SINGLE RF CHAIN HYBRID ANALOG/DIGITAL BEAMFORMING FOR MMWAVE MASSIVE-MIMO

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ABSTRACT

Hybrid beamforming has attracted considerable attention in recent years as an efficient and promising technique for the practical implementation of millimeter-Wave (mmWave) massive multiple-input multiple-output (MIMO) wireless systems. In this paper, we investigate hybrid analog/digital beamforming designs based on a single RF chain architecture (SRCA) for mmWave massive-MIMO. We first revisit the SRCA and then explore its shortcomings. Subsequently, we present three novel beamformer designs which achieve the performance of fully-digital precoding systems while eliminating the drawbacks of SRCA. We further explore the applications of these designs for optimal precoding in both single-user and multi-user scenarios. Finally, we present simulation results which confirm the superiority of our proposed designs to recently published works.

Index Terms— Hybrid beamforming, hybrid analog/digital beamforming, massive-MIMO, mmWave.

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) and mmWave communications are now established as key technologies for fifth generation (5G) networks [1,2]. Nevertheless, the practical implementation of mmWave massive-MIMO systems remains challenging. Conventional MIMO systems are implemented using the fully-digital (FD) architecture, in which signal processing is performed in the digital domain by means of dedicated processors and/or digital circuitry. In the downlink scenario, the digital baseband output signals are then converted to analog signals for transmission, which requires a dedicated RF chain per antenna element [3–5]. For the large-scale antenna arrays envisaged for massive-MIMO systems, however, a FD architecture is impractical due to the huge power consumption and production costs [6].

One the most effective solutions to this problem is the hybrid analog/digital beamforming (HBF) [6–13]. In this approach, an additional layer of signal processing in the analog domain, referred to as analog beamformer (or precoder), is inserted between the RF chains and the antenna elements. In effect, by properly designing the analog beamformer, it becomes possible to reduce the number of RF chains while achieving a performance comparable to the FD architecture. Since the building blocks of the analog beamformer are phase-shifters, designing the associated beamforming (BF) matrix is challenging as the ensuing optimization problem is non-convex. [11].

In recent years, several optimization techniques have been introduced to directly design the analog and digital beamforming matrices of an HBF system [6–9]. Alternatively, some works have focused on the exact realization of optimal FD design within the HBF architecture [10–13]. In [10], a technique was proposed for the exact HBF realization of a single stream FD MIMO system using two RF chains. This approach was later extended to multi-stream transmissions in [11, 12], where the number of RF chains must be equal to the number of data streams. In [13], we proposed a single RF chain architecture (SRCA) for multi-stream HBF which can be used to realize any FD MIMO precoder.

In this paper, we revisit the SRCA in [13] and after discussing its shortcomings, present novel HBF schemes which use SRCA as building blocks. The main issue with SRCA is the required update-rate of the phase-shifters. We therefore propose three HBF schemes which facilitate the practical implementation of the SRCA concept. While these designs can be employed for the realization of various FD techniques, our discussions focus on the optimal precoding for single-user and multi-user MIMO scenarios. Simulation results confirm that the proposed HBF schemes achieve the performance of FD optimal precoder systems and outperform recent direct HBF designs from the literature.

2. SYSTEM MODEL

We consider a massive-MIMO transmitter with N_T antennas and N_{RF} RF chains where $N_T \gg N_{RF}$. We further assume that the massive-MIMO base station (BS) either is in MU mode which serves K single antenna users simultaneously or in SU mode which in this case is serving one multi-antenna user with N_R antennas. Without loss of generality, $N_s = K$ symbols are transmitted for SU and in case of MU one symbol per user is transmitted. Therefore, the BS performs precoding on the symbol vector $\mathbf{s} = [s_1, s_2, ..., s_K]^T \in \mathscr{A}^K$, where s_i is taken from a discrete constellation $\mathscr{A}$ (such as M-QAM or M-PSK).

2.1. Fully-Digital Architecture

In a fully-digital architecture, each antenna element is connected to a dedicated RF chain as shown in Fig. 1a. Thus, the transmitted signal can be written as

$$\mathbf{x}_{\mathrm{T}}^{\mathrm{FD}} = \mathbf{P}_{\mathrm{FD}}\mathbf{s}. \tag{1}$$

where $\mathbf{P}_{\mathrm{FD}}$ is the FD precoding (FDP) matrix of size $N_T \times K$.

