

# EFFECTS OF DIRECTIONAL INTERFERENCE ON THE ESTIMATION ACCURACY OF TIME-VARYING DELAYS

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## ABSTRACT

This paper investigates the effects of directional interference on the accuracy of time-varying delay estimates obtained with a passive array of sensors. To this end, the output of the  $i^{\text{th}}$  sensor is modeled as  $x_i(t) = a(t - d_i(t)) + \eta(t - \delta_i) + v_i(t)$ , where  $a(t)$ ,  $\eta(t)$  and  $v_i(t)$  are uncorrelated Gaussian random processes representing a moving-source signal, the directional interference, and the background noise, respectively. The time-varying delays  $d_i(t)$  for the moving-source signal are slowly-varying functions of time parametrized by an unknown vector  $\theta$ . The time delays  $\delta_i$  for the interference signal are known. Using the general results on array processing in semi-stationary environments recently derived by the authors, new expressions for the Cramér-Rao lower bound (CRLB) on the error covariance matrix of estimates of  $\theta$  are obtained. These expressions are used to investigate the effects of directional interference on the estimation accuracy of differential Doppler shift under simplifying assumptions. Both qualitative and numerical results are presented.

Key-words: array processing, time delay estimation, Cramér-Rao lower bound

## I. INTRODUCTION

Several methods proposed for the localization of a moving source with a passive array of sensors are based on the estimation of the time-varying delay functions between the source signal components received at the various sensors [1]. In this respect, the Cramér-Rao lower bound (CRLB) on time-varying delay estimator variance provides a benchmark against which the performance of the various localization methods can be compared. Results currently available in the literature for the CRLB assume that the noise field on which is superposed the signal radiated by the moving source is *spatially uncorrelated* [2]. In practical applications, however, the noise field may contain spatially correlated components due to the presence of directional sources of interference in the environment. Because of a lack of a general approach for evaluating the CRLB in this case, the effects of directional interference on the estimation accuracy of time-varying delays have not been investigated yet.

In this paper, we use the general results on array processing in semi-stationary environments recently derived by the authors [3] to conduct such an investigation. To this end, we consider an arbitrary array configuration of  $M$  sensors and we model the output of the  $i^{\text{th}}$  sensor as  $x_i(t) = a(t - d_i(t)) + \eta(t - \delta_i) + v_i(t)$ , where  $a(t)$ ,  $\eta(t)$  and  $v_i(t)$  are zero-mean uncorrelated stationary Gaussian random processes representing the moving-source signal, the directional interference, and the background noise, respectively. The time-varying delays  $d_i(t)$  for the moving-source signal are slowly-varying functions of time parametrized by an unknown vector  $\theta$ . The condition of slow variation imposed on the delays is not very restrictive and is satisfied in most practical applications. The time delays  $\delta_i$  for the interference signal are assumed to be known. Using the results of [3], new expressions for the CRLB on the error covariance matrix of estimates of  $\theta$

are obtained. These expressions are used to investigate the effects of directional interference on the estimation accuracy of differential Doppler shift under certain simplifying assumptions.

## II. BACKGROUND

In this section, we briefly review some of the concepts and results of [3] on array processing in semi-stationary environments. The role of the CRLB in the context of processor performance evaluation is also given further consideration.

Consider the physical situation described in Fig. 1, where a passive array of  $M$  sensors is used to monitor a "slowly" moving source (S) in the presence of additive noise which may originate from various sources, including possibly a localized source of interference (I).

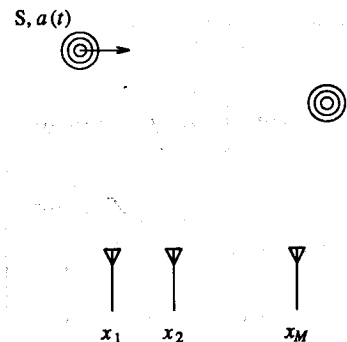


Fig. 1.  $M$ -sensor array monitoring a moving source in the presence of noise.

