MMSE-Based Non-Regenerative Parallel MIMO Relaying with Simplified Receiver

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***Abstract*—This paper considers a cooperative MIMO relaying system in which the source sends information to the destination with the aid of multiple relays, each equipped with multiple antennas. We design optimal linear relaying matrices that minimize the mean square error between the source and the received signal vectors under a total transmitted power constraint imposed on all the relays. The optimal matrices are obtained by solving the corresponding Karush-Kuhn-Tucker (KKT) conditions. Simulation results show significant advantages of the newly designed relaying matrices over other competing approaches in terms of bit-error rate (BER) performance. Moreover, this strategy enables the self-demultiplexing of the spatial substreams without the need for a MIMO combiner at the destination, so that a simplified receiver structure can be used without performance loss. In addition, the new design can serve as a suboptimal solution for multiuser MIMO relaying applications.**

I. Introduction

MIMO wireless relaying can increase system throughput, overcome shadowing and expand network coverage more efficiently than its single-antenna counterpart. The multi-antenna relays can either decode information bits, or simply apply a linear matrix to the received baseband signals before retransmitting them. These approaches are known as decode-and-forward (DF) and amplify-and-forward (AF), respectively. The latter *non-regenerative* approach benefits from shorter processing delay, lower complexity and better security. Moreover, the source can forward its signal through a single relay (selection-type), or use multiple relays whose retransmitted signals superimpose at the destination (combining-type) [1].

The optimal relaying matrices for the one-source–one-relay–one-destination (1S-1R-1D) configurations are well established for a wide variety of criteria under a transmitted power constraint at the relay station [2]. These matrices share a common structure based on singular value decomposition (SVD) which diagonalizes the backward and forward channels, but with different diagonal entries [3]–[6]. However, the joint design of multiple optimal relaying matrices remains an open problem for combining-type, one-source–multiple-relays–one-destination (1S-MR-1D) systems. Herein, the main difficulty comes from the block-diagonal constraint imposed on the compound AF matrix of the multiple parallel relays.

By enforcing the power constraint on the signals received at the destination, instead of the signals transmitted from the relays, it is possible to circumvent this difficulty and design matrices that maximize mutual information (MMI) or minimize the mean square error (MMSE) [7], [8]. Alternatively, an approximate MMI strategy is derived in [9] by specifying a special diagonal structure and relaxing the objective function.

In 1S-MR-1D systems, a good relaying method should ensure that the signals transmitted by all the relays are coherently combined at the destination. In this regard, some heuristic strategies have been proposed that “borrow” ideas from MIMO transceiver design, including matched filtering (MF), zero-forcing (ZF), linear MMSE [7], [10] and QR decomposition [11]. Although inferior to SVD-based approaches in 1S-1R-1D systems, these methods achieve higher distributed array gain in 1S-MR-1D systems. In [12], we proposed a unified hybrid framework in which the relaying matrices serve as equalizers for the backward channels and precoders for the forward channels. By considering various combinations of MF, ZF, MMSE as well as a new cooperative MMSE (CMMSE) scheme, we were able to demonstrate significant performance improvement along with added flexibility in system design.

In this paper, we design optimal relaying matrices for combining-type parallel MIMO relay systems. The objective is to minimize the sum of mean square errors (MSE) between the received and transmitted signal samples with a transmitted power budget imposed on the relay stations. We derive the optimal matrices based on Karush-Kuhn-Tucker (KKT) conditions [13] and the properties of Kronecker product. Simulation results show that the proposed strategy outperforms competing methods in terms of bit-error rate (BER). This design enables the self-demultiplexing of the spatial substreams at the relaying layer without the need of a MIMO combiner at the destination, so that a low-complexity receiver structure can be used without performance loss. This is particularly suitable for mobile terminals or sensor nodes with limited battery capacity and computational power. Interestingly, simulation results illustrate that the proposed design, together with the simplified receiver, still outperforms competing methods even if they employ MIMO combiners. Moreover, since the multiple streams are decoded independently, the proposed design can be readily generalized to the multiuser MIMO relaying networks investigated in [14].

The organization of this paper is as follows: Section II introduces the system model and Section III derives the optimal relaying matrix form and also proves the optimality of the duality parameter. Simulation results are shown in Section IV followed by a brief conclusion. Throughout this paper, $(\cdot)^H$ and $(\cdot)^T$ represents Hermitian and transpose of a matrix.

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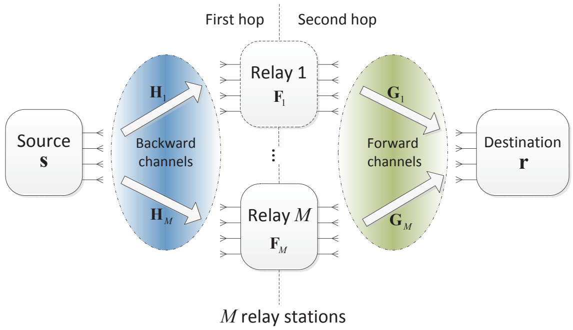

Fig. 1. A 1S-MR-1D MIMO relaying channel.