2.2. Conventional Hybrid Architecture

Fig. 1b depicts the conventional HBF architecture massive-MIMO transmitters. In such systems [10, 12], since a limited number of RF chains are available, the symbol vector is first precoded with the digital beamformer and then analog beamforming is performed by the means of analog circuits. The transmitted signal is therefore given as

$$\mathbf{x}_{\mathrm{T}}^{\mathrm{HY}} = \mathbf{P}_{\mathrm{A}}\mathbf{P}_{\mathrm{D}}\,\mathbf{s}, \tag{2}$$

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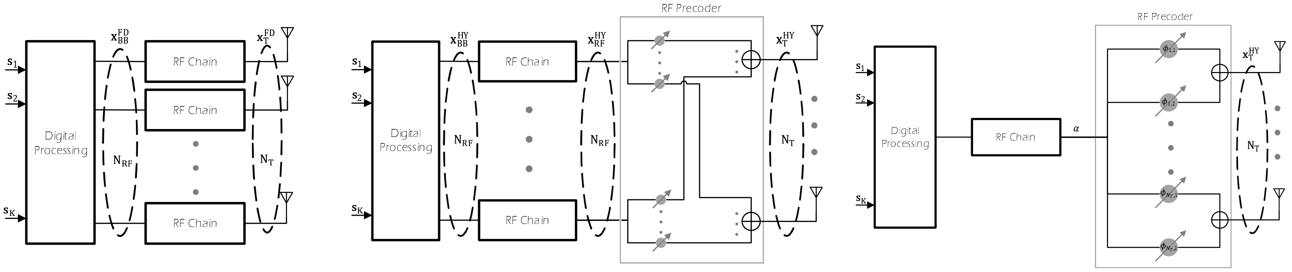
Fig. 1: Different precoder architectures



(a) FD beamforming architecture. (b) HBF architecture. (c) HBF with SRCA.

where $\mathbf{P}_{\text{D}} \in \mathbb{C}^{N_{RF} \times K}$ and $\mathbf{P}_{\text{A}} \in \mathbb{U}^{N_T \times N_{RF}}$ are the digital and analog precoders, respectively, with $\mathbb{U}$ being the set of complex numbers with unit norm:

$$\mathbb{U} = \{z \in \mathbb{C}\colon |z| = 1\}. \tag{3}$$

2.3. Review of Single RF Chain Architecture

Here, we briefly review our proposed SRCA to realize any given FD precoding with HBF. It is shown in [13] that any FDP can be realized by a SRCA HBF as shown in Fig. 1c with the following parameters:

$$\alpha \geq \frac{1}{2}|x|_{max} \tag{4a}$$

$$\phi_{i,1} = \vartheta_i - \cos^{-1}\big(\frac{|x_i|}{2\alpha}\big) \tag{4b}$$

$$\phi_{i,2} = \vartheta_i + \cos^{-1}\big(\frac{|x_i|}{2\alpha}\big). \tag{4c}$$

where $x_i = |x_i|e^{j\vartheta_i}$ denotes the polar representation of the ith entry of the vector $\mathbf{x}_{\text{T}}^{\text{FD}}$ in (1) and $|x|_{max}$ is defined as the maximum value of $|x_i|$ for $i = 1, \ldots, N_T$.

2.4. Implementation Aspects

It is discussed in [13] that except for its reduced number of RF chains, SRCA is implemented by the same hardware used in the conventional HBF architecture [6,10–12,14]. However, the required phase-shifter update period (PUP) limits the potential use of such architecture. In a conventional HADP system, to allow for accurate tracking of the wireless channel conditions, the RF precoder coefficients are updated according to the channel coherence time, T_c. In SCRA however, the RF precoder coefficients are also influenced by the transmit symbol vector $\mathbf{s}$ and as such, they must be updated according to the symbol duration T_s. Since $T_s < T_c$ in slowly-varying channels, this means that in this case, the RF precoder needs be updated at a higher rate, i.e., by a factor T_c/T_s. Consequently, in the following section, we present three HBF schemes based on the SRCA which remedy this shortcoming.