Let the sensor output vector  $x(t) = [x_1(t), \dots, x_M(t)]^T$ , where the superscript  $T$  denotes transposition, be given by

$$x(t) = \int_{-\infty}^{\infty} L(t, u) a(u) du + n(t), \quad (1)$$

where  $a(t)$  is the signal transmitted by the moving source,  $n(t) = [n_1(t), \dots, n_M(t)]^T$  is the sensor noise vector, and  $L(t, u)$  is the impulse response of the transmission channel between the source and the sensors. Finally, assume that  $a(t)$  and  $n(t)$  are zero-mean, uncorrelated, stationary Gaussian random processes with power spectral densities  $A(\omega)$  and  $N(\omega)$ , respectively.

In a typical source localization problem, the position and track of the moving source are defined in terms of a parameter vector  $\theta$ . The components of  $\theta$  might be, for example, the bearing and bearing rate of the source relative to a chosen orientation. This information is embedded in the channel impulse response  $L(t, u)$  which explicitly depends on the parameter vector  $\theta$ . The localization problem then consists in processing the sensor outputs in such a way as to obtain an estimate of  $\theta$ , generally denoted by  $\hat{\theta}(x)$ .

The CRLB sets a lower bound on the error covariance matrix of any unbiased estimator  $\hat{\theta}(x)$  of  $\theta$ . More precisely, it asserts that

$$E_{\theta} \{ [\hat{\theta}(x) - \theta][\hat{\theta}(x) - \theta]^T \} \geq J(\theta)^{-1} \quad (2)$$

where  $E_{\theta}$  is the expectation conditioned on  $\theta$ ,  $\partial_i \equiv \partial/\partial\theta_i$  denotes a partial derivative with respect to  $\theta_i$ , the  $i^{\text{th}}$  component of  $\theta$ , and  $J(\theta)$  is a square matrix known as the Fisher information matrix (FIM). The practical value of the CRLB lies mostly in the following property [2]: under the assumption of long observation interval, it is theoretically possible to construct an estimator (the maximum likelihood estimator) whose error covariance matrix reaches the absolute minimum predicted by the CRLB. In this respect, the CRLB provides a benchmark against which the performance of the various localization methods can be compared.

In the present context, however, the evaluation of the CRLB is considerably complicated by the fact that the channel impulse response  $L(t, u)$  in (1) is not time-invariant. This follows because of the relative source-receiver motion in Fig. 1. This problem is partially resolved in [2] by assuming that the sensor noise is spatially uncorrelated, i.e. that  $E[n_i(t)n_j(u)] = 0$  whenever  $i \neq j$ . But in many applications, this assumption is not satisfied. This occurs, for example, when the noise field contains a directional plane wave component such as in Fig. 1.

In [3], a new approach to array processing in non-stationary environments which is not based on the assumption of spatially uncorrelated noise is presented. The main idea is to exploit the slowly-varying nature of  $L(t, u)$  encountered in most applications. In this approach,  $L(t, u)$  is characterized by a time-varying transfer function known as the *system function* and defined by

$$C(t, \omega) = \int_{-\infty}^{\infty} L(t, u) e^{-j\omega(t-u)} du \quad (3)$$

It is then required that the time variations of  $C(t, \omega)$  occur over intervals much larger than the channel correlation time. Accordingly, the channel impulse response  $L(t, u)$  is referred to as semi-stationary. This assumption results in tremendous analytical simplifications and makes it possible to obtain relatively simple expressions for the components of the FIM. More precisely, it can be shown that

$$J_{ij}(\theta) = \frac{1}{2\pi} \iint A \partial_i C^H N^{-1} [\partial_j C G C^H + C G \partial_j C^H - C G \partial_j \Omega G C^H] N^{-1} C d\omega dt \quad (4)$$

where the superscript  $H$  denotes complex conjugate transposition and

$$G = A/(1 + A \Omega), \quad (5)$$

$$\Omega = C^H N^{-1} C. \quad (6)$$

Note that in (4)-(6), the dependence of  $A$ ,  $C$ ,  $G$ ,  $N$  and  $\Omega$  on  $t$ ,  $\omega$  and  $\theta$  has been omitted for convenience. In the next section, we shall specialize (4) to the problem of time-varying delay estimation in the presence of directional interference.