II. System Model and Problem Definition

Fig. 1 illustrates a 1S-MR-1D MIMO relaying channel between a source, M parallel relays and a destination. The number of antennas for these nodes are respectively N_S, N_R and N_D satisfying $N_R \geq N_S = N_D$[1]. We consider a half-duplex mode in which the receiver and transmitter work in different time intervals. This can prevent the relay receivers from being saturated by their transmitters. We assume that the direct link between the source and destination is unavailable or too weak to be taken into consideration.

In order to introduce a suitable system model for the above MIMO relaying channel, we make the following general assumptions: (1) the wireless channels undergo frequency non-selective block-fading, which means that the system bandwidth is less than the coherence bandwidth and the channels remain constant during the transmission period of interest; (2) all the channels and signal representations are baseband equivalent to their narrowband counterparts and, as a result, all the processing takes place on complex-valued baseband signal samples; (3) the source has no channel state information (CSI), each relay knows its own backward and forward channels, and the destination has CSI of all the channels.

The above narrowband assumption enables a matrix description of MIMO channels. In fact, this is also possible for broadband frequency-selective channels if the system employs a multicarrier transceiver architecture such as filter banks, e.g. orthogonal frequency-division multiplexing (OFDM) [15], [16]. With this in mind, the linear MIMO relaying matrices designed in this paper are also suitable for wideband systems.

Under the above assumptions, the received baseband signal vector $\mathbf{x}_j \in \mathbb{C}^{N_R \times 1}$ at the jth relay takes the form

$$\mathbf{x}_j = \mathbf{H}_j \mathbf{s} + \mathbf{w}_j \ , \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_S \times 1}$ is the vector of N_S transmitted source symbol streams, $\mathbf{H}_j \in \mathbb{C}^{N_R \times N_S}$ is the matrix corresponding to the *backward* channel between the source user and the jth relay and $\mathbf{w}_j \in \mathbb{C}^{N_R \times 1}$ is an additive noise component. The signal and noise vectors, $\mathbf{s}$ and $\mathbf{w}_j$ for $j = 1, \ldots, M$, are modeled as independent, zero-mean circularly symmetric complex Gaussian (ZMCSCG) random vectors with covariance matrices $\mathbf{R}_\mathbf{s} = \mathrm{E}\{\mathbf{s}\mathbf{s}^H\}$ and $\mathbf{R}_{\mathbf{w}_j} = \mathrm{E}\{\mathbf{w}_j \mathbf{w}_j^H\}$, respectively.

With linear processing defined by matrices $\mathbf{F}_j \in \mathbb{C}^{N_R \times N_R}$, the jth relay retransmits its received noisy signal $\mathbf{x}_j$ as

$$\mathbf{y}_j = \mathbf{F}_j \mathbf{x}_j \ , \qquad j = 1, \ldots, M. \tag{2}$$

The total transmitted power of all the relays should satisfy

$$\sum_{j=1}^{M} \mathrm{tr}(\mathbf{F}_j \mathbf{R}_{\mathbf{x}_j} \mathbf{F}_j^H) \leq P_R \ , \tag{3}$$

where $\mathbf{R}_{\mathbf{x}_j} = \mathrm{E}\{\mathbf{x}_j \mathbf{x}_j^H\} = \mathbf{H}_j \mathbf{R}_\mathbf{s} \mathbf{H}_j^H + \mathbf{R}_{\mathbf{w}_j}$. Note that P_R represents the power used to transmit both the desired signal $\mathbf{s}$ and the additive relay noises $\{\mathbf{w}_j\}$ to the destination.

The received signal vector at the destination, denoted by $\mathbf{r} \in \mathbb{C}^{N_D \times 1}$, can be expressed as

$$\mathbf{r} = \sum_{j=1}^{M} \mathbf{G}_j \mathbf{F}_j \mathbf{H}_j \mathbf{s} + \sum_{j=1}^{M} \mathbf{G}_j \mathbf{F}_j \mathbf{w}_j + \mathbf{n} \ , \tag{4}$$

where $\mathbf{G}_j \in \mathbb{C}^{N_D \times N_R}$ is the baseband *forward* channel matrix from relay j to the destination user and $\mathbf{n} \in \mathbb{C}^{N_D \times 1}$ is an additive noise component. The latter, independent from $\mathbf{s}$ and $\{\mathbf{w}_j\}$, is also modeled as a ZMCSCG vector with covariance matrix $\mathbf{R}_\mathbf{n} = \mathrm{E}\{\mathbf{n}\mathbf{n}^H\}$. Eq. (4) can also be expressed in a *block-diagonal* form as