3. HADP SCHEMES BASED ON SRCA

The proposed SRCA enables us to generate any desired signal vector $\boldsymbol{x}$ of size M in RF domain with one RF chain. Let us first introduce $\mathcal{S}_M(\boldsymbol{x}, \alpha)$ as a primary building block shown in Fig. 2 which can be configured to generate a given signal $\boldsymbol{x}$. In what follows three HBF schemes are presented based on SRCA:

3.1. Phase-Shifter Bank

As discussed earlier, direct implementation of SRCA for realizing FDP realization requires analog precoder to be updated at each symbol period PUP $= T_s$. However, in practice, meeting this requirement is not possible; therefore, we present the following alternative design based on phase-shifter banks. Assuming the minimum update period of the chosen phase-shifter is T_p, this architecture requires phase-shifter bank of size $q \geq \lceil \frac{T_p}{T_s} \rceil$ as is shown in Fig. 4a. Output of the RF chain is connected to an analog switch (multiplexer) and the switch selects each of the analog beamformers in turn. Each antenna is connected to an analog switch which selects the active analog precoder. The switches are synchronized, i.e., operate simultaneously, and are controlled by the digital processor. Since power consumption is the key challenge, adding extra hardware is acceptable in the system design. Assuming the q consecutive symbol vectors are precoded to obtain the desired output signals $\mathbf{x}_{\text{T}}^1, \mathbf{x}_{\text{T}}^2, ..., \mathbf{x}_{\text{T}}^q$ each vector $\mathbf{x}_{\text{T}}^i$ for $i = \{1, 2, ..., q\}$ is then used to configure $\mathcal{S}_M(\boldsymbol{x}, \alpha)^i$ accordingly based on the SRCA and all the multiplexers are then switched to state i to transmit the precoded signal. Step-by-step FDP realization by phase-shifter bank scheme can be summarized as follows:

- Calculate the desired transmit signal by: $\mathbf{x}_{\text{T}}^{\text{FD}} = \mathbf{P}_{\text{FD}}\, \mathbf{s}^i$.
- Choose α as (4a).
- Set $\mathcal{S}_M(\boldsymbol{x}, \alpha)^i$ using (4).
- Feed α to the RF chain.
- Set all switches to the ith position

Note that additional memory and delay is *not* imposed on the system in this scheme as each symbol vector is precoded and transmitted immediately and since there are q RF blocks available, the ith block has time to be updated when it is required to be reconfigured again.

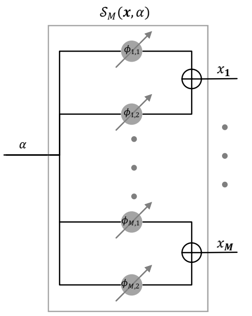
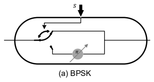
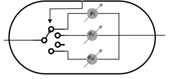


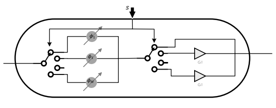
Fig. 2: $\mathcal{S}_M(\boldsymbol{x}, \alpha)$ block.



(a) BPSK



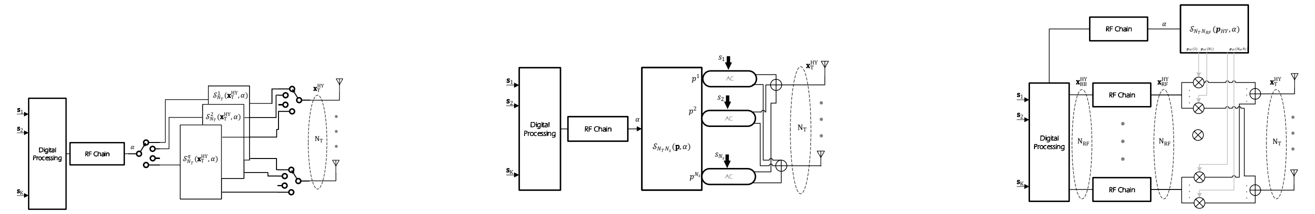
(b) M-PSK



(c) QAM

Fig. 3: AC block for different constellations.

