Optimal Power Allocation Based on Success Probability of SIC Detection in MWRC PNC

Hao Li
Dept. of Elec. and Comp. Eng.
McGill university, Montreal, Canada
hao.li3@mail.mcgill.ca

Xiao-Wen Chang
School of Comp. Sci.
McGill Univeristy, Montreal, CA
xiao-wen.chang@mcgill.ca

Benoit Champagne
Dept. of Elec. and Comp. Eng.
McGill university, Montreal, CA
benoit.champagne@mcgill.ca

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***Abstract*—In this paper, we propose a novel power allocation scheme for physical-layer network coding (PNC) in uplink multi-way relay channels (MWRC). The power allocation is formulated as a constrained optimization problem under the transmitting power constraint of user terminals, aiming at maximizing the success probability of the successive interference cancellation (SIC) detection at the relay. Optimizing over such a metric maximizes the probability of correctly detecting all user signals, which is critical to the network code generation at the relay. Specifically, we first develop a generalized closed-form success probability of the SIC detection on signals with pulse-amplitude modulation (PAM) at the relay. A constraint optimization is formulated over this probability subject to the power constraints of user terminals. We implement an evolutionary particle swarm optimization (PSO) algorithm to solve the problem whose cost function is complicated and not necessarily concave. The numerical results show that the proposed power allocation method can improve the quality of network code extraction at the relay.**

I. Introductions

Physical layer network coding (PNC), first proposed in [1], is a throughput improvement scheme compared to the conventional network coding (NC) [2]. Since then, a large volume of research on PNC has been mainly conducted in two-way relay channels (TWRC) [3]–[5]. To generalize and further extend the study on PNC, the use of PNC in multi-way relay channels (MWRC), where multiple users share information through a single relay, has gradually received attention [6]–[8]. In an MRWC PNC system among N users, the relay mixes N pieces of received user signals and generates $N-1$ network codes. These codes are usually designed to be strongly correlated with each other like a chain [8], whose robustness highly depends on the detection accuracy of all received user signals at the relay. For this reason, successfully detecting every piece of user signal from the superimposed signals at the relay will significantly improve the efficacy of network code generation. Hence, it is critical to devise mechanisms that ensure the detection accuracy at the relay for PNC in MWRC.

Successive interference cancellation (SIC) [9], which mitigates the interfering effects by discriminating superimposed signals based on their relative power levels, is a widely adopted detection scheme for multi-user communications. Existing studies on the efficiency of the SIC mainly emphasize improving the sum rate of the system, where the aim is to enhance the average detection accuracy of each user signal. For instance, a relay selection strategy for an NC scheme is proposed in [10], where the goal is to choose the best relay that preserves the maximized signal-to-interference-plus-noise ratio (SINR). In [11], a beamforming scheme for a superposition code of PNC is designed to achieve an acceptable symbol error rate (SER) in the SIC detection. Many works exploiting the SIC detection for non-orthogonal multiple access (NOMA) systems, such as [12]–[15], also address their respective problems from the perspective of the sum-rate maximization.

Unfortunately, a limited number of works explicitly focus on improving the accuracy of the entire chain of correlated signals during the SIC process, as is critical for PNC in MWRC. In this regard, the work in [16] analyzes the closed-form word error rate (WER) for the SIC decoders, which characterizes the success probability of detecting all signals in the successive process. Thus, it provides a valuable metric for evaluating the quality of the entire chain of detected signals. However, this work mainly emphasizes the theoretical analysis and does not involve further consideration on utilizing the metric for practical system design.

Inspired by the work of [16], we propose a novel power allocation for the SIC detection on signals with pulse-amplitude modulation (PAM) at the relay for PNC in MWRC. To be specific, we first extend the work in [16] and develop a generalized closed-form success probability of the SIC detection on signals with PAM signaling. We then formulate a constrained optimization problem, where the aim is to maximize the success probability of the SIC detection at the relay under the transmitting power constraint of user terminals. Optimizing over this metric maximizes the probability of correctly detecting all signals from the superimposed signals, which improves the efficacy of the network code generation at the relay. To solve this optimization problem where the cost function is complicated and not necessarily concave, we implement an evolutionary particle swarm optimization (PSO) algorithm. The simulation results verify the derivation of the success probability and demonstrate the effectiveness of the proposed power allocation in improving the relay's ability to extract network codes from the superimposed signals.

