit from the fact that the echo channel estimate comprehends the low pass FIR channel used in the DAC. Indeed, while in [2,6], the same importance was given to all frequency components of the echo channel, the idea here is to ignore the effect of the high frequency components. The motivation is that the leakage coming from the DAC low pass filtering action is negligible at high frequencies. We begin by presenting a new method for the initialization of the frequency echo channel taps.

3.1 Initialization

It is possible to accelerate the convergence of the echo canceller by replacing the first LMS iteration (equation (6)) by another adaptation equation. The idea is to consider the convolution of the echo channel with the transmit signal as if it were circular. By doing so, it is possible to write and easily solve the following MMSE equation. Because the adaptation equation is applied only one time at the beginning, we will omit including symbol indexes.

$$\min |S-\hat{S}|^2 = \min |S-(V-\mathrm{diag}(U)W)|^2. \qquad (7)$$

Here S and $\hat{S}$ are vectors containing the frequency samples of, respectively, the far-end transmit symbol, and the estimated far-end transmit symbol just before entering into the slicer. V is the far-end transmit symbol including the echo interference. U is the frequency samples of the transmit symbol on the near-end side. One should note that in the aim of making the previous equation clearer, thermal noise was not taken into account because it does not affect the derivation. Therefore, solving the linear equation, we get the first iteration value of the echo channel estimated taps

$$W = \mathrm{diag}(U)^{-1}(V-S) \simeq \mathrm{diag}(U)^{-1}E. \qquad (8)$$

Here we assume that the received far-end signal V has sufficiently small interference so that the slicer decodes the proper transmit signal and thus allows to state $V-S=E$, where E is the same as defined in the previous section. The computation of $\mathrm{diag}(U)^{-1}$ is very simple; it corresponds to the element by element inverse of matrix U put on a diagonal. Therefore, applying the new equation in the model takes only four multiply operations per frequency coefficient as compared to six for one conventional least mean square (LMS) adaptation iteration (equation 6). This new adaptation equation is applicable for symmetric rate DSL as well as for multi-rate DSL. The performance of this algorithm depends on the closeness of the approximation of the linear convolution by a circular one. Therefore the fact that DMT modulation includes a prefix tends to favor the algorithm. For the same reason, the algorithm will perform better for shorter echo channel impulse response.

3.2 Frequency echo canceller simplifications

As mentioned previously, the echo channel estimated by the FEC consists in the low pass filter FIR of the near-end DAC convolved with the hybrid channel FIR. Due to the fact that convolution in time corresponds to multiplication in frequency, the magnitude frequency response of the echo channel follows that of the DAC low pass filter. Therefore, given that the frequency magnitude components of echo channel decay above the cutoff frequency, then once the components have reached a sufficiently low value, it is possible to approximate the remaining frequency magnitude components to zero.

Instead of adapting the whole $kN/2$ frequency taps, the idea that we propose is to adapt solely the first s frequency taps and approximate to zero the remaining taps. The starting frequency sample to begin the zeroing has to be higher than the frequency bandwidth of the near-end signal (which corresponds to the $\frac{N}{2}^{th}$ taps) and has to take into account the leakage implied by the DAC filter. Indeed, depending on the filter performance, more or less leakage occurs. Big leakage results in higher value of s.

Applying such algorithms in a noise-free environment would reduce the system performance. However, since it is not the case in real environment, mainly because of the near-end crosstalk (NEXT) interference, as long as the residual echo level is below the actual interference plus noise level, the algorithm has no effect on the system performance. Consequently, the possibility of fixing the high frequency taps to zero, depends on the possible residual echo level achievable, which depends on the NEXT interference level.