Fig. 4: Different precoder architectures



(a) Phase-Shifter Bank SRCA implementation of DNT

(b) Analog Constellation SRCA implementation of DNT

(c) Hybrid beamformer with RF multipliers.

3.2. Analog Constellation

To realize a given FDP in HBF, the output of the hybrid beamformer must be equal to the output of the FDP, i.e.,

$$\mathbf{x}_{\text{T}}^{\text{HY}} = \mathbf{P}_{\text{FD}}\mathbf{s}. \tag{5}$$

Let us express the FDP matrix as

$$\mathbf{P}_{\text{FD}} = [\mathbf{p}^1, \mathbf{p}^2, ..., \mathbf{p}^{N_S}] \tag{6}$$

where $\mathbf{p}^j$'s are columns of the precoder matrix. Having $\mathbf{s} = [s_1, s_2, ..., s_{N_s}]^T$, and vectorizing $\mathbf{P}_{\text{FD}}$ as:

$$\mathbf{p} = [{\mathbf{p}^1}^T, {\mathbf{p}^2}^T, ..., {\mathbf{p}^{N_S}}^T]^T \tag{7}$$

by defining:

$$\mathbf{S} = [s_1\mathbf{I}_{N_T}, s_2\mathbf{I}_{N_T}, ..., s_{N_s}\mathbf{I}_{N_T}] \in \mathbb{C}^{N_T \times N_T K}, \tag{8}$$

it can be seen that, (5) can be also written as:

$$\mathbf{x}_{\text{T}}^{\text{HY}} = \mathbf{S}\mathbf{p}. \tag{9}$$

Using SRCA, vector $\mathbf{p}$ can be generated by one RF chain and matrix $\mathbf{S}$ can be implemented by analog constellation (AC) blocks as illustrated in Fig. 4b. The AC blocks are designed based on the constellation (such as QAM or PSK) and are comprised of fixed phase-shifters and switches as shown in Fig. 3. It can be observed that, BPSK for instance only requires a single switch and the π phase-shifter is just a negative sign which can be implemented by natively in the analog adders. Step-by-step FDP realization by Analog Constellation scheme can be summarized as follows:

- Construct $\mathbf{S}$ from (8).
- Update all AC modules according to $\mathbf{S}$.
- Calculate $\mathbf{P}_{\text{FD}}$ and construct $\mathbf{p}$ from (7).
- Set $\mathbb{S}_M(\mathbf{p}, \alpha)$ using (4).

3.3. Hybrid Beamformer with RF Multiplier

Using RF multipliers, we present a new architecture for HBF which relaxes the unit modulus constraints of the analog precoder. RF multipliers are not useful in conventional FDP and even modern HBF designs because *Implementation of conventional FDP with RF multipliers requires more RF chains than baseband FDP, i.e., $(N_T+1)K$ RF chains.*

However, using RF multipliers in SRCA simplifies the RF precoder design. We introduce a general architecture which can be used for designing various hybrid signal processing systems. This technique also relaxes the unit modulus constraint of analog precoders. Fig. 4c illustrates the system architecture for this technique.

The output of the system is:

$$\mathbf{x}_{\text{T}}^{\text{HY}} = \mathbf{P}_{HY}\mathbf{x}_{\text{RF}}^{\text{HY}} \tag{10}$$

where $\mathbf{x}_{\text{RF}}^{\text{HY}}$ is the output of RF chains and

$$\mathbf{P}_{HY} = [\mathbf{p}_{HY}^1, \mathbf{p}_{HY}^2, ..., \mathbf{p}_{HY}^{N_S}] \in \mathbb{C}^{N_T \times N_{RF}} \tag{11}$$

is the new analog precoder Vectorizing the matrix $\mathbf{P}_{HY}$, we have:

$$\mathbf{p}_{HY} = [{\mathbf{p}_{HY}^1}^T, {\mathbf{p}_{HY}^2}^T, ..., {\mathbf{p}_{HY}^{N_s}}^T]^T \tag{12}$$