$$\mathbf{r} = \mathbf{G}\mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{G}\mathbf{F}\mathbf{w} + \mathbf{n} \ , \tag{5}$$

where $\mathbf{G} = [\mathbf{G}_1, \ldots, \mathbf{G}_M]$, $\mathbf{H} = \left[\mathbf{H}_1^H, \ldots, \mathbf{H}_M^H\right]^H$, $\mathbf{F} = \mathrm{diag}(\mathbf{F}_1, \ldots, \mathbf{F}_M)$, and $\mathbf{w} = \left[\mathbf{w}_1^H, \ldots, \mathbf{w}_M^H\right]^H$. When $M = 1$, this signal model reduces to the 1S-1R-1D case [7].

To ensure constructive enhancement of the signals from different relays and minimize the effect of noise amplification, in this work, we design the linear relaying matrices to minimize the sum of MSE's between a scaled version of received signal vector $\mathbf{r}$ and the transmitted vector $\mathbf{s}$ under the power constraint in (3). Although BER performance also depends on nonlinear components such as channel coding, interleaving and constellation mapping, this MSE criterion still serves as a good indicator and is more mathematically tractable.

III. MMSE-based Cooperative Relaying

The sum of all substream MSE's is expressed as

$$\mathrm{MSE}(\mathbf{F}, \eta) = \mathrm{E}\left\{ \left\| \eta^{-1}\mathbf{r} - \mathbf{s} \right\|^2 \right\} , \tag{6}$$

where $\eta > 0$ is a gain scaling factor and $\| \cdot \|$ denotes the Euclidean norm of a vector. The expectation is taken with respect to $\mathbf{s}$, $\mathbf{w}$ and $\mathbf{n}$. This expression can be expanded as

$$\mathrm{MSE}(\mathbf{F}, \eta) = \eta^{-2} \sum_{j=1}^{M} \mathrm{tr}\left(\mathbf{G}_j \mathbf{F}_j \mathbf{R}_{\mathbf{w}_j} \mathbf{F}_j^H \mathbf{G}_j^H\right) + \eta^{-2} \mathrm{tr}\left(\mathbf{R}_\mathbf{n}\right) + \mathrm{tr}\Big(\big(\eta^{-1}\sum_{j=1}^{M} \mathbf{G}_j \mathbf{F}_j \mathbf{H}_j - \mathbf{I}\big)\mathbf{R}_\mathbf{s}\big(\eta^{-1}\sum_{j=1}^{M} \mathbf{G}_j \mathbf{F}_j \mathbf{H}_j - \mathbf{I}\big)^H\Big). \tag{7}$$

[1]This is only for notational simplicity and the generalization to different numbers of antennas at the relays, i.e. $N_{R,j}$, is straightforward.

The problem of interest here is to minimize (7) subject to the power constraint in (3). First, we derive the closed-form solution of this convex optimization problem by solving the KKT conditions based on the properties of Kronecker product. Then we prove the optimality of the optimal duality parameter, which turns out to be independent of the channel matrices. These mathematical techniques were widely used before; however, our main theoretical contribution is to provide an alternative but tractable MMSE formulation whose *closed-form* solution outperforms previous strategies by a wide margin, when the joint MMSE approach with Wiener MIMO combiner involves a non-convex problem [17].

A. Optimum Solution Based on KKT Conditions

Define the Lagrangian function as

$$\mathrm{MSE}(\mathbf{F},\eta,\lambda') = \mathrm{MSE}(\mathbf{F},\eta) + \lambda' \sum_{j=1}^{M} \mathrm{tr}(\mathbf{F}_j \mathbf{R}_{\mathbf{x}_j} \mathbf{F}_j^H) \ . \quad (8)$$

Without changing the solution of this problem, we define $\lambda = \lambda' \eta^2$. The optimal KKT conditions are expressed as

$$\sum_{j=1}^{M} \mathrm{tr}(\mathbf{F}_{j,o} \mathbf{R}_{\mathbf{x}_j} \mathbf{F}_{j,o}^H) - P_R \leq 0 \ , \quad (9a)$$

$$\lambda_o \geq 0 \ , \quad (9b)$$

$$\lambda_o \Big(\sum_{j=1}^{M} \mathrm{tr}(\mathbf{F}_{j,o} \mathbf{R}_{\mathbf{x}_j} \mathbf{F}_{j,o}^H) - P_R \Big) = 0 \ , \quad (9c)$$

$$\frac{\partial \mathrm{MSE}(\mathbf{F},\eta,\lambda)}{\partial \mathbf{F}_k} = \mathbf{0} \ , \quad (9d)$$

$$\frac{\partial \mathrm{MSE}(\mathbf{F},\eta,\lambda)}{\partial \eta} = 0 \ , \quad (9e)$$