### III. CRLB FOR TIME-VARYING DELAY ESTIMATION IN DIRECTIONAL NOISE FIELD

Consider again the situation represented in Fig. 1. Suppose that the signal transmission from the source to the sensors is ideal and that the noise field consists of a single, localized source of interference superposed on a spatially uncorrelated background noise. Under these conditions, the output of the  $i^{\text{th}}$  sensor can be written as

$$x_i(t) = a(t - d_i(t)) + \eta(t - \delta_i) + v_i(t). \quad (7)$$

where  $\eta(t)$  is the interference signal,  $v_i(t)$  is the spatially uncorrelated background noise component,  $d_i(t)$  is the time-varying transmission delay between the source and the  $i^{\text{th}}$  sensor, and  $\delta_i$  is the transmission delay for the interference. In the following analysis, it is assumed that  $\eta(t)$  and  $v_i(t)$  ( $i = 1, \dots, M$ ) are zero-mean uncorrelated stationary Gaussian random processes with power spectral densities  $N_I(\omega)$  and  $N_B(\omega)$ , respectively. Note that  $N_B(\omega)$ ,  $N_I(\omega)$  and the  $\delta_i$  are assumed known, while the delays  $d_i(t)$  depend explicitly on the unknown parameter vector  $\theta$  to be estimated.

In light of the observation model (1), the channel impulse response  $L(t, u)$  corresponding to (7) is given by

$$L(t, u) = \phi(t) [\delta(t - d_1(t) - u), \dots, \delta(t - d_M(t) - u)]^T \quad (8)$$

where  $\delta(t)$  represents the Dirac delta function and

$$\phi(t) = \begin{cases} 1, & -T/2 < t < T/2 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

is introduced to account for the finite observation window. In this respect,  $T$  represents the total observation interval.

The conditions under which  $L(t, u)$  (8) is semi-stationary have been investigated in [3] and are summarized below. Let  $B$  denote the system bandwidth and suppose that  $|d_i(t)| \leq D_0$  and  $|d_i'(t)| \leq D_1$ , where  $D_0$  and  $D_1$  are the maximum delay and delay rate, respectively. Then, for  $L(t, u)$  to be semi-stationary, the following asymptotic conditions must be satisfied:  $BT \gg 1$ ,  $D_0/T \ll 1$ ,  $D_1 \ll 1$  and  $BD_0D_1 \ll 1$ . These conditions, whose interpretation can be found in [3], are satisfied in most practical applications because of the usually low source speed when compared to the signal transmission velocity.

The first step in the specialization of (4) to the above observation model is to evaluate the system function  $C(t, \omega)$  (3), the inverse noise power spectral density  $N(\omega)^{-1}$  and the functions  $G(t, \omega)$  (5) and  $\Omega(t, \omega)$  (6). In this case, the evaluation of  $C(t, \omega)$  is trivial and the result is

$$C(t, \omega) = \phi(t) [e^{j\omega d_1(t)}, \dots, e^{j\omega d_M(t)}]^H \quad (10)$$

From (7) and the assumptions made on the noise field, we have

$$N(\omega) = N_I(\omega)D(\omega)D^H(\omega) + N_B(\omega)I_M \quad (11)$$

where  $I_M$  is the  $M \times M$  identity matrix and

$$D(\omega) = [e^{j\omega \delta_1}, \dots, e^{j\omega \delta_M}]^H \quad (12)$$

is the steering vector of the interference source. The inverse of  $N(\omega)$  (11) can be found by applying the matrix inversion lemma. The result is

$$N^{-1}(\omega) = \frac{1}{N_B(\omega)} [I_M - \frac{g(\omega)}{M} D(\omega)D^H(\omega)] \quad (13)$$

where

$$g(\omega) = \frac{\text{INR}_0}{1 + \text{INR}_0}, \quad (14)$$

$$\text{INR}_0 = M \frac{N_I(\omega)}{N_B(\omega)}. \quad (15)$$

The quantity  $\text{INR}_0$  (15) is the interference-to-noise ratio at the output of a conventional beamformer steered at the interference. The quantity  $g(\omega)$  (14) is a Wiener type shaping function satisfying  $0 \leq g(\omega) \leq 1$ , with  $g(\omega) = 1$  only in the limit  $\text{INR}_0 = \infty$ .