The paper is organized as follows: In Section II, we in-

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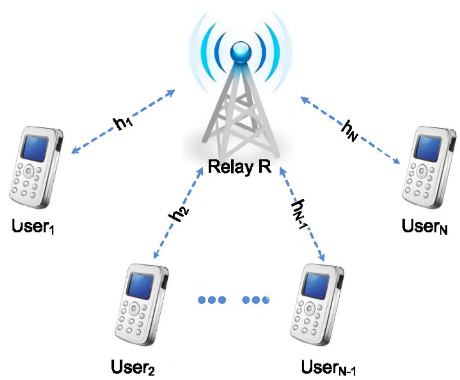

Fig. 1. Illustration of the PNC in MWRC.

troduce the system model of the PNC in MWRC. Section III proposes the power allocation scheme, including the derivation of the general closed-form success probability, the formulation of the optimization problem, and the solution using the PSO algorithm. In Section IV, we provide simulation results to demonstrate the effectiveness of the proposed methods. Finally, section V concludes the paper.

II. System model

As illustrated in Fig. 1, we consider a half-duplex multiway relay network where N users share information with each other through a common relay R. User terminals are equipped with a single antenna while the relay is equipped with $K < N$ antennas. We assume that there is no direct link among users, i.e., information exchange between two users needs to go through the relay. We assume all radio transmissions are over narrow-bands, i.e., frequency flat, slow fading channels. For simplicity, we also assume perfect channel estimation and time synchronization are available for any node in the network.

The superimposed signals received at the relay can be given as:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{A}\mathbf{s} + \tilde{\mathbf{n}}, \tag{1}$$

where $\mathbf{s} \in \mathbb{R}^{N\times 1}$ is the user signal vector whose i-th entry s_i is an M_i-PAM signal that is independently and uniformly distributed over a real set $\mathcal{B}_i = \{b_i^{(1)}, \dots, b_i^{(M_i)}\}$ with $b_i^{(1)} < \cdots < b_i^{(M_i)} \in \mathbb{R}$, $\mathbf{y} \in \mathbb{C}^{K\times 1}$ is a received signal vector, $\tilde{\mathbf{H}} = [\tilde{\mathbf{h}}_1, \dots, \tilde{\mathbf{h}}_N] \in \mathbb{C}^{K\times N}$ is the channel matrix, $\mathbf{A} = \mathrm{diag}(\sqrt{P_1}, \dots, \sqrt{P_N})$ with P_i being the power allocated to s_i, and $\tilde{\mathbf{n}} \in \mathbb{C}^K$ is the noise vector with each $\tilde{n}_i$ being independently distributed following $\mathcal{CN}(0, \tilde{\sigma}^2)$. For simplicity, we assume the distance between any two consecutive elements in each $\mathcal{B}_i$ is constant and denote it by $2d_i$, where

$$d_i = \frac{b_i^{(M_i)} - b_i^{(1)}}{2(M_i - 1)},$$

Since $\mathbf{s}$ has real-valued constellations, the estimation solely depends on the real domain. To simplify the analysis, we can transform the system in to an equivalent real-valued system model, i.e.:

$$\mathbf{y} = \mathbf{H}\mathbf{A}\mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = \Re(\tilde{\mathbf{y}})$, $\mathbf{H} = \Re(\tilde{\mathbf{H}})$, and $\mathbf{n} = \Re(\tilde{\mathbf{n}})$.

III. The proposed method

In this section, we first present the derivation of a generalized closed-form success probability of the SIC detection on the PAM signaling. We then formulate a constrained optimization over this metric subject to the transmitting power constraint of user terminals and implement a particle swarm optimization algorithm for the solution.