for all $k = 1, \ldots, M$, where the values in (9d) and (9e) are evaluated at $\eta = \eta_o, \lambda = \lambda_o, \mathbf{F}_j = \mathbf{F}_{j,o}, \forall j = 1, \ldots, M$. The condition in (9d) leads to the following equation

$$\begin{aligned} &\left(\mathbf{G}_k^H \mathbf{G}_k + \lambda_o \mathbf{I}\right) \mathbf{F}_{k,o} \left(\mathbf{H}_k \mathbf{R}_\mathbf{s} \mathbf{H}_k^H + \mathbf{R}_{\mathbf{w}_k}\right) \\ &\quad + \sum_{j \neq k} \mathbf{G}_k^H \mathbf{G}_j \mathbf{F}_{j,o} \mathbf{H}_j \mathbf{R}_\mathbf{s} \mathbf{H}_k^H = \eta_o \mathbf{G}_k^H \mathbf{R}_\mathbf{s} \mathbf{H}_k^H . \quad (10) \end{aligned}$$

By setting $\mathbf{f}_{k,o} = \mathrm{vec}(\mathbf{F}_{k,o})$ and applying $\mathrm{vec}(\mathbf{ABC}) = \left(\mathbf{C}^T \otimes \mathbf{A}\right) \mathrm{vec}(\mathbf{B})$, this equation is equivalent to

$$\begin{aligned} &\left((\mathbf{H}_k \mathbf{R}_\mathbf{s} \mathbf{H}_k^H + \mathbf{R}_{\mathbf{w}_k})^T \otimes (\mathbf{G}_k^H \mathbf{G}_k + \lambda_o \mathbf{I})\right) \mathbf{f}_{k,o} + \\ &\sum_{j \neq k} \left((\mathbf{H}_j \mathbf{R}_\mathbf{s} \mathbf{H}_k^H)^T \otimes (\mathbf{G}_k^H \mathbf{G}_j)\right) \mathbf{f}_{j,o} = \eta_o \, \mathrm{vec}\left(\mathbf{G}_k^H \mathbf{R}_\mathbf{s} \mathbf{H}_k^H\right) (11) \end{aligned}$$

where $\mathrm{vec}(\cdot)$ stacks columns of a matrix in a single column vector and $\otimes$ represents Kronecker product. These equations can be written in a compact block-matrix form as following

$$\begin{bmatrix} \mathbf{R}_{\mathbf{x}_1}^T \otimes (\mathbf{G}_1^H \mathbf{G}_1 + \lambda_o \mathbf{I}) & \cdots & (\mathbf{H}_M \mathbf{R}_\mathbf{s} \mathbf{H}_1^H)^T \otimes (\mathbf{G}_1^H \mathbf{G}_M) \\ \vdots & \ddots & \vdots \\ (\mathbf{H}_1 \mathbf{R}_\mathbf{s} \mathbf{H}_M^H)^T \otimes (\mathbf{G}_M^H \mathbf{G}_1) & \cdots & \mathbf{R}_{\mathbf{x}_M}^T \otimes (\mathbf{G}_M^H \mathbf{G}_M + \lambda_o \mathbf{I}) \end{bmatrix}$$

$$\times \begin{bmatrix} \mathbf{f}_{1,o} \\ \vdots \\ \mathbf{f}_{M,o} \end{bmatrix} = \eta_o \begin{bmatrix} \mathrm{vec}\left(\mathbf{G}_1^H \mathbf{R}_\mathbf{s} \mathbf{H}_1^H\right) \\ \vdots \\ \mathrm{vec}\left(\mathbf{G}_M^H \mathbf{R}_\mathbf{s} \mathbf{H}_M^H\right) \end{bmatrix} , \quad (12)$$

Let $\mathbf{\Psi}(\lambda_o)$ and $\mathbf{b}$ be the coefficient matrix and right-sided vector of (12), respectively. If $\lambda_o \neq 0$, $\mathbf{\Psi}(\lambda_o)$ is nonsingular and the optimal solution of this equation is

$$\mathbf{f}_o = [\mathbf{f}_{1,o}^T, \ldots, \mathbf{f}_{M,o}^T]^T = \eta_o \mathbf{\Psi}^{-1}(\lambda_o) \mathbf{b} \ ; \quad (13)$$

if $\lambda_o = 0$, there exist infinitely many solutions. The main drawback of this expression is that the dimension of $\mathbf{\Psi}(\lambda_o)$, $MN_R^2 \times MN_R^2$, is significantly larger than those of the original matrices. We can use properties of the Kronecker product and matrix inverse lemma[2] to simplify the solution in (13) as