Using (10) and (13), the following expressions for  $\Omega(t, \omega)$  (6) and  $G(t, \omega)$  (5) can be obtained:

$$\Omega(t, \omega) = \frac{M}{N_B(\omega)} [1 - g(\omega) |\rho(t, \omega)|^2], \quad (16)$$

$$G(t, \omega) = \frac{A(\omega)}{1 + \text{SNR}_0 [1 - g(\omega) |\rho(t, \omega)|^2]}, \quad (17)$$

where

$$\text{SNR}_0 = M \frac{A(\omega)}{N_B(\omega)}, \quad (18)$$

$$\rho(t, \omega) = \frac{1}{M} \sum_{\mu=1}^M e^{j\omega[d_{\mu}(t) - \delta_{\mu}]}. \quad (19)$$

The quantity  $\text{SNR}_0$  represents the signal-to-noise ratio at the output of a conventional beamformer aimed at a fixed source with PSD  $A(\omega)$ . For a fixed value of  $t = t_0$ , the quantity  $\rho(t_0, \omega)$  is analogous to the complex response of a conventional beamformer steered along  $C(t_0, \omega)$  to a directional wavefront with steering vector  $D(\omega)$ . Observe that  $0 \leq |\rho(t, \omega)| \leq 1$ , with  $|\rho(t, \omega)| = 1$  only when  $d_{\mu}(t) - \delta_{\mu}$

is constant for all values of  $\mu$ , i.e. when the source and the interference are perfectly aligned in the delay space.

To simplify the evaluation of the FIM, it is assumed that

$$\sum_{\mu=1}^M d_{\mu}(t) = \sum_{\mu=1}^M \delta_{\mu} = 0. \quad (20)$$

In practice, this condition can always be satisfied by choosing the reference signals for the moving source and the directional interference at appropriate locations. Using (10), (13), (16), (17) and (20), a tedious but otherwise straightforward calculation reveals that for the observation model considered in this application, the elements  $J_{ij}(\theta)$  (4) of the FIM are given by

$$J_{ij}(\theta) = \frac{1}{2\pi} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} \frac{\text{SNR}_o^2}{1 + \text{SNR}_o(1-g|\rho|^2)} \left\{ E_{ij} + \frac{F_{ij}}{1 + \text{SNR}_o(1-g|\rho|^2)} \right\} d\omega dt, \quad (21)$$

where

$$E_{ij} = \frac{\omega^2}{M} (1-g|\rho|^2) \sum_{\mu=1}^M \partial_i d_{\mu} \partial_j d_{\mu} - g \partial_i \rho \partial_j \rho^*, \quad (22)$$

$$F_{ij} = g^2 \rho^* \partial_i \rho \partial_j |\rho|^2. \quad (23)$$

In the absence of interference,  $g(\omega)$  (14) is null and these expressions reduce to those given in [2], as expected.

#### IV. EFFECTS OF INTERFERENCE ON DIFFERENTIAL DOPPLER SHIFT ESTIMATION

Equations (21)-(23) can be used to evaluate the CRLB (2) regardless of the particular parametrization  $\theta$  used to characterize the delay functions  $d_i(t)$  in (7). However, to obtain a better understanding of the effects of directional interference on the CRLB, as predicted by these general expressions, it is preferable to focus our attention on a simple example involving only a few parameters. To this end, consider a two-sensor array ( $M=2$ ) and suppose that the delay functions  $d_i(t)$  vary linearly with time, so that

$$d_1(t) = -d_2(t) = \theta_0 + \theta_1 t, \quad (24)$$

$$\delta_1 = -\delta_2 = \delta_0, \quad (25)$$

where  $\theta_0$  and  $\theta_1$  are unknown parameters to be estimated. In physical terms, the parameter  $\theta_0$  represents the time average value of  $d_1(t)$  over the observation interval  $-T/2 \leq t \leq T/2$ , while  $\theta_1 T$  represents the total variation in  $d_1(t)$  over this interval. The parameter  $\theta_1$  is also referred to as the differential Doppler shift [2]. For  $d_i(t)$  (24) and  $\delta_i$  (25), the complex response (19) is given by

