A NOVEL APPROACH TO ARRAY STEERING VECTOR ESTIMATE IMPROVEMENT

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ABSTRACT

In this paper, we present a method for estimating the signal sources steering vector using an arbitrary planar array with omnidirectional elements. The proposed method improves the initial estimation of the signal steering vector in two steps. In the first step of this algorithm we minimize of the distance between the steering vector and the signal subspace. The second step improves the estimation of the first step using a defined cost function which is based on a structural criterion for signal steering vector. Simulation results show the capability of the proposed signal steering vector estimate improvement.

1 INTRODUCTION

Adaptive beamforming is used to extract a desired signal immersed in noise and interference. Several algorithms in array processing are based on maximizing the array output signal to interference and noise ratio (SINR). In such cases, one should know the array steering response in the direction of the desired source. Many reports have indicated that a small error in the steering vector can severely degrade the performance of the adaptive beamformer. The error can induce a pattern with nulls in the direction of the desired signal and accentuate sidelobes in the direction of interference [1, 2, 3].

Steering vectors are directly related to the direction of arrival (DOA) of the signals and can be used to either estimate the DOAs or –if the DOAs are known a priori– to calibrate the array [4]. Array steering vector error occurs because of the DOA estimation error and/or array geometry imperfection. Mechanical stroke, objects in the near field, and temperature all affect calibration precision.

Several robust array processing techniques have been proposed in the literature. However, the methods exert additional constraints on the weight vector, resulting in a degradation on the produced pattern. Since in practice, maintaining a precisely calibrated array is difficult, steering vector estimation (SVE) is of interest [5, 6].

Here, we introduce a new method for improving the SVE based on the eigen–decomposition of array output correlation matrix. The proposed method minimizes the distance between the true and estimated steering vectors in two steps. The first step projects the erroneous steering vector to the so–called signal subspace, and the second step rotates and scales the result of the first step to the direction of true steering vectors on the signal subspace.

2 SIGNAL MODEL AND EIGENVECTORS PROPERTIES

Consider a scenario with an L–omnidirectional element planar array with arbitrary geometry receiving the wavefronts of p narrowband point sources. The baseband complex representation of the array output at each snapshot is given by

$$\mathbf{x}(k) = \mathbf{A}(k)\mathbf{s}(k) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{A} = [\mathbf{a}_1 \ \cdots \ \mathbf{a}_p]$ is the $L \times p$ steering matrix with $\mathbf{a}_i$ (for $1 \leq i \leq p$) being the steering vector related to the ith signal source, $\mathbf{s}$ is a $p \times 1$ signal vector, and $\mathbf{n}$ is an $L \times 1$ vector of spatially and temporally white noise with variance σ^2. Non–white case can also be handled with pre-whitening. We assume uncorrelated signals. Using (1), the autocorrelation matrix of the array output is

$$\mathbf{R}(k) = E\{\mathbf{x}(k)\mathbf{x}^H(k)\} = \mathbf{A}(k)\mathbf{\Gamma}\mathbf{A}^H(k) + \sigma^2\mathbf{I}, \tag{2}$$

$$\mathbf{\Gamma} = \mathrm{diag}(\gamma_1, \cdots, \gamma_p). \tag{3}$$

Diagonal elements of $\mathbf{\Gamma}$ represent the received power of the signal sources.

Given N snapshots, the autocorrelation matrix can be estimated as

$$\hat{\mathbf{R}}(k) = \frac{1}{N}\sum_{i=0}^{N-1} \mathbf{x}(k-i)\mathbf{x}^H(k-i). \tag{4}$$