$$\begin{aligned} \mathbf{f}_{k,o} = \eta_o \mathbf{\Gamma}_k^{-1}(\lambda_o) \left(\mathbf{H}_k^* \otimes \mathbf{G}_k^H\right) \Big(\mathbf{R}_\mathbf{s}^{-T} \otimes \mathbf{I} + \\ \sum_{j=1}^{M} \left(\mathbf{H}_j^T \otimes \mathbf{G}_j\right) \mathbf{\Gamma}_j^{-1}(\lambda_o) \left(\mathbf{H}_j^* \otimes \mathbf{G}_j^H\right) \Big)^{-1} \mathrm{vec}(\mathbf{I}), (14) \end{aligned}$$

where

$$\mathbf{\Gamma}_j(\lambda_o) = \mathbf{R}_{\mathbf{x}_j}^T \otimes \lambda_o \mathbf{I} + \mathbf{R}_{\mathbf{w}_j}^T \otimes \mathbf{G}_j^H \mathbf{G}_j \ . \quad (15)$$

Here, $\mathbf{H}^*$ represents entrywise complex conjugation of $\mathbf{H}$.

The main limitation of this strategy is that a fusion center (FC) has to compute the sum term in (14) and then feedback this information perfectly to all the relays via broadcasting. This necessiates the knowledge of all $\mathbf{G}_k$'s and $\mathbf{H}_k$'s at the FC, which might require additional resources dedicated for this purpose. In addition, imposing a total power constraint on the physically separate relays is less practical than individual power constraints, in spite of the fact that the latter might not lead to a closed-form solution.

B. Optimum Duality Parameter λ_o

Although (14) gives the solution form for the optimization problem, the scaling coefficient η_o and the duality parameters λ_o are still unknown. They should satisfy the other conditions in (9). To simplify the notation, we define the following terms:

$$\mathbf{D}_1 = \mathrm{diag}\left(\mathbf{R}_{\mathbf{w}_1}^T \otimes \mathbf{G}_1^H \mathbf{G}_1, \ldots, \mathbf{R}_{\mathbf{w}_M}^T \otimes \mathbf{G}_M^H \mathbf{G}_M\right) ,$$

$$\mathbf{D}_2 = \mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_1}^T \otimes \mathbf{I}, \ldots, \mathbf{R}_{\mathbf{x}_M}^T \otimes \mathbf{I}\right) ,$$

$$\mathbf{\Phi} = \begin{bmatrix} (\mathbf{H}_1^* \mathbf{R}_\mathbf{s}^T \mathbf{H}_1^T) \otimes (\mathbf{G}_1^H \mathbf{G}_1) & \cdots & (\mathbf{H}_1^* \mathbf{R}_\mathbf{s}^T \mathbf{H}_M^T) \otimes (\mathbf{G}_1^H \mathbf{G}_M) \\ \vdots & \ddots & \vdots \\ (\mathbf{H}_M^* \mathbf{R}_\mathbf{s}^T \mathbf{H}_1^T) \otimes (\mathbf{G}_M^H \mathbf{G}_1) & \cdots & (\mathbf{H}_M^* \mathbf{R}_\mathbf{s}^T \mathbf{H}_M^T) \otimes (\mathbf{G}_M^H \mathbf{G}_M) \end{bmatrix} .$$

and hence $\mathbf{\Psi}(\lambda_o) = \mathbf{\Phi} + \mathbf{D}_1 + \lambda_o \mathbf{D}_2$. Using the formula [18, p.252]

$$\mathrm{tr}(\mathbf{A}^T \mathbf{Y}^T \mathbf{B} \mathbf{X}) = [\mathrm{vec}(\mathbf{Y})]^T \left(\mathbf{A} \otimes \mathbf{B}\right) \mathrm{vec}(\mathbf{X}) \ , \quad (16)$$

the MSE expression (7) can be simplified as

$$\begin{aligned} \mathrm{MSE}(\mathbf{f}_o, \eta) = \eta^{-2} \mathbf{f}_o^H \mathbf{D}_1 \mathbf{f}_o + \eta^{-2} \mathrm{tr}(\mathbf{R_n}) + \eta^{-2} \mathbf{f}_o^H \mathbf{\Phi} \mathbf{f}_o \\ - \eta^{-1} \mathbf{b}^H \mathbf{f}_o - \eta^{-1} \mathbf{f}_o^H \mathbf{b} + \mathrm{tr}(\mathbf{R_s}) \ , (17) \end{aligned}$$

and the power constraint is $\mathbf{f}_o^H \mathbf{D}_2 \mathbf{f}_o \leq P_R$. The condition in (9e) is equivalent to

$$\eta_o = \frac{2 \, \mathbf{f}_o^H (\mathbf{\Phi} + \mathbf{D}_1) \mathbf{f}_o + 2 \, \mathrm{tr}(\mathbf{R_n})}{\mathbf{b}^H \mathbf{f}_o + \mathbf{f}_o^H \mathbf{b}} \ . \quad (18)$$