$$\rho(t, \omega) = \cos \omega(\theta_0 - \delta_0 + \theta_1 t). \quad (26)$$

Substituting (24) and (26) in (21)-(23), we obtain

$$J_{ij}(\theta) = \frac{1}{2\pi} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} \omega^2 t^{i+j} \frac{\text{SNR}_o^2}{1 + \text{SNR}_o(1-g \cos^2 \alpha)} \left\{ 1 - g + \frac{2g^2 \sin^2 \alpha \cos^2 \alpha}{1 + \text{SNR}_o(1-g \cos^2 \alpha)} \right\} d\omega dt \quad (27)$$

where

$$\alpha = \omega(\theta_0 - \delta_0 + \theta_1 t). \quad (28)$$

Equation (27) clearly shows one of the effects of interference on the FIM: the introduction of "oscillations" in the integrand of  $J_{ij}(\theta)$ , which can be interpreted as the time-frequency distribution of mutual information. These oscillations, characterized by the trigonometric functions of  $\alpha$ , become more and more important as the level of interference  $g(\omega)$  increases from 0 to 1. In this respect, it is interesting to compare the expressions of  $J_{ij}(\theta)$  obtained from (27) in the limiting cases  $g(\omega) = 0$  (i.e.  $\text{INR}_o = 0$ ) and  $g(\omega) = 1$  (i.e.  $\text{INR}_o = \infty$ ).

When  $g(\omega) = 0$ ,

$$J_{ij}(\theta) = \frac{1}{2\pi} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} \omega^2 t^{i+j} \frac{\text{SNR}_o^2}{1 + \text{SNR}_o} d\omega dt. \quad (29)$$

In this case, the time-frequency distribution of information is smooth (i.e. no oscillations). Since  $\text{SNR}_o$  is independent of  $t$ , the integral over  $t$  in (29) can be performed separately. In particular, we find that  $J_{10} = 0$  regardless of the shape of the spectral function  $\text{SNR}_o$ . As already indicated in [2], this means that there is no mutual information between  $\theta_0$  and  $\theta_1$ , or equivalently, that the estimation errors associated with  $\theta_0$  and  $\theta_1$ , as predicted by the CRLB, are uncorrelated. Also of interest is the behavior of  $J_{ij}(\theta)$  (29) as a function of  $\text{SNR}_o$ : at low  $\text{SNR}_o$  ( $\text{SNR}_o \ll 1$ ),  $J_{ij}(\theta)$  varies like  $\text{SNR}_o^2$ , while at high  $\text{SNR}_o$  ( $\text{SNR}_o \gg 1$ ),  $J_{ij}(\theta)$  varies like  $\text{SNR}_o$ .

The situation is quite different in the limit  $g(\omega) = \infty$  where (27) reduces to

$$J_{ij}(\theta) = \frac{1}{\pi} \int_{-T/2}^{T/2} \int_{-\infty}^{\infty} \omega^2 t^{i+j} \left\{ \frac{\text{SNR}_o \sin \alpha \cos \alpha}{1 + \text{SNR}_o \sin^2 \alpha} \right\}^2 d\omega dt. \quad (30)$$

In this case, the time-frequency distribution of mutual information is completely dominated by oscillations. Because of this complex behavior, it is no longer possible to perform the integral over  $t$  separately and in general,  $J_{10} \neq 0$ . The presence of non-zero off-diagonal elements in the FIM means that the estimation errors of  $\theta_0$  and  $\theta_1$  will now be correlated. Another very interesting feature of (30) is its behavior as a function of  $\text{SNR}_o$ . Indeed, at high  $\text{SNR}_o$  ( $\text{SNR}_o \gg 1$ ), (30) becomes independent of  $\text{SNR}_o$ , and reduces to

$$J_{ij}(\theta) = \frac{1}{\pi} \int_{-T/2}^{T/2} \int_{-B}^B \omega^2 t^{i+j} \cot^2 \alpha d\omega dt, \quad (31)$$

where  $B$  represents the signal bandwidth. Observe that (31) can also be obtained from (27) by letting  $N_B(\omega) \rightarrow 0$ . In this case, the background noise is null and the received signal is contaminated by directional interference alone. The CRLB obtained by inverting the FIM (31) therefore represents the performance limit on time delay estimation variance due solely to the presence of directional interference.