The vector $\mathbf{p}_{HY}$ is generated using SRCA. The most trivial design is to set $\mathbf{P}_{HY} = \mathbf{P}_{\text{FD}}$ and $\mathbf{x}_{\text{RF}}^{\text{HY}} = \mathbf{s}$. However, more sophisticated optimizations and/or decompositions can be used to minimize the number of RF chains. Step-by-step FDP realization by RF multiplier SRCA scheme can be summarized as:

- Feed symbol vector $\mathbf{s}$ to RF chains number 1 to K.
- Calculate $\mathbf{P}_{\text{FD}}$ and construct $\mathbf{P}_{HY}$ from (12).
- Setup $\mathbb{S}_M(\mathbf{P}_{HY}, \alpha)$ using (4).
- Feed α to RF chain number $K+1$ to drive $\mathbb{S}_M(\mathbf{P}_{HY}, \alpha)$

Remark 1 *Comparison of the proposed SRCA techniques: The conventional hybrid architecture was particularly designed for BF and cannot innately achieve the performance of FD systems. However, the proposed SRCA techniques are purposefully designed to produce and deliver any desired signal to the large scale antenna array. Therefore, the applications are not limited to BF and and other wireless techniques can be implemented in hybrid analog/digital form with limited RF chains such as FD channel estimation (CE), space-time coding (STC). In table 1, proposed techniques, FD systems and existing hybrid structure are compared. While, the proposed schemes achieve the performance of FD systems, the required RF chains are even less than existing hybrid structures.*

4. PRECODING APPLICATIONS

In this section, we present the realization of the optimal FD precoding for SU and MU scenarios as an example. The proposed schemes are able to realize any given FDP and various design criteria can be used for designing the precodering matrix. Here, we focus on optimal FD precoding for SU and MU transmission modes.

4.1. Single-User

According to the requirements and constraints of the system, various objective functions can be used to design the precoder for SU setup. In what follows, we explore spectral efficiency as a criterion to design the precoder. In order to maximize the spectral efficiency of a point-to-point MIMO, the optimal FD beamformer is obtained from the following optimization problem:

$$\max_{\mathbf{D}_{su},\mathbf{P}} \quad \log_2(|\mathbf{I}_K| + \frac{\rho}{K}\mathbf{R}_n^{-1}\mathbf{D}^H\mathbf{H}_{su}\mathbf{P}_{su}\mathbf{P}_{su}^H\mathbf{H}^H\mathbf{D}) \tag{13a}$$

$$s.t. \; Tr(\mathbf{P}_{su}\mathbf{P}_{su}^H) \le P_T \tag{13b}$$

where P_T is the power budget at the transmitter, $\mathbf{H}_{su}$ is the mmWave MIMO channel matrix, $\mathbf{P}_{su}$ and $\mathbf{D}$ are precoder and

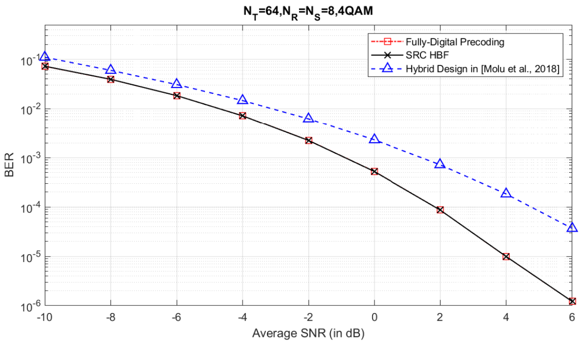


Fig. 5: BER versus SNR for [8], fully digital precoding and our design in a 64 × 8 massive-MIMO system.

combiner matrices and $\mathbf{R}_n = \mathbf{D}^H\mathbf{D}$. Then, the optimal solution can be analytically calculated [15] using the singular value decomposition of $\mathbf{H}_{su}$, i.e.,

$$\mathbf{H}_{su} = \mathbf{U}_{su}\mathbf{\Sigma}_{su}\mathbf{V}_{su}{}^H, \tag{14}$$

as:

$$\mathbf{P}_{su} = \mathbf{V}_{su}\mathbf{W}, \tag{15}$$

where the matrix $\mathbf{W}$ is calculated via water-filling [15].