[2]Matrix inverse lemma: $(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1} \mathbf{B} (\mathbf{C}^{-1} + \mathbf{D} \mathbf{A}^{-1} \mathbf{B})^{-1} \mathbf{D} \mathbf{A}^{-1}$

From (9a)-(9c), if the power constraint is not satisfied tightly, λ_o must be zero. In this case, substituting (12) into (18) would lead to

$$\eta_o = \eta_o + \frac{\mathrm{tr}(\mathbf{R_n})}{\mathbf{f}_o^H \mathbf{b}} , \tag{19}$$

which implies that $\mathbf{f}_o^H \mathbf{b}$ goes to infinity. Henceforth, $\|\mathbf{f}_o\|$ would be unbounded and this would contradict with the finite power constraint (9a). Therefore, $\lambda_o > 0$ and the power constraint should be tightly satisfied as

$$P_R = \mathbf{f}_o^H \mathbf{D}_2 \mathbf{f}_o = \eta_o^2 \, \mathbf{b}^H \mathbf{\Psi}^{-1} \mathbf{D}_2 \mathbf{\Psi}^{-1} \mathbf{b} \, . \tag{20}$$

Left-multiplying both sides of (12) by $\mathbf{f}_o^H$ leads to

$$\mathbf{f}_o^H(\mathbf{\Phi}+\mathbf{D}_1+\lambda_o\mathbf{D}_2)\mathbf{f}_o = \frac{2\,\mathbf{f}_o^H(\mathbf{\Phi}+\mathbf{D}_1)\mathbf{f}_o + 2\,\mathrm{tr}(\mathbf{R_n})}{\mathbf{b}^H\mathbf{f}_o + \mathbf{f}_o^H\mathbf{b}} \mathbf{f}_o^H \mathbf{b} \, , \tag{21}$$

Since $\mathbf{f}_o^H \mathbf{b} = \mathbf{b}^H \mathbf{f}_o = \mathbf{b}^H \mathbf{\Psi}^{-1} \mathbf{b}$, we have

$$\lambda_o \mathbf{f}_o^H \mathbf{D}_2 \mathbf{f}_o = \mathrm{tr}(\mathbf{R_n}) \, , \tag{22}$$

which, in turn, gives

$$\lambda_o = \frac{\mathrm{tr}(\mathbf{R_n})}{P_R} = \frac{1}{M}\frac{N_D}{N_R}\frac{1}{\rho_2} \, , \tag{23}$$

where ρ_2 is a SNR parameter representing the quality of the second hop as explained in Section IV. Surprisingly, the optimal λ_o is independent of channel characteristics and SNR value of the first hop.

IV. Numerical Results

We compare the BER performance of the proposed method with previous relaying strategies, including MMSE-MMSE [10], simplistic AF (SAF) which uses identity matrices [10], SVD with uniform power allocation [1], and CMMSE-MMSE as proposed in our previous work [12]. In the simulations, each source antenna transmits independent uncoded 16-QAM symbol streams. The relay stations apply one of the above linear relaying matrix schemes to their input signals and retransmit them. The destination scales its received signal vector with a proper factor, and then employs single-stream maximum likelihood decoding. The number of relays is set to $M = 3$ and the numbers of antennas are $N_S = N_R = N_D = 4$. The channel matrices have normalized, independent and identically distributed ZMCSCG entries. We assume $\mathbf{R_s} = \sigma_s^2\mathbf{I}$, $\mathbf{R}_{\mathbf{w}_j} = \sigma_w^2\mathbf{I}$, $\mathbf{R_n} = \sigma_n^2\mathbf{I}$, and $\sigma_w^2 = \sigma_n^2$.

To discuss the behavior of the proposed MIMO relaying strategies, it is convenient to introduce two SNR parameters as follows. The first SNR represents the link quality between the source and the relay, and is defined as $\rho_1 = \sigma_s^2/\sigma_w^2$, i.e. the ratio of transmitted signal power per antenna to the received noise power per antenna. The second SNR parameter, defined in terms of P_R as $\rho_2 = P_R/(MN_R\sigma_n^2)$, gives the ratio of average transmitted power per relay antenna to the power of the noise induced at the individual destination antennas. Here, we set the first-hop SNR ρ_1 to 15dB and change ρ_2.