The eigendecomposition of $\mathbf{R}$ has the following form

$$\mathbf{R} = \sum_{i=1}^{L} \lambda_i \mathbf{q}_i \mathbf{q}_i^H = \mathbf{Q}_s \Lambda_s \mathbf{Q}_s^H + \mathbf{Q}_n \Lambda_n \mathbf{Q}_n^H, \tag{5}$$

where $\mathbf{q}_i$s are the eigenvectors and $\lambda_1 \geq \cdots \lambda_p > \lambda_{p+1} = \cdots = \lambda_L = \sigma$ are the associated eigenvalues, the columns of $\mathbf{Q}_s = [\mathbf{q}_1, \cdots, \mathbf{q}_p]$ span the signal subspace and the columns of $\mathbf{Q}_n = [\mathbf{q}_{p+1}, \cdots, \mathbf{q}_L]$ span the noise subspace. The column span of the array manifold matrix $\mathbf{A}$ is also the signal subspace. In other word,

$$\mathbf{A} = \mathbf{Q}_s \mathbf{K}, \tag{6}$$

where $\mathbf{K} \in \mathcal{C}^{p\times p}$ with $\mathcal{C}^{p\times p}$ being the $p \times p$ complex vector space.

3 SIGNAL STEERING VECTOR IMPROVEMENT

Taking $\hat{\mathbf{A}}_0$ as an erroneous estimation of array manifold matrix, we propose the following two–step algorithm to improve the estimated signal steering vectors.

3.1 Step–1: Projection to Signal Subspace

Knowing that columns of $\mathbf{A}$ span the signal subspace, we solve

$$\min_{\hat{\mathbf{A}}_1} ||\hat{\mathbf{A}}_1 - \hat{\mathbf{A}}_0||. \tag{7}$$
$$\text{S.T. } \hat{\mathbf{A}}_1 \text{ in signal subspace.}$$

Matrix $\hat{\mathbf{A}}_1$, being in the signal subspace, is equivalent to $\hat{\mathbf{A}}_1 = \mathbf{Q}_s\mathbf{K}_1$ where $\mathbf{K}_1 \in \mathcal{C}^{p\times p}$. Thus, minimization (7) is equivalent to

$$\min_{\mathbf{K}_1} ||\mathbf{Q}_s\mathbf{K}_1 - \hat{\mathbf{A}}_0||. \tag{8}$$

This minimization has the well–known least squares solution

$$\mathbf{K}_1 = \mathbf{Q}_s^{\dagger}\hat{\mathbf{A}}_0 = (\mathbf{Q}_s^H\mathbf{Q}_s)^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}}_0 = \mathbf{Q}_s^H\hat{\mathbf{A}}_0, \tag{9}$$

where $\mathbf{Q}_s^{\dagger}$ represents the pseudo–inverse of $\mathbf{Q}_s$ (note that $\mathbf{Q}_s^H\mathbf{Q}_s = \mathbf{I}$). Thus

$$\hat{\mathbf{A}}_1 = \mathbf{Q}_s\mathbf{Q}_s^H\hat{\mathbf{A}}_0. \tag{10}$$

In fact, $\hat{\mathbf{A}}_1$ is the projection of $\hat{\mathbf{A}}_0$ onto the signal subspace, and thereby the noise subspace components of the erroneous p steering vectors $\tilde{\mathbf{a}}_i$ are removed.

3.2 Step–2: Rotation in Signal Subspace

In [7] we prove

$$\mathbf{A}^H\mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}\mathbf{Q}_s^H\mathbf{A} = \mathbf{\Gamma}^{-1}. \tag{11}$$

Let us define the cost function

$$\begin{aligned}\Psi(\hat{\mathbf{A}}) &= ||\hat{\mathbf{A}}^H\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}} - \mathbf{\Gamma}^{-1}||^2 \\ &= \operatorname{tr}(\hat{\mathbf{A}}^H\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}} - \mathbf{\Gamma}^{-1})^2 \end{aligned} \tag{12}$$

where $\mathbf{\Lambda}_{s,NF} \stackrel{\Delta}{=} \mathbf{\Lambda}_s - \sigma^2\mathbf{I}$ is the noise–free version of $\mathbf{\Lambda}_s$, $\hat{\mathbf{A}}$ is an erroneous estimate of the steering vector, and tr(.) represents the trace of matrix.