A. Success Probability of the SIC Detection

Assuming that the successive process goes from user index N to 1. For the i^{th} iteration, an estimator s_i^{sd} of s_i can be obtained after the removal of the previously detected signals s_j^{sd}, $j = i+1, \dots, N$, from $\mathbf{y}$. Suppose that s_j^{sd} has been obtained for $j = i+1, \dots, N$ and we define:

$$\mathbf{y}^{(i)} = \mathbf{y} - \sum_{j=i+1}^{N} \mathbf{w}_j s_j^{sd}. \tag{3}$$

where $\mathbf{W} = \mathbf{H}\mathbf{A} = [\mathbf{w}_1, \dots, \mathbf{w}_N] \in \mathbb{R}^{K\times N}$, $\mathbf{w}_i = \sqrt{P_i}\mathbf{h}_i$. Then we solve:

$$\min_{s_i \in \mathcal{B}_i} \|\mathbf{y}^{(i)} - \mathbf{w}_i s_i\|_2. \tag{4}$$

The solution s_i^{sd} can then be obtained by:

$$c_i = \frac{\mathbf{w}_i^H \mathbf{y}^{(i)}}{\|\mathbf{w}_i\|_2^2}, \qquad s_i^{sd} = \lfloor c_i \rceil_{\mathcal{B}_i}. \tag{5}$$

where $\lfloor c_i \rceil_{\mathcal{B}_i}$ denotes the nearest element to c_i in $\mathcal{B}_i$. From (5), (3) and (2), we obtain:

$$c_i = \frac{\mathbf{w}_i^H \mathbf{y}^{(i)}}{\|\mathbf{w}_i\|_2^2} = \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j s_j}{\|\mathbf{w}_i\|_2^2} + s_i + \sum_{j=i+1}^{N} \frac{\mathbf{w}_i^H \mathbf{w}_j (s_j - s_j^{sd})}{\|\mathbf{w}_i\|_2^2} + \frac{\mathbf{w}_i^H \mathbf{n}}{\|\mathbf{w}_i\|_2^2}. \tag{6}$$

When previous signals are successfully detected, i.e., $\mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}$, from (6) we obtain:

$$c_i = \frac{\mathbf{w}_i^H \mathbf{y}^{(i)}}{\|\mathbf{w}_i\|_2^2} = \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2^2} s_j + s_i + \frac{\mathbf{w}_i^H \mathbf{n}}{\|\mathbf{w}_i\|_2^2}, \tag{7}$$

where $\mathbf{w}_i^H \mathbf{n}$ follows a normal distribution with the mean and the variance as:

$$\mathrm{E}[\mathbf{w}_i^H \mathbf{n}] = \mathbf{w}^H \mathrm{E}[\mathbf{n}] = 0, \mathrm{Var}[\mathbf{w}_i^H \mathbf{n}] = \mathbf{w}_i^H \mathrm{Cov}[\mathbf{n}] \mathbf{w}_i = \sigma^2 \|\mathbf{w}_i\|_2^2,$$

where $\sigma = \frac{\sqrt{2}}{2}\tilde{\sigma}$. Thus, based on (7), we have:

$$\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 (c_i - s_i) - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j \right) \sim \mathcal{N}(0, 1), \tag{8}$$

Our goal is to derive a formula for $\Pr(\mathbf{s}^{sd} = \mathbf{s})$. Note that by the chain rule for conditional probability,

$$\Pr(\mathbf{s}^{sd} = \mathbf{s}) = \prod_{i=1}^{N} \Pr(s_i^{sd} = s_i | \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}). \tag{9}$$

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Since events $(s_i = b_i^{(1)})$, $(b_i^{(1)} < s_i < b_i^{(M_i)})$, and $(s_i = b_i^{(M_i)})$ are mutually exclusive,

$$\begin{aligned}
&\Pr(s_i^{sd} = s_i \,|\, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}) \\
&= \underbrace{\Pr(s_i = b_i^{(1)}, c_i \le b_i^{(1)} + d_i \,|\, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N})}_{P_{i,l}} \\
&+ \underbrace{\Pr(b_i^{(1)} < s_i < b_i^{(M_i)}, s_i - d_i < c_i < s_i + d_i \,|\, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N})}_{P_{i,m}} \\
&+ \underbrace{\Pr(s_i = b_i^{(M_i)}, c_i \ge b_i^{(M_i)} - d_i \,|\, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N})}_{P_{i,u}}.
\end{aligned} \tag{10}$$

In the derivation, we need to use the error function: $\text{erf}(\zeta) = \frac{2}{\sqrt{\pi}} \int_0^{\zeta} \exp(-t^2)dt$. Given l and u with $l \le u$, if $x \sim \mathcal{N}(0,1)$, then

$$\Pr(x \le l) = 1/2\left(1 + \text{erf}\left(l/\sqrt{2}\right)\right), \tag{11}$$

and similarly $\Pr(x \ge u) = 1/2\left(1 - \text{erf}\left(u/\sqrt{2}\right)\right)$, $\Pr(l \le x \le u) = 1/2\left(\text{erf}\left(u/\sqrt{2}\right) - \text{erf}\left(l/\sqrt{2}\right)\right)$.