4.2. Multi-User

In case of single antenna MU, the optimal precoder is calculated using the ergodic sum-rate:

$$\max_{\mathbf{P}_{mu}} \sum_{k=1}^{K} \log_2(1 + \mathit{SINR}_k) \tag{16a}$$

$$s.t.\ \mathit{SINR}_k = \frac{|\mathbf{h}_k\mathbf{P}_{mu_k}|^2}{\sum_{m\neq k}|\mathbf{h}_k\mathbf{P}_{mu_k}|^2 + \frac{1}{\rho}} \tag{16b}$$

where $\mathbf{P}_{mu_k}$ is the beamforming vector of kth user and consequently kth column of beamforming matrix $\mathbf{P}_{mu}$. Moreover, $\mathbf{h}_k$ is the channel vector between the user k and BS; thus, the channel matrix seen by the BS can be written as

$$\mathbf{H}_{\text{mu}} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_K]. \tag{17}$$

The zero-force (ZF) beamforming with optimal power allocation is obtained by

$$\mathbf{P}_{mu} = \mathbf{H}_{mu}(\mathbf{H}_{mu}\mathbf{H}_{mu}^H)^{-1}\mathbf{\Gamma}^{\frac{1}{2}} \tag{18}$$

where Γ is the diagonal weight matrix obtained by water-filling.

4.3. Conventional HBF and SRCA HBF

In conventional hybrid structures, the following constraints are added to all of the above optimization problems:

$$\mathbf{P} = \mathbf{P}_{\text{A}}\mathbf{P}_{\text{D}} \tag{19a}$$

$$\mathbf{P}_{\text{A}} \in \mathbb{U}^{N_T \times N_{RF}} \tag{19b}$$

which makes the consequent problems non-convex and therefore very difficult to solve. All the proposed schemes, on the other hand, are capable of realizing any given fully digital precoding scheme. For the optimal precoding, we first need to design the optimal precoders using (15) and (18) for SU and MU, respectively. Then, as discussed in section III, the proposed schemes can be accordingly configured to achieve the performance of FD systems. In the next section we perform computer simulations to illustrate the performance of the proposed schemes compared to recent HBF designs.

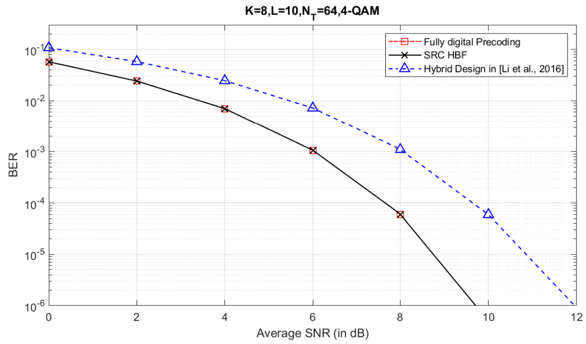


Fig. 6: BER versus SNR for [9], fully digital precoder and our design in a MU setup with a 64 massive-MIMO BS.

Table 1: Comparison of different structures

-	N_{RF}	PUP	Applications
Phase-shifter bank	1	T_p	BF, STC, CE
Analog constellation	1	T_c	BF
SRCA with multiplier	2 to $K+1$	T_c	BF,STC,CE
Fully digital	N_T	-	BF,STC,CE
Existing hybrid designs	K to N_T	T_c	BF

5. SIMULATION RESULTS

In this section, simulation results for both SU and MU cases are presented. We perform simulations for a massive-MIMO BS with $N_T = 64$ antennas and uniform linear configuration at the center of a single-cell wireless communication system.

In a SU scenario, 4-QAM constellation is used and we consider the channel model for mmWave massive-MIMO with sparse scattering environments as in [8, 11, 13]. For UE with $N_T = 8$ transmit antennas, $K = 8$ symbols per transmission, Fig. 5 depicts the BER performance versus SNR (SNR=P_T) for fully digital beamforming described in section 4.1, our three proposed SRCA schemes in Section 3, and hybrid robust design in [8]. While our scheme matches the performance of fully digital beamforming, our proposed schemes outperforms [8] by more than 3 dB for instance ar BER 10^{-4}.