Let the estimate of the steering vector at Step 2 be represented by $\hat{\mathbf{A}}_2$. Taking into account that $\hat{\mathbf{A}}_2$ belongs to the signal subspace, we can write

$$\hat{\mathbf{A}}_2 = \mathbf{Q}_s\mathbf{K}_2. \tag{13}$$

Thus, for the second step we seek for

$$\mathbf{K}_2 = \mathbf{K}_1 + \Delta\mathbf{K}. \tag{14}$$

Within the above framework, we express $\mathbf{K}_2$ using the gradient of (12) so as to reduce (12), as

$$\mathbf{K}_2 = \mathbf{K}_1 - \mu \left.\frac{\partial\Psi(\mathbf{Q}_s\mathbf{K})}{\partial\mathbf{K}}\right|_{\mathbf{K}=\mathbf{K}_1}. \tag{15}$$

We can show that

$$\left.\frac{\partial\Psi(\mathbf{Q}_s\mathbf{K})}{\partial\mathbf{K}}\right|_{\mathbf{K}=\mathbf{K}_1} = \mathbf{\Lambda}_{s,NF}^{-1}\mathbf{K}_1\Delta\mathbf{\Gamma}^{-1}, \tag{16}$$

where

$$\Delta\mathbf{\Gamma}^{-1} = \hat{\mathbf{A}}_1^H\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}}_1 - \mathbf{\Gamma}^{-1}. \tag{17}$$

Now, find $\Delta\mathbf{K}$ such that

$$\Psi(\mathbf{Q}_s\mathbf{K}_2) = 0. \tag{18}$$

It can be shown that the optimum μ for the above constraint is

$$\mu = \frac{\Psi(\hat{\mathbf{A}}_1)}{4\operatorname{tr}[\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\mathbf{A}_1\Delta\mathbf{\Gamma}^{-1}\mathbf{A}_1^H\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}]}, \tag{19}$$

Thus

$$\mathbf{K}_2 = \mathbf{K}_1 - \mu\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\mathbf{A}_1\Delta\mathbf{\Gamma}^{-1}, \tag{20}$$

and $\hat{\mathbf{A}}_2$ can be expressed as

$$\hat{\mathbf{A}}_2 = \hat{\mathbf{A}}_1 - \frac{\Psi(\hat{\mathbf{A}}_1)\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}}_1\Delta\mathbf{\Gamma}^{-1}}{4\operatorname{tr}[\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}}_1\Delta\mathbf{\Gamma}^{-1}\hat{\mathbf{A}}_1^H\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}]}. \tag{21}$$

We use the following formula to estimate $\Delta\mathbf{\Gamma}^{-1}$

$$\Delta\hat{\mathbf{\Gamma}}^{-1} = (\hat{\mathbf{A}}_1^H\mathbf{Q}_s\mathbf{\Lambda}_{s,NF}^{-1}\mathbf{Q}_s^H\hat{\mathbf{A}}_1) \odot \tilde{\mathbf{I}}^{-1}, \tag{22}$$

where $\tilde{\mathbf{I}}$ is a $p\times p$ matrix with zero diagonal elements and the rest of elements equal to 1, and $\odot$ is the Hadamard element–by–element multiplication.

In Fig. 1 a simplified model of the proposed method is shown.

4 SIMULATION EXPERIMENTS

For simulations, we have assumed an 8 element uniform circular array (UCA) with the interelement spacing $\lambda/2$ where λ is the wavelength of the carrier frequency of the received signal.

To simulate uncalibrated sensors, we use a randomly displaced array. The array elements are distributed uniformly around the nominal position within the maximum distance r. Fig. 2 shows the position of the array elements at ten different trials using $r = 0.08\lambda$ (array elements are shown by circles).