In addition, for notational convenience, we label $b_t^{(1)}, \ldots, b_t^{(M_t)}$ in $\mathcal{B}_t$ by $0, 1, \ldots, M_t - 1$, respectively. Specifically, we define the bijection $\beta_t : \{b_t^{(1)}, \ldots, b_t^{(M_t)}\} \to \{0, \ldots, M_t - 1\}$. Let $\mathbf{s}_{1:i}^{(k_i)} = [s_1^{(k_i)}, \ldots, s_i^{(k_i)}]^T$ be the k_i-th possible instance of $\mathbf{s}_{1:i}$, where the index k_i is defined by $k_i = 1 + \sum_{t=1}^{i} \left(\beta_t(s_t) \prod_{j=t+1}^{i} M_j\right)$. For example, given $\mathcal{B}_1 = \{-1, 1\}, \mathcal{B}_2 = \{-3, -2, 0\}, \mathcal{B}_3 = \{2, 3, 4, 6, 7\}, \mathcal{B}_4 = \{-1, 0, 1, 2\}$, $\mathbf{s}_{1:4}^{(99)} = [-1, -2, 7, 1]^T$ represents the 99^{th} instance of $\mathbf{s}_{1:4}$ where k_i is computed as: $k_i = 1 + 1 \times (3 \times 5 \times 4) + 1 \times (5 \times 4) + 4 \times (4) + 2 \times (1) = 99$. Note that $k_i = 1, 2, \ldots, \prod_{t=1}^{i} M_t$ and $\mathbf{s}_{1:i}$ has a total of $\mathcal{M}_i = \prod_{t=1}^{i} M_t$ instances. Since s_i is independently uniformly distributed over $\mathcal{B}_i$ for $i = 1, \ldots, N$,

$$\Pr(\mathbf{s}_{1:i} = \mathbf{s}_{1:i}^{(k_i)}) = \frac{1}{\mathcal{M}_i}. \tag{12}$$

a) Derivation of $P_{i,l}$: According to Bayes's theorem, we have:

$$\begin{aligned}
P_{i,l} &= \Pr(s_i = b_i^{(1)}) \Pr(c_i \le b_i^{(1)} + d_i \,|\, s_i = b_i^{(1)}, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}) \\
&= \Pr(s_i = b_i^{(1)}) \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} \Pr\left(\mathbf{s}_{1:i-1} = \mathbf{s}_{1:i-1}^{(k_{i-1})}\right) \\
&\quad \Pr\left(c_i \le b_i^{(1)} + d_i \,|\, s_i = b_i^{(1)}, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}, \mathbf{s}_{1:i-1} = \mathbf{s}_{1:i-1}^{(k_{i-1})}\right)
\end{aligned} \tag{13}$$

We plug (12) into (13) and obtain:

$$\begin{aligned}
P_{i,l} &= \frac{1}{M_i} \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} \frac{1}{\mathcal{M}_{i-1}} \Pr\left(c_i \le b_i^{(1)} + d_i \,|\, s_i = b_i^{(1)}, \right. \\
&\quad \left. \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}, \mathbf{s}_{1:i-1} = \mathbf{s}_{1:i-1}^{(k_{i-1})}\right) && \text{(14a)} \\
&= \frac{1}{\mathcal{M}_i} \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} \Pr\left(c_i \le b_i^{(1)} + d_i \,|\, s_i = b_i^{(1)}, \right. \\
&\quad \left. \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}, \mathbf{s}_{1:i-1} = \mathbf{s}_{1:i-1}^{(k_{i-1})}\right) && \text{(14b)}
\end{aligned}$$

Now we find a formula for the probability in (14b). Note that when $s_i = b_i^{(1)}$ and $\mathbf{s}_{1:i-1} = \mathbf{s}_{1:i-1}^{(k_{i-1})}$, the inequality $c_i \le b_i^{(1)} + d_i$ is equivalent to

$$\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2(c_i - s_i) - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right) \le \frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right).$$