3.3 Modified IFFT

One of the most costly operations involved in echo canceller regards the IFFT of echo channel estimate. In [7], Wu *et al.*

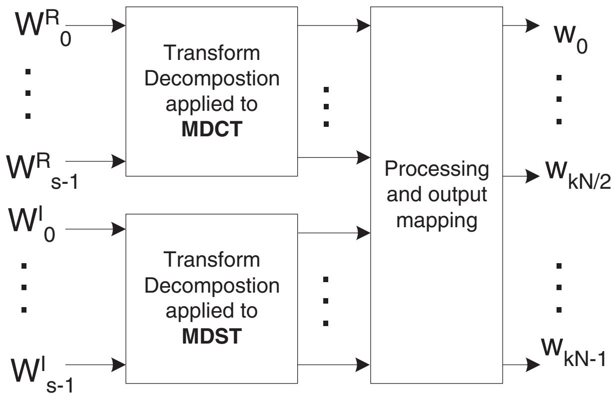

Figure 3: IFFT for the non-zero taps

have addressed this issue by developing a low computational complexity IFFT algorithm especially designed for DMT communication. The benefit comes from the fact that the IFFT output is real, which confers a special geometry structure to the signal frequency samples. More precisely for a frequency symbol vector X containing kN samples, $X_n = X^*_{kN-n}$ for $n = [1, 2, \ldots, kN/2-1]$. X_0 and $X_{kN/2}$ are not included in the previous equation but needs to be real. The algorithm proposed in [7] only needs the first half of the input and uses a modified discrete cosine transform (MDCT) as well as a modified discrete sinus transform (MDST).

The fact of zeroing the frequency taps can reduce the computational complexity of the IFFT. The idea is to exploit the low number of non-zero frequency samples. In [8], Sorensen *et al.* developed a method, named transform decomposition, that benefit from this particularity. The special form of the frequency echo channel estimate allow to merge some of the IFFT innovations described in [8] and [7].

The input vector is defined by $W = [W_0 \ldots W_{s-1}]$. To make the equation clearer, we will omit specifying the symbol index in the following equations. Due to the conjugate-symmetric property of the vector, it is possible to define the IFFT by

$$w_m = 2\left[\sum_{n=0}^{s-1} W_n^R \cos\left(\frac{2\pi nm}{kN}\right) + \sum_{n=0}^{s-1} W_n^I \sin\left(\frac{2\pi nm}{kN}\right)\right] \tag{9}$$

$$= 2[MDCT(m) + MDST(m)] \tag{10}$$

where W^R and W^I represent respectively the real and imaginary part of W, and $m = \{0, 1, \ldots, kN-1\}$. Furthermore, by using the symmetrical and anti-symmetrical properties, the computational complexity is getting reduced through

$$MDCT(m) = MCDT(kN-m) \tag{11}$$

$$MDST(m) = -MDST(kN-m) \tag{12}$$

Figure 3 illustrates the global IFFT scheme. Defining $C^i_{kN} = \cos(\frac{2\pi i}{kN})$ and similarly $S^i_{kN} = \sin(\frac{2\pi i}{kN})$, it is possible to write

$$MDCT(m) = \sum_{n=0}^{s-1} W_n^R C_{kN}^{nm}. \tag{13}$$

Using the transform decomposition method, we can introduce the variables P and Q such that $kN/2 = PQ$ and find the following new indices:

$$n = Pn_1 + n_2 \qquad \begin{array}{l} n_1 = 0, 1, ..., Q-1 \\ n_2 = 0, 1, ..., P-1 \end{array} \tag{14}$$

and

$$m = m_1 + Qm_2 \qquad \begin{array}{l} m_1 = 0, 1, ..., Q-1 \\ m_2 = 0, 1, ..., P-1. \end{array} \tag{15}$$

Now on, we can insert these new indices into (13) and we obtain:

$$MDCT(m_1, m_2) = \sum_{n_2=0}^{P-1}\sum_{n_1=0}^{Q-1} W^R_{Pn_1+n_2} C_{kN}^{(Pn_1+n_2)(m_1+Qm_2)}. \tag{16}$$

Using trigonometrical identities, we can simplify the above equation to

$$MDCT(m_1, m_2) = \sum_{n_2=0}^{P-1} W^R_{n_2} C_{kN}^{n_2m_1} C_{2P}^{n_2m_2} - \sum_{n_2=0}^{P-1} W^R_{n_2} S_{kN}^{n_2m_1} S_{2P}^{n_2m_2}. \tag{17}$$