For simulations, two autocorrelation matrix are computed, the first one is the exact $\mathbf{R}$ computed based on (2), and the second one is the estimated autocorrelation matrix using (4) with $N = 20$. The white noise power is estimated from

$$\hat{\sigma}^2 = \frac{1}{L-p} \sum_{i=p+1}^{L} \hat{\lambda}_i, \qquad (23)$$

where $\{\hat{\lambda}_i\}$ is the set of eigenvalues of the sample correlation matrix arranged in non-increasing order. White noise was assumed at the output of all the array elements and three signals were received by the array with the parameter summarized in Table 1.

We define the following formula

$$D_i = \left|\left| \frac{\mathbf{A}}{||\mathbf{A}||} - \frac{\hat{\mathbf{A}}_i}{||\hat{\mathbf{A}}_i||} \right|\right|^2 \quad \text{for} \quad i = 0,\ 1,\ 2 \qquad (24)$$

which we call the normalized proximity factor. Table 2 and Table 3 show the normalized proximity factors (D_i for $i = 0,\ 1,\ 2$) using different values of random displacement of the array elements (r) respectively for true and estimated correlation matrix. A total of 4000 runs were performed for each simulation.

As an application for the proposed method, consider the minimum variance (MV) beamformer. In MV, the array weight vector $\mathbf{w}_i$ in order to extract the ith signal is computed as $\mathbf{w}_i = \mathbf{R}^{-1}\mathbf{a}_i$. Fig. 3 represents the normalized beam pattern intended to extract the signal located at 180° applying the true and estimated $\mathbf{R}$, using initial, Step–1 ,and Step–2 steering vectors for a typical case. Here $r = 0.02\lambda$.

Table 4 and Table 5 summarize the noise gain ($G_N = \mathbf{w}^H\mathbf{w}$), the produced gain in the direction of interferers (G_1 and G_3) and signal (G_2), and the output SINR (signal to interference and noise ratio) for the patterns shown is Fig. 3. Here, we assume that the input SINR is -10.09dB.

5 Conclusion

This paper has proposed a two–step algorithm for signal steering vectors estimation. The approach assumes that the signals are uncorrelated, number of signal sources and the power of the received noise are known. The proposed algorithm use the eigenvector decomposition of the correlation matrix. In the first step of the algorithm we project the steering vector onto the signal subspace. The second step uses a cost function for minimization. Simulation experiments show the effectiveness of the method.

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Table 1: SOURCE PARAMETERS USED IN SIMULATIONS.

Source	DOA	Signal level	SNR level
s_1	50°	-10dBm	0dB
s_2	180°	0dBm	10dB
s_3	250°	10dBm	20dB

Table 2: PROXIMITY FACTOR USING THE TRUE $\mathbf{R}$ IN THE PROPOSED METHOD.

r (λ)	0.02	0.04	0.08	0.16
D_0	.00263	.01052	.04197	.16282
D_1	.00100	.00394	.01541	.05665
D_2	.00062	.00246	.00973	.03750

Table 3: PROXIMITY FACTOR USING ESTIMATED **R** IN THE PROPOSED METHOD.

r (λ)	0.02	0.04	0.08	0.16
D_0	.00263	.01052	.04197	.16282
D_1	.00228	.00523	.01675	.05820
D_2	.00130	.00316	.01046	.03835

Table 4: GAIN AND SINR OF THE PRODUCED MV BEAM–PATTERN USING TRUE **R**.

	G_N	G_1 (dB)	G_2 (dB)	G_3 (dB)	$SINR_o$ (dB)
$\hat{\mathbf{a}}_0$	0dB	-18.722	-2.036	-32.959	7.694
$\hat{\mathbf{a}}_1$	0dB	-8.100	8.586	-27.649	17.358
$\hat{\mathbf{a}}_2$	0dB	-18.884	8.370	-55.774	18.313
$\mathbf{a}$	0dB	-32.924	8.754	-60.982	18.751

Table 5: GAIN AND SINR OF THE PRODUCED MV BEAM–PATTERN USING ESTIMATED **R**.