Then, by (8) and (11) we have:

$$\begin{aligned}
&\Pr\left(c_i \le b_i^{(1)} + d_i \,|\, s_i = b_i^{(1)}, \mathbf{s}_{i+1:N}^{sd} = \mathbf{s}_{i+1:N}, \mathbf{s}_{1:i-1} = \mathbf{s}_{1:i-1}^{(k_{i-1})}\right) \\
&= \frac{1}{2}\left(1 + \text{erf}\left(\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right).
\end{aligned} \tag{15}$$

Therefore, from (14b) and (15), we obtain:

$$P_{i,l} = \frac{1}{2\mathcal{M}_i} \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} \left\{1 + \text{erf}\left(\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right\}. \tag{16}$$

b) Derivation of $P_{i,m}$ and $P_{i,u}$: Similarly to the derivation of (16), we have:

$$\begin{aligned}
P_{i,m} = \frac{M_i - 2}{\mathcal{M}_i} \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} &\left\{\text{erf}\left(\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right. \\
&\left. - \text{erf}\left(\frac{1}{\sigma}\left(-\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right\}.
\end{aligned} \tag{17}$$

$$P_{i,u} = \frac{1}{2\mathcal{M}_i} \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} \left\{1 - \text{erf}\left(\frac{1}{\sigma}\left(-\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right\}. \tag{18}$$

Eventually, we plug (16), (17), and (18) into (10) and then into (9). The success probability $\Pr(\mathbf{s}^{sd} = \mathbf{s})$ is thus given as:

$$\begin{aligned}
\Pr(\mathbf{s}^{sd} = \mathbf{s}) &= \prod_{i=1}^{N} \{P_{i,l} + P_{i,m} + P_{i,u}\} \\
&= \prod_{i=1}^{N} \left\{\frac{1}{M_i} + \frac{M_i - 1}{2\mathcal{M}_i} \sum_{k_{i-1}=1}^{\mathcal{M}_{i-1}} \right. \\
&\quad \left[\text{erf}\left(\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 d_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right. \\
&\quad \left.\left. + \text{erf}\left(\frac{1}{\sigma}\left(\|\mathbf{w}_i\|_2 d_i + \sum_{j=1}^{i-1} \frac{\mathbf{w}_i^H \mathbf{w}_j}{\|\mathbf{w}_i\|_2} s_j^{(k_{i-1})}\right)\right)\right]\right\}
\end{aligned} \tag{19}$$

B. Problem Formulation

We formulate an optimization problem based on (19) subject to the transmitting power constraint P_T of user terminals to maximize the success probability of SIC detection at the relay side, i.e.:

$$\max_{P_1, \cdots, P_N} \Pr(\mathbf{s}^{sd} = \mathbf{s}) \tag{20a}$$

$$\text{s.t.} : 0 \le P_j \le P_T, \text{ for j=1,..., N.} \tag{20b}$$

However, the function in (19) is not necessarily concave with respect to P_j, and the corresponding proof is difficult

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to obtain due to the complication of its form. We will use a numerical computing method to solve such a problem in (20).

C. Particle Swarm Optimization Approach

The particle swarm optimization (PSO) algorithm starts with random initializations of a swarm of individuals, called particles, within the problem feasible region [17]. Then each particle iteratively approaches better and better approximations to the optimal solution with moving directions and step length that are coordinated by the entire swarm's motion. A detailed process is given in Algorithm 1.

a) Velocity and position: At iteration $t \in \mathbb{N}$, each particle in a swarm of size S is characterized by its position vector $\mathbf{X}_m^t = [P_{m1}^t, \ldots, P_{mK}^t]^T \in \mathbb{R}_+^K$ and a velocity vector $\mathbf{V}_m^t = [v_{m1}^t, \ldots, v_{mK}^t]^T$, where $m \in \{1, \ldots, S\}$ is the particle index, P_{mj}^t is the particle m's current solution to power P_j, and v_{mj}^t is the jth velocity component, $j \in \{1, \ldots, K\}$. Each particle adjusts its trajectory towards its own previous best position, called O_{best}, and towards a global best position attained by any member within the swarm, called $\mathcal{G}_{best}$. O_{best} and $\mathcal{G}_{best}$ are determined by evaluating the cost function $f(\mathbf{X}_m^t) \equiv \Pr(\mathbf{s}^{sd} = \mathbf{s})$ in (20a) during the particle's motion.