Similarly for the MDST, the same set of operations can be pursued to get

$$MDST(m_1, m_2) = \sum_{n_2=0}^{P-1} W^I_{n_2} S_{kN}^{n_2m_1} C_{2P}^{n_2m_2} + \sum_{n_2=0}^{P-1} W^I_{n_2} C_{kN}^{n_2m_1} S_{2P}^{n_2m_2}. \tag{18}$$

We can then sum up together the MDCT and the MDST and we obtain

$$w(m_1, m_2) = 2[MDCT(m_1, m_2) + MDST(m_1, m_2)] \tag{19}$$

$$= 2[\sum_{n_2=0}^{P-1} \left(W^R_{n_2} C_{kN}^{n_2m_1} + W^I_{n_2} S_{kN}^{n_2m_1}\right) C_{2P}^{n_2m_2} + \sum_{n_2=0}^{P-1} \left(W^I_{n_2} C_{kN}^{n_2m_1} - W^R_{n_2} S_{kN}^{n_2m_1}\right) S_{2P}^{n_2m_2}]. \tag{20}$$

By analyzing the previous equation for a fixed m_1 value, we can deduce that it corresponds to a P-points MDCT added of a P-points MDST. Therefore, given that $m_1 = 0, 1, ..., Q-1$, it follows that we have Q length-P MDCT added with Q length-P MDST. The operations regarding the new IFFT scheme are resumed in Figure 4. In this scheme $w_{kn/2}$ is a special case and has to be computed by $\sum_{n=0}^{kN/2-1} \left(W_n^R \cos(\frac{n\pi}{2}) + W_n^I \sin(\frac{n\pi}{2})\right)$. The only requirement for this scheme is that $W_0 = W_{kN/2} = 0$.

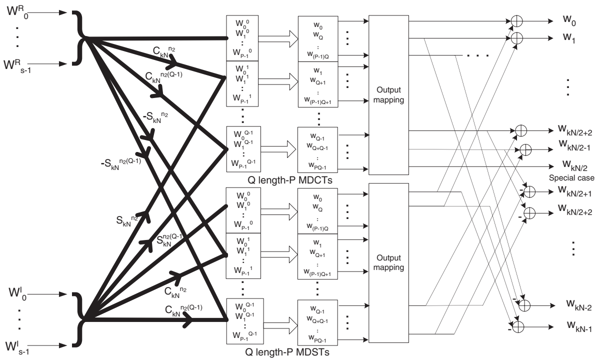

Figure 4: Transform decomposition applied to a MDCT

TABLE I
COMPUTATIONAL COMPLEXITY

	Scheme I	Scheme II
Emul.	$\frac{M^2}{4k} + 2kN + \delta$	$\frac{M^2}{4k} + 4s + \delta$
Adapt.	$5kN + \zeta$	$10s + \zeta$
δ	$N(log_2 \frac{N}{2} + 2)$	
ζ	$kN(log_2 \frac{kN}{2} + 2)$	$\frac{kN}{2} log_2 \frac{kN}{2} - kN + 2$ or $2Q(\frac{P}{2} log_2 P - P + 1) + 4s(Q-1)$

4 Complexity and simulation results

The aim of the echo cancellation papers in the literature is either to improve convergence speed and/or to reduce computational complexity. Table I shows the complexity equations, in term of real multiplies for two RT schemes; scheme I is the algorithm presented by [6], and scheme II is our proposed scheme when taps are zeroed. δ and ζ represent respectively the computational complexity of a FFT and IFFT.

As it can be noticed, scheme II has a reduction of the computational complexity for both the emulation and adaptation processes. In scheme II, depending on the ratio of s over kN (length of the FFT), the optimized IFFT algorithm is either the one described by [7] or the one presented in this paper. As for scheme I, the IFFT calculation is based on [9].