	G_N	G_1 (dB)	G_2 (dB)	G_3 (dB)	$SINR_o$ (dB)
$\hat{\mathbf{a}}_0$	0dB	-18.390	-5.032	-64.606	4.905
$\hat{\mathbf{a}}_1$	0dB	-8.772	8.600	-56.218	18.058
$\hat{\mathbf{a}}_2$	0dB	-19.368	8.314	-57.992	18.263
$\mathbf{a}$	0dB	-32.924	8.754	-60.982	18.751

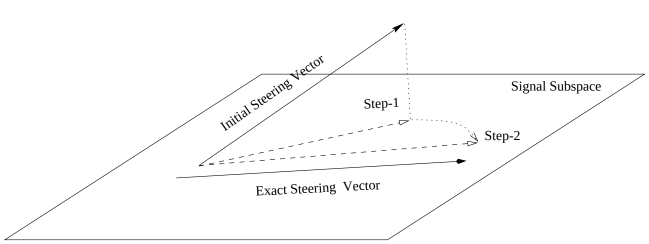


Figure 1: A simplified geometrical representation of the proposed method.

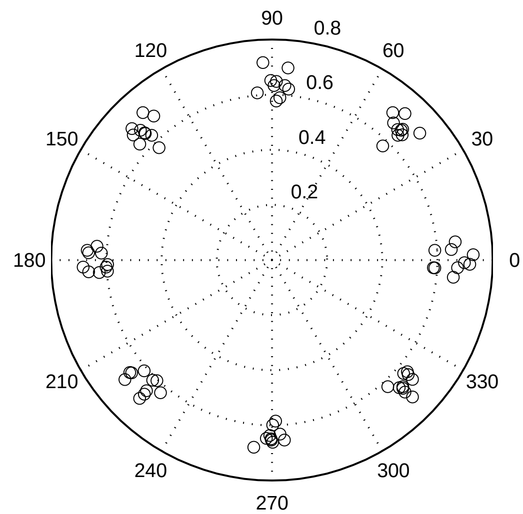
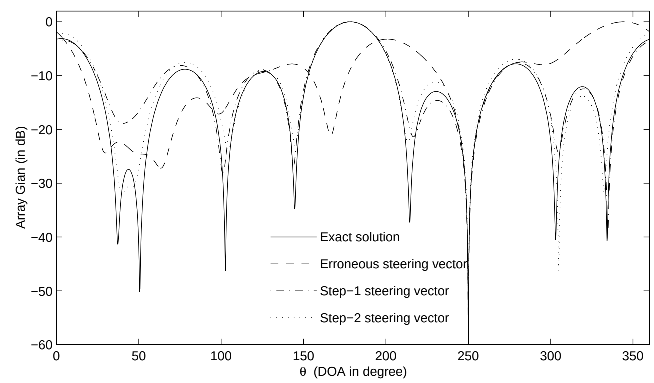
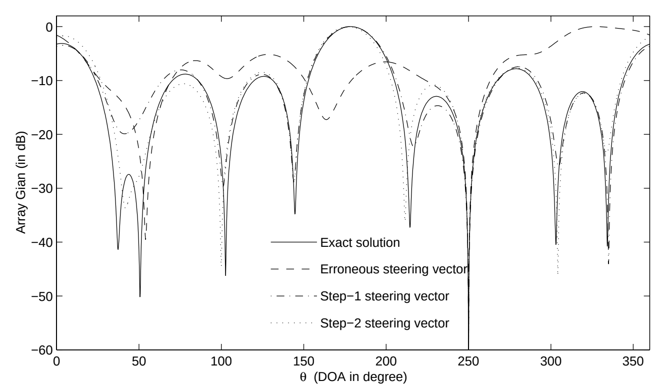


Figure 2: Location of array elements in 10 independent trials.



(a)



(b)

Figure 3: Produced beam pattern using the MV method, a) exact correlation matrix b) estimated correlation matrix (maximum array elements position error $r = 0.02\lambda$).