The BER values obtained from simulations based on the above settings, without using a MIMO combiner at the destination, are plotted versus ρ_2 in Fig. 2. As explained earlier, SVD

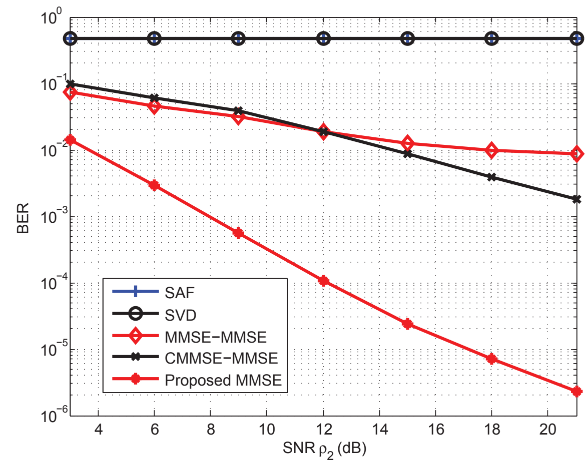

Fig. 2. BER performance without a MIMO equalizer.

and SAF cannot achieve distributed array gain and accordingly, they have the worst performance. The hybrid strategies offer some improvements in BER, but in the absence of the MIMO combiner, these are very limited. In contrast, the proposed strategy significantly outperforms these methods. In Fig. 3, we plot the BER performance when a Wiener MIMO equalizer is used to better separate the multiple symbol streams. One can see that the addition of a MIMO equalizer improves the BER performance of SVD, SAF and CMMSE-MMSE. In this case, CMMSE-MMSE performs significantly better than MMSE-MMSE but is still inferior to the proposed strategy. Furthermore, comparison of the results in Fig. 2 and 3 confirms that the new MMSE-based approach does not suffer from performance loss when a simplified receiver structure is used at the destination. In particular, its performance without MIMO combiner remains superior to that of the other schemes *with* Wiener equalizer. In a nutshell, the proposed strategy achieves both lower BER and reduced receiver complexity at the price of CSI exchange between relays and higher design complexity.

We also studied the BER performance of these strategies under correlated Rayleigh-fading channels. Specifically, we used the Kronecker model [19] to generate the mutually independent random matrices $\mathbf{H}_j$ and $\mathbf{G}_j, j = 1, \ldots, M$ in Fig. 1. The underlying correlation matrices in the Kronecker model were chosen as Toeplitz matrices whose (i, j) entry is equal to $\beta^{|i-j|}$ where $\beta = 0.7$. The corresponding BER are plotted versus ρ_2 in Fig. 4. Herein, a Wiener MIMO equalizer is used for all the methods; in addition, we also show results of the proposed method without such an equalizer. The same general conclusions as above can be made in this case although here, correlation between antennas incurs higher BER values due to loss in the diversity gain.

We emphasize that the new relaying strategy proposed in this work is also suitable for multiuser MIMO relaying networks. Since the source antennas transmit independent substreams and the relay processing enables self-demultiplexing of the substreams at the destination, the source and destination antennas can be either collocated or spatially distributed.

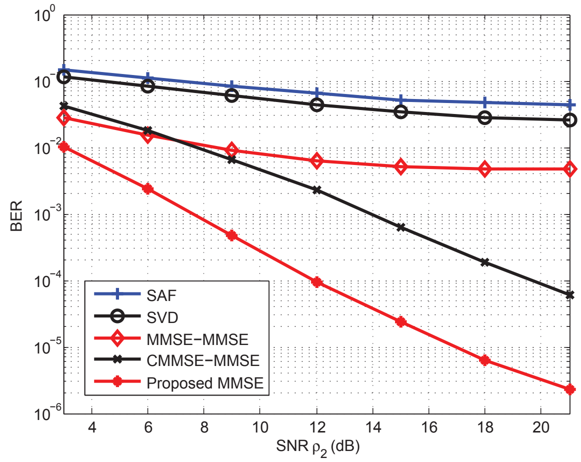

Fig. 3. BER performance with a linear Wiener MIMO equalizer.

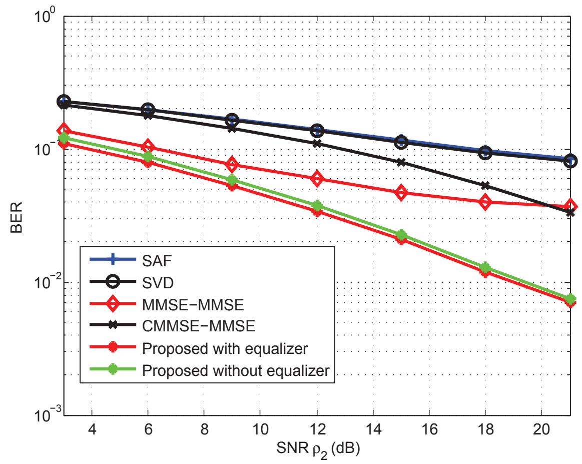

Fig. 4. BER performance under correlated channels.

In fact, it is this simplified receiver structure that provides additional flexibility so that our work can be readily extended to relay-assisted broadcast channels (BC), multiple access channels (MAC) and interference channels [14].