The impact of zeroing taps on the system performance is assessed through an ADSL like simulator in which 32 tones are used in the upstream and 256 tones in the downstream. The frequency bandwidth of a tone is 4.3125 KHz. The power of the transmit signal is -40dBm/Hz. The NEXT interference signal is calculated as if 10 ADSL interferers were present. The zeroing of the taps begins at $s = 64$, i.e. the length of two sub-bands, where a sub-band corresponds to the upstream signal frequency bandwidth. The echo channel is modelled as dictated in [10] and the filter representing the ADC is modelled as a fifth order Butterworth. To reduce the complexity of the simulator, the modulation scheme used is 4-QAM and the propagation channel is unitary.

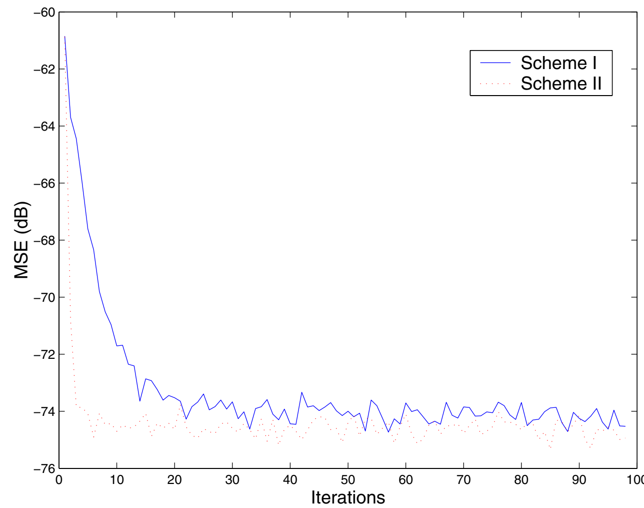

Figure 5: Comparison of MSE

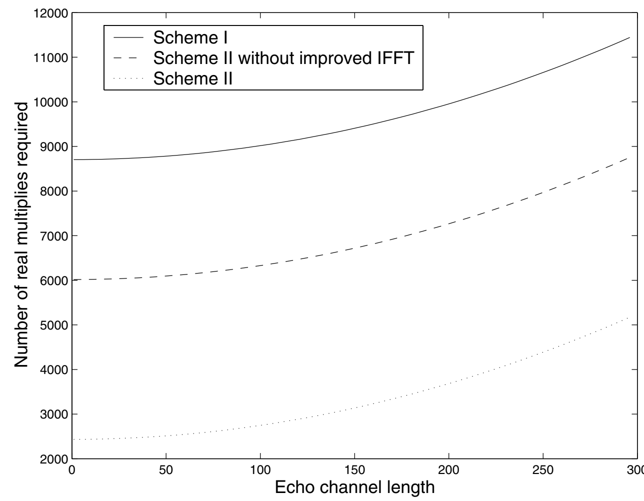

Figure 6: Comparison of complexity in term of multiply operations

Figure 5 compares the MSE performance of the zeroing tap case versus usual one. Once convergence has been reached, no performance difference can be observed. Moreover, the quick initialization algorithm allows the convergence to be reached in less than 5 iterations as opposed to 20 iterations for the standard LMS, which represents 75% fewer iterations. In term of computational complexity gain, Figure 6 illustrates the improvement brought by our proposed scheme; first by the simple fact of avoiding the calculation of high frequency taps and, secondly, by the addition of the IFFT simplifications (in this particular case, [7] represent the best IFFT scheme). For the ADSL like configuration mentioned above, 270 taps are used to represent the echo channel, thus 67% fewer multiplies are required for our proposed scheme.

5 conclusion

In this paper, a modified structure for asymmetrical-rate echo cancellation at the RT is presented. The structure exploits the fact that the high frequency components of the estimated echo are very small. The method consist of avoiding the calculation of high frequency taps by fixing these values to zero. Further, the new frequency representation of the echo allows a reduction of the computational complexity in the IFFT involved in the adaptation process of the EC. In an ADSL like configuration system, simulations have proven to reduce as much as 67% the number of multiply operations when compared with [6]. Furthermore, the paper also demonstrates a new initialization algorithm to improve the convergence speed of the EC. It has been shown that 15 iterations of usual LMS algorithm can be saved without affecting the computational complexity.

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