To investigate further the effects of interference on the CRLB, we must have recourse to numerical evaluation of (27). To simplify the discussion, we assume that  $\theta_0$  is known and we concentrate on the effects of directional interference on the estimation of the differential Doppler shift  $\theta_1$  alone. When  $\theta_0$  is known, the CRLB (2) reduces to

$$E_{\theta} \{ (\hat{\theta}_1 - \theta_1)^2 \} \geq 1/J_{11}(\theta). \quad (32)$$

The effects of interference on the CRLB are more easily interpreted in terms of the *degradation ratio* DR, simply defined as the CRLB (32) normalized by its value in the absence of interference. That is,

$$\text{DR} = \frac{J_{11}(\theta)|_{g=0}}{J_{11}(\theta)} \quad (33)$$

where the subscript  $g=0$  indicates that  $g(\omega)$  is set equal to zero in (27).

For the numerical evaluation of DR (33), the functions  $\text{INR}_o(\omega)$  (15) and  $\text{SNR}_o(\omega)$  (18) are modeled as flat low-pass spectra with bandwidth  $B$  and amplitude  $\text{INR}_o$  and  $\text{SNR}_o$ , respectively. The results of the computations are shown in Fig. 2-5. In each of these figures, which correspond to different values of  $\text{SNR}_o$  ranging from -10dB to +10dB, DR is plotted as a function  $B(\theta_0 - \delta_0)$  for  $BT\theta_1 = 1$  and  $\text{INR}_o = -10\text{dB}, 0\text{dB}, 10\text{dB}$  and  $+\infty$ . The quantity  $B(\theta_0 - \delta_0)$  represents the delay separation between the moving source and the directional interference at time  $t=0$ , normalized by the source signal correlation time  $1/B$ , while  $BT\theta_1$  represents the total variation in  $d_1(t)$  (24), again normalized by  $1/B$ .

To begin, consider Fig. 2, which corresponds to  $\text{SNR}_o = -10\text{dB}$ . At very low interference-to-noise ratio, i.e.  $\text{INR}_o = -10\text{dB}$ , the effect of interference on the CRLB (32) is negligible and DR remains close to 1. As expected, DR increases with  $\text{INR}_o$  and for  $\text{INR}_o = 0\text{dB}$ , DR reaches a value close to 1.7 (2.3dB). It is interesting to note that DR remains close to this value even when  $B(\theta_0 - \delta_0)$  is large. In other words, the negative effects of directional interference on the estimator variance do not disappear as the distance between the source and the interference in the delay space increases. The presence of oscillations

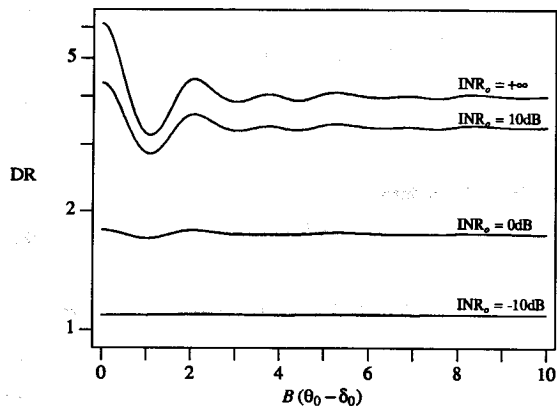


Fig. 2. DR as a function of  $B(\theta_0 - \delta_0)$  for  $\text{INR}_0 = -10\text{dB}, 0\text{dB}, 10\text{dB}$  and  $+\infty$ ,  $\text{SNR}_0 = -10\text{dB}$ , and  $BT\theta_1 = 1$ .

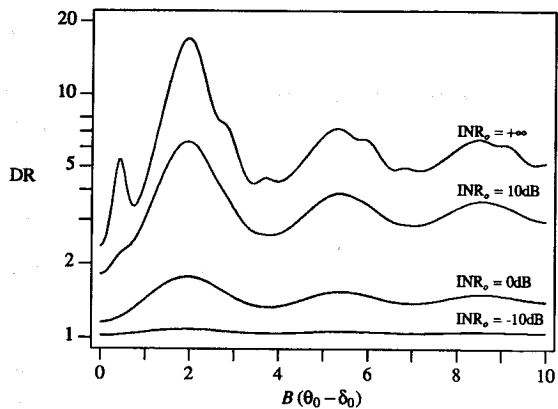


Fig. 5. DR as a function of  $B(\theta_0 - \delta_0)$  for  $\text{INR}_0 = -10\text{dB}, 0\text{dB}, 10\text{dB}$  and  $+\infty$ ,  $\text{SNR}_0 = 10\text{dB}$ , and  $BT\theta_1 = 1$ .