b) Penalized cost function: In order to confine the particles' motion within the feasible region, we incorporate a penalty function to the cost function f, i.e.:

$$F(\mathbf{X}_m^t) = f(\mathbf{X}_m^t) - \Omega \max\{0, P_{m1}^t - P_T, \ldots, P_{mN}^t - P_T, -\mathbf{X}_m^t\}, \tag{21}$$

where Ω is a penalty factor with a large positive value. Once a particle motion violates the constraints, $F(\mathbf{X}_m^t)$ deteriorates dramatically to a small value. The result of this motion will thus be discarded.

c) Motion updates: Velocity $\mathbf{V}_m^{t+1}$ directs the particle to the next new position. Its component on dimension j is given as:

$$v_{mj}^{t+1} = \omega^t v_{mj}^t + c_1 r_1^t (O_{best,mj} - P_{mj}^t) + c_2 r_2^t (\mathcal{G}_{best,mj} - P_{mj}^t), \tag{22}$$

where ω^t is an inertia weight, c_1 and c_2 are the constant cognitive and social parameters respectively, and r_1^t and r_2^t are the randomly generated numbers. The new position on dimension j is accordingly given as:

$$P_{mj}^{t+1} = P_{mj}^t + v_{mj}^{t+1}, \tag{23}$$

d) Convergence and termination: The algorithm eventually comes to a stop when the motion of swarm stalls. This occurs when the largest change in the objective value for the swarm, i.e.: $\max\{\Delta_m = |F(\mathbf{X}_m^t) - F(\mathcal{F}_m^{t-1})|, m = 1, \ldots, S\}$, is less than a certain small value ϵ. In addition, based on our experience, the algorithm can always solve problem (20) within a certain number of iterations for a specific swarm size. Thus, we set a maximum number of iterations T_{max} as an additional stopping criterion for the algorithm. The eventual $\mathcal{G}_{best}$ is thus considered as the solution to the power allocation problem in (20).

Algorithm 1 PSO algorithm to solve problem (20)

Step 1: Input swarm size S; number of max iterations T_{max}; penalized cost function $F(\cdot)$; stopping criteria ϵ.
Step 2: Initialize parameters c_1, c_2, r_1^0, r_2^0.
Step 3:: For each particle $m = 1$ to S, initialize random particle positions $\mathbf{X}_m^0$ in the feasible region and velocity $\mathbf{V}_m^0 = \mathbf{0}$. Set particle best known position $O_{best,m} = X_m^0$ and valuate each particle's cost $F(\mathbf{X}_m^0)$.
Step 4: Initialize swarm's best known position $\mathcal{G}_{best}$, where $F(\mathcal{G}_{best}) = \max\{F(\mathbf{X}_m^0)|m = 1, \ldots, S\}$.
Step 5: Initialize t=0.
Step 6: $t = t + 1$.
Step 7: Update the velocity of particles according to (22) and the position of particles according to (23).
Step 8: Evaluate $F(\mathbf{X}_m^t)$ for $m = 1, \ldots, S$ and determine $O_{best,m}$, where $F(O_{best,m}) = \max\{F(\mathbf{X}_m^i)|i = 1, \ldots, t\}$.
Step 9: Update $\mathcal{G}_{best}$, where $F(\mathcal{G}_{best})= \max\{F(O_{best,m})|m = 1, \ldots, S\}$.
Step 10: Randomize parameter r_1^t and r_2^t.
Step 11: If $t \leq T_{max}$ or $\max\{\Delta_m = |F(X_m^t) - F(X_m^{t-1})|, m = 1, \ldots, S\} > \epsilon$, return to **Step 6**. Otherwise, stop the iteration and output $\mathcal{G}_{best}$.

IV. Simulation Results

In this section, numerical results are provided to demonstrate the performance of the proposed success-probability-based power allocation scheme for SIC detection. We assume the transmitting power constraint of the MWRC PNC is normalized to $P_T = 1$ for different system scales specified in the following experiments. We assume the various radio links to be Rayleigh fading, i.e., the entries of the channel matrix $\tilde{\mathbf{H}}$ are modeled as independent complex circular Gaussian random variables with zero mean and unit variance. The noise variance at receiving antennas is adjusted accordingly to obtain the desired SNR levels.