V. Conclusions

We considered the joint design of multiple linear relaying matrices in parallel MIMO AF relay systems, by minimizing the sum of MSEs between the received and transmitted symbol streams, under a constraint of total transmit power. The optimal matrices are obtained by solving the KKT conditions based on the properties of Kronecker product. The optimal duality parameter is inversely proportional to the second-hop SNR but does not depend on the first-hop SNR. This strategy achieves a better distributed array gain than previous methods and hence yields significant improvement in BER performance. Furthermore, by allowing for a self-demultiplexing of the spatial substreams at the destination, it does not require a MIMO combiner at the destination, so that a simpler receiver structure can be used without performance loss, as demonstrated by numerical simulations. Henceforth, since the multiple substreams can be decoded independently, the new strategy is readily applicable to multiuser MIMO relaying networks. Further works that can strengthen this paper include a detailed complexity analysis, imposing power constraints on individual relays, and investigating simpler, distributed and adaptive implementations.

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References

[1] Y. J. Fan and J. Thompson, “MIMO configurations for relay channels: Theory and practice,” *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1774–1786, May 2007.

[2] Y. Rong, X. Tang, and Y. Hua, “A unified framework for optimizing linear nonregenerative multicarrier MIMO relay communication systems,” *IEEE Trans. Signal Process.*, vol. 57, pp. 4837–4851, Dec. 2009.

[3] O. Muñoz-Medina, J. Vidal, and A. Agustín, “Linear transceiver design in nonregenerative relays with channel state information,” *IEEE Trans. Signal Process.*, vol. 55, pp. 2593–2604, Jun. 2007.

[4] X. Tang and Y. Hua, “Optimal design of non-regenerative MIMO wireless relays,” *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1398–1407, Apr. 2007.

[5] W. Guan and H. Luo, “Joint MMSE transceiver design in non-regenerative MIMO relay systems,” *IEEE Commun. Lett.*, vol. 12, pp. 517–519, Jul. 2008.

[6] C. Xing, S. Ma, and Y.-C. Wu, “Robust joint design of linear relay precoder and destination equalizer for dual-hop amplify-and-forward MIMO relay systems,” *IEEE Trans. Signal Process.*, vol. 58, pp. 2273–2283, Apr. 2010.

[7] A. S. Behbahani, R. Merched, and A. M. Eltawil, “Optimizations of a MIMO relay network,” *IEEE Trans. Signal Process.*, vol. 56, pp. 5062–73, Oct. 2008.

[8] A. Toding, M. R. A. Khandaker, and R. Yue, “Optimal joint source and relay beamforming for parallel MIMO relay networks,” in *Proc. Int. Conf. Wireless Commun. Netw. and Mobile Comput.*, 2010, 4 pages.

[9] Y. Fu, L. Yang, and W.-P. Zhu, “A nearly optimal amplify-and-forward relaying scheme for two-hop MIMO multi-relay networks,” *IEEE Commun. Lett.*, vol. 14, pp. 229–231, Mar. 2010.

[10] O. Oyman and A. J. Paulraj, “Design and analysis of linear distributed MIMO relaying algorithms,” *IEE Proc. Commun.*, vol. 153, pp. 565–572, Apr. 2006.

[11] H. Shi, T. Abe, T. Asai, and H. Yoshino, “A relaying scheme using QR decomposition with phase control for MIMO wireless networks,” in *Proc. 2005 IEEE Int. Conf. Commun.*, vol. 4, pp. 2705–2711.

[12] C. Zhao and B. Champagne, “Non-regenerative MIMO relaying strategies — from single to multiple cooperative relays,” in *Int. Conf. Wireless Commun. and Signal Process.*, Suzhou, China, Oct. 2010, 6 pages.

[13] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, UK: Cambridge Univ. Press, 2004.

[14] S. Ma, C. Xing, Y. Fan, Y.-C. Wu, T.-S. Ng, and H. Poor, “Iterative transceiver design for MIMO AF relay networks with multiple sources,” in *Proc. 2010 Military Commun. Conf.*, Nov. 2010, pp. 369–374.

[15] A. Scaglione, G. B. Giannakis, and S. Barbarossa, “Redundant filterbank precoders and equalizers. I. Unification and optimal designs,” *IEEE Trans. Signal Process.*, vol. 47, pp. 1988–2006, Jul. 1999.

[16] H. Sampath, P. Stoica, and A. Paulraj, “Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion,” *IEEE Trans. Commun.*, vol. 49, pp. 2198–2206, Dec. 2001.

[17] E. Park, K.-J. Lee, and I. Lee, “Joint MMSE transceiver design for MIMO amplify-and-forward relay systems with multiple relays,” in *Proc. IEEE 70th Veh. Tech. Conf.*, Sep. 2009, 5 pages.

[18] R. A. Horn and C. R. Johnson, *Topics in matrix analysis*. Cambridge: Cambridge Univ. Press, 1994.

[19] W. Weichselberger, M. Herdin, H. Ozcelik, and E. Bonek, “A stochastic MIMO channel model with joint correlation of both link ends,” *IEEE Trans. Wireless Commun.*, vol. 5, pp. 90–100, Jan. 2006.