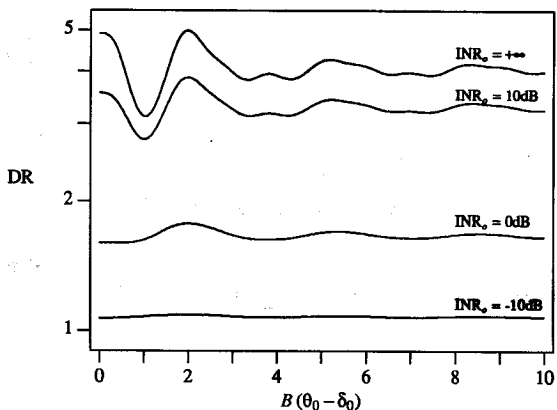


Fig. 3. DR as a function of  $B(\theta_0 - \delta_0)$  for  $\text{INR}_0 = -10\text{dB}, 0\text{dB}, 10\text{dB}$  and  $+\infty$ ,  $\text{SNR}_0 = -3\text{dB}$ , and  $BT\theta_1 = 1$ .

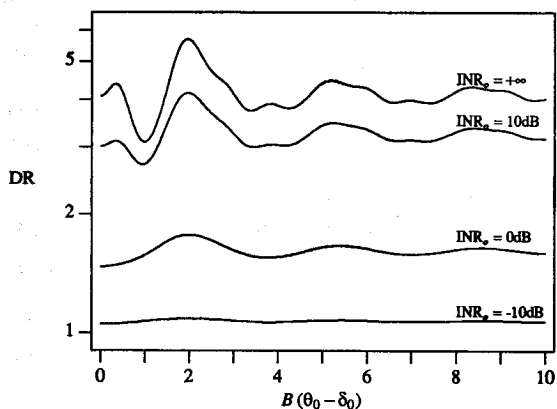


Fig. 4. DR as a function of  $B(\theta_0 - \delta_0)$  for  $\text{INR}_0 = -10\text{dB}, 0\text{dB}, 10\text{dB}$  and  $+\infty$ ,  $\text{SNR}_0 = 0\text{dB}$ , and  $BT\theta_1 = 1$ .

in the curves corresponding to larger values of  $\text{INR}_0$  is a direct consequence of the oscillatory nature of the integrand in (27). The exact shapes of these oscillations as well as the positions of the relative maximum and minimum depend on the values of  $\text{SNR}_0$  and  $BT\theta_1$ . For instance, in Fig. 2, the curves corresponding to  $\text{INR}_0 = 10\text{dB}$  and  $\text{INR}_0 = +\infty$  have a main peak at  $B(\theta_0 - \delta_0) = 0$  and a secondary one at  $B(\theta_0 - \delta_0) \approx 2$ . However, as  $\text{SNR}_0$  increases while  $BT\theta_1$  remains constant (Fig. 3-7), the amplitude of the peaks changes so that the secondary peak becomes the dominant one and vice-versa. This result is indeed remarkable for it means that the effects of directional interference on the CRLB can be worst when the moving source remains distinct from the interference than when it actually crosses the interference, which is the case when  $B(\theta_0 - \delta_0) = 0$ . This very interesting and previously unknown phenomenon can be attributed to the complex behavior of the integrand in (27).

## V. CONCLUSIONS

In this paper, the effects of directional interference on the accuracy of time-varying delay estimates obtained with a passive array of sensors were investigated. First, the results of [3] were used to derive new expressions for the Fisher information matrix of the sensor outputs when the latter contain a directional noise component. These expressions, which apply to arbitrary parametrizations of the time-varying delay functions, were then specialized to the problem of differential Doppler shift estimation with an array of 2 sensors.

Besides the overall deterioration in performance caused by the introduction of interference, two important conclusions can be reached. First, this deterioration does not disappear completely as the average source interference separation is increased. Secondly, for relatively small values of the separation, the deterioration behaves in an oscillatory pattern and can actually be worst when the moving source remains distinct from the interference than when it crosses the interference during the observation interval.

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