We first validate the derivation of (19) by comparing the theoretical analysis with the numerical results from Monte Carlo experiments in Fig. 2. The validation is conducted under the system consisting of $N = 4$ user terminals and $K = 3$ relay antennas. In order to have a straightforward comparison and eliminate any potential distraction from optimization process, we simply allocate equal power to the transmitting user terminals with 3 groups of different PAM modulations, i.e., 2-PAM $\mathbf{s} \in \{-1, +1\}$, 4-PAM $\mathbf{s} \in \{-3, -1, +1, +3\}$, and 6-PAM $\mathbf{s} \in \{-5, -3, -1, +1, +3, +5\}$. Based on the results of three groups of comparison, we observe that the numerical results are in accordance with their respective theoretical values.

We then demonstrate the effect of the power allocation strategy on the rate of correctly generated network code chains. In this experiment, we compare the proposed success-probability-based strategy with a conventional sum-rate-based strategy that maximizes user signals' minimal SINR. The equal power allocation strategy is also provided as a reference. The sequential coding strategy as in [8] is adopted at the

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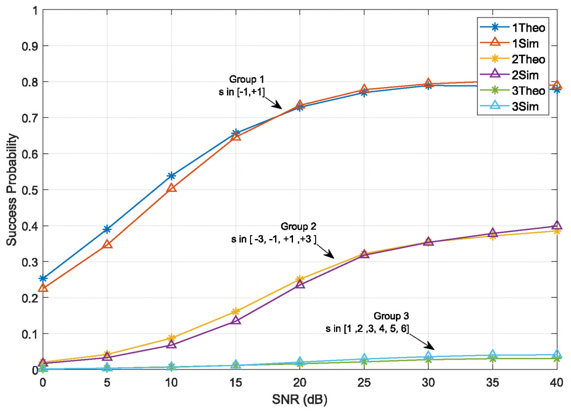

Fig. 2. Comparison between theoretical analysis and simulation results.

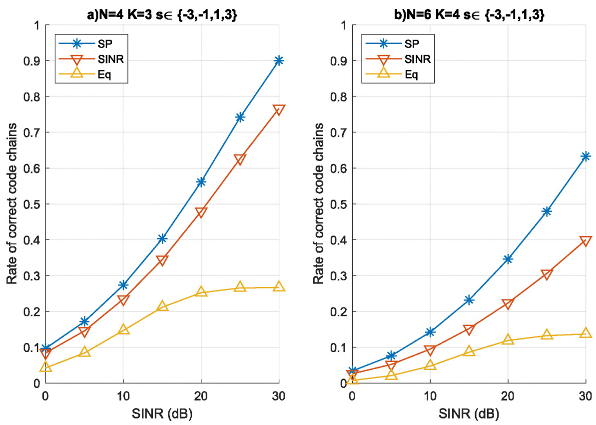

Fig. 3. The effect of power allocation strategy on the rate of correctly generated network code chains with $N = 4$, $K = 3$ and $N = 6$, $K = 4$.

relay for the code chain generation. We transmit $10,000$ signals with 4-PAM signaling from each user and compare the generated code chains to their expected results at the relay. In Fig. 3 a) and b), we present the comparison among the proposed strategy (indicated by 'SP'), the conventional strategy ('SINR'), and the equal power allocation strategy ('Eq') with two different system scales, i.e., $N = 4$, $K = 3$ and $N = 6$, $K = 4$ respectively. From the result, we can see that the proposed method effectively improves the rate of correct code chains with an advantage of around $5dB$ over the conventional sum-rate based method. Hence, the result demonstrates the effectiveness of the proposed in improving the relay's ability to extract network codes from the superimposed signals.

V. Conclusion

In this paper, we proposed a novel power allocation scheme for PNC in uplink MWRC. The power allocation was formulated as a constrained optimization problem under the transmitting power constraint of user terminals, aiming at maximizing the success probability of the SIC detection at the relay. Optimizing over such a metric maximizes the probability of correctly detecting all user signals, which is critical to the network code generation at the relay. Specifically, we first developed a generalized closed-form success probability of the SIC detection on signals with PAM at the relay. We then formulated a constraint optimization over this probability subject to the power constraints of user terminals. We implemented an evolutionary PSO algorithm to solve the problem whose cost function is complicated and not necessarily concave. The numerical results verified the success probability derivation and demonstrated the effectiveness of the proposed power allocation scheme in improving the relay's ability to extract network codes from the superimposed signals.

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