For multi-user case, 4-QAM constellation and independent multipath channel model [9] is used. We compared our scheme with hybrid design in [9] as well as FD beamforming scheme presented in section 4.2 . For $K = 8$ single antenna users, and independent mmWave channels with $L_k = 10$, Figs. 6 illustrates the BER performance versus SNR. It can be observed that our scheme has a margin of more than 2 dB to the hybrid design in [9] while achieving the same performance as fully digital precoding.

6. CONCLUSION

In this paper, we investigated hybrid/analog beamforming schemes based on the SRCA for mmWave massive-MIMO. We first reviewed the SRCA architecture and then discussed its shortcomings. Then, we proposed three novel beamformer schemes which eliminate the defects of SRCA, and can achieve the performance of FD beamforming schemes. Moreover, the applications of these systems for optimal precoding in both SU and MU cases were studied. Finally, we presented simulation results which confirm the superiority of our proposed designs to presently published works.

7. REFERENCES

[1] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, “Massive MIMO for next generation wireless systems,” *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[2] S. A. Busari, K. M. S. Huq, S. Mumtaz, L. Dai, and J. Rodriguez, “Millimeter-Wave Massive MIMO Communication for Future Wireless Systems: A Survey,” *IEEE Commun. Surv. Tut.*, vol. PP, no. 99, pp. 1–1, 2017.

[3] S. Tofigh, H. M. Kermani, and A. Morsali, “A New Design Criterion for Linear Receiver STBCs Based on Full-Rank Spaces,” *IEEE Commun. Lett.*, vol. 19, no. 2, pp. 207–210, Feb. 2015.

[4] M. Samavat, A. Morsali, and S. Talebi, “Delay Interleaved Cooperative Relay Networks,” *IEEE Commun. Lett.*, vol. 18, no. 12, pp. 2137–2140, Dec. 2014.

[5] A. Morsali and S. Talebi, “On permutation of space-time-frequency block codings,” *IET Commun.*, vol. 8, no. 3, pp. 315–323, Feb. 2014.

[6] A. Garcia-Rodriguez, V. Venkateswaran, P. Rulikowski, and C. Masouros, “Hybrid analog-digital precoding revisited under realistic RF modeling,” *IEEE Commun. Lett.*, vol. 5, no. 5, pp. 528–531, Oct. 2016.

[7] A. Alkhateeb, J. Mo, N. Gonzalez-Prelcic, and R. W. Heath, “MIMO Precoding and Combining Solutions for Millimeter-Wave Systems,” *IEEE Commun. Mag.*, vol. 52, no. 12, pp. 122–131, Dec. 2014.

[8] M. M. Molu, P. Xiao, M. Khalily, K. Cumanan, L. Zhang, and R. Tafazolli, “Low-Complexity and Robust Hybrid Beamforming Design for Multi-Antenna Communication Systems,” *IEEE Trans. Wireless Commun.*, vol. 17, no. 3, pp. 1445–1459, Mar. 2018.

[9] J. Li, L. Xiao, X. Xu, and S. Zhou, “Robust and low complexity hybrid beamforming for uplink multiuser mmwave MIMO systems,” *IEEE Commun. Lett.*, vol. 20, no. 6, pp. 1140–1143, June 2016.

[10] X. Zhang, A. F. Molisch, and S.-Y. Kung, “Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection,” *IEEE Trans. Signal Processing*, vol. 53, no. 11, pp. 4091–4103, Nov. 2005.

[11] F. Sohrabi and W. Yu, “Hybrid digital and analog beamforming design for large-scale antenna arrays,” *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 501–513, Apr. 2016.

[12] T. E. Bogale, L. B. Le, A. Haghighat, and L. Vandendorpe, “On the Number of RF Chains and Phase Shifters, and Scheduling Design With Hybrid Analog Digital Beamforming,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3311–3326, May 2016.

[13] A. Morsali, A. Haghighat, and B. Champagne, “Realizing Fully Digital Precoders in Hybrid A/D Architecture With Minimum Number of RF Chains,” *IEEE Commun. Lett.*, vol. 21, no. 10, pp. 2310–2313, Oct. 2017.

[14] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, “Spatially sparse precoding in millimeter wave MIMO systems,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, Mar. 2014.

[15] E. Torkildson, U. Madhow, and M. Rodwell, “Indoor Millimeter Wave MIMO: Feasibility and Performance,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4150–4160, Dec. 2011.