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MAXIMUM LIKELIHOOD ESTIMATION OF TIME-VARYING DELAYS IN THE PRESENCE OF DIRECTIONAL INTERFERENCE

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ABSTRACT

When a moving acoustic source is monitored with a passive array of sensors, the differential propagation delays between the signal components received at the array are time-varying. This paper investigates both the structure and performance of the maximum likelihood (ML) estimator of slowly-varying time delays when the received signals are contaminated by additive noise containing a strongly directional component, due to the presence of a fixed, localized source of interference in the acoustic environment. The ML estimator is obtained by maximizing the output of the log-likelihood processor. The latter is shown to consist of a slowly-varying noise canceller followed by a minimum mean square error estimator of the source signal and a correlator. Closed form expressions are obtained for the Cramer-Rao lower bound (CRLB) on the error covariance matrix of time-varying delay estimators. Finally, the effects of directional interference on the CRLB are investigated numerically for a simplified configuration consisting of two sensors and linearly varying time delays.

I. INTRODUCTION

When a moving acoustic source is monitored with a passive array of spatially distributed sensors, the differential propagation delays between the signal components received at the individual sensors are time-varying. By estimating these time-varying delay functions, it is possible to obtain important information about the location and track of the moving source [1]. The structure and performance of the maximum likelihood (ML) estimator of time-varying delay parameters have been studied extensively under the assumptions of slowly moving sources and spatially uncorrelated environmental noise field [2]-[3]. While the first assumption is generally satisfied, because of the relatively large sound propagation velocity, this is not the case for the latter. Indeed, in many applications, the noise field contains one or more spatially correlated components due to the presence of directional sources of interference in the acoustic environment. Because of mathematical difficulties related to the non-stationary nature of the problem, the structure and performance of the ML estimator of time-varying delays in the presence of directional interference have not yet been investigated. In this paper, we use the general results on array processing in semi-stationary environments recently derived by the authors [4] to conduct such an investigation.

II. OBSERVATION MODEL AND SEMI-STATIONARITY

Consider the physical situation shown in Fig. 1, where a passive array of M sensors is used to monitor a "slowly" moving acoustic source (S) in the presence of additive environmental noise. The total noise field consists of a directional component, originating from a fixed source of interference (I), superimposed on a spatially uncorrelated background noise (not represented in the figure).

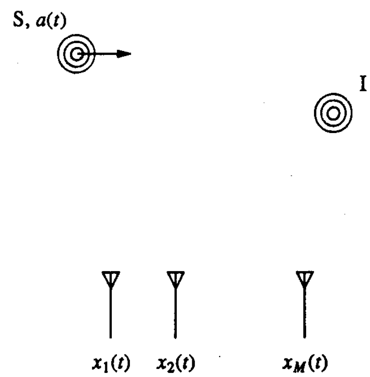

Fig. 1. M-sensor array monitoring a slowly-moving source (S) in the presence of directional interference (I).

Under the assumption that the signal transmission from the source (and the interference) to the sensors is ideal, i.e. pure time delay, the output of the i^{th} sensor can be written as

$$x_i(t) = a(t - d_i(t)) + \eta(t - \delta_i) + v_i(t), \quad -T/2 \le t \le T/2, \qquad (1)$$

where $a(t)$ is the moving-source signal, $\eta(t)$ is the interference signal, $v_i(t)$ is a spatially uncorrelated background noise component, $d_i(t)$ is the time-varying transmission delay between the source and the i^{th} sensor, δ_i is the corresponding transmission delay for the interference, and T is the duration of the observation interval. In the following analysis, it is assumed that, $a(t)$, $\eta(t)$ and $v_i(t)$ ($i=1,\ldots,M$) are zero-mean uncorrelated stationary Gaussian random processes with power spectral densities $A(\omega)$, $N_I(\omega)$ and $N_B(\omega)$, respectively. Note that $A(\omega)$, $N_B(\omega)$, $N_I(\omega)$ and the δ_i are assumed known, while the delays $d_i(t)$ are unknown.

In a typical source localization problem, the position and track of the moving source are defined in terms of a parameter vector θ. The components of θ might be, for example, the bearing and bearing rate of the source relative to a chosen orientation. This information is embedded in the time-varying delay functions $d_i(t)$ which depend explicitly on the parameter vector θ, which is unknown *a priori*. The localization problem then consists in processing the sensor outputs in such a way as to obtain an estimate of θ, generally denoted $\hat{\theta}(x)$. In this paper, we shall be concerned with the maximum likelihood (ML) estimator of θ.

The evaluation of the ML estimator of θ and the statistical analysis of its performance are considerably complicated by the fact that the linear transformation between the source signal $a(t)$ and the received signal component $a(t-d_i(t))$, in (1), is not time-invariant. While this problem can be partially resolved in the case of spatially uncorrelated noise [2]-[3] (i.e. when $\eta(t)=0$ in (1)), it

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remained until very recently a major obstacle in the detailed treatment of more general observation models such as (1).

In [4], a new approach to array processing in non-stationary environment was presented which does not rely on the assumption of spatially uncorrelated noise. This approach exploits the fact that in most applications of array processing, the transmission channel between the source and the sensors varies only "slowly" over time. More specifically, if we denote by $L(t,u)$ the impulse response of the channel and by $C(t,\omega)$ its *system function*, defined as

$$C(t,\omega) = \int L(t,u)\, e^{-j\omega(t-u)}\, du, \tag{2}$$

we find that in many applications, $C(t,\omega)$ remains almost contant over time intervals on the order of the correlation time of $L(t,u)$. In [4], the concept of *semi-stationarity* is introduced to characterize formally impulse responses satisfying this condition. Then, by requiring that the transmission channel in an arbitrary array processing problem be semi-stationary, new and very general expressions are obtained for the log-likelihood processor of the sensor outputs and for the associated Cramer-Rao lower bound on the error covariance matrix of parameter estimates. In this paper, we shall use these general expressions to analyse both the structure and performance of the ML estimator of θ. Before doing this, however, we shall summarize the conditions that must be satisfied by the time-varying delay functions $d_i(t)$ in order for the corresponding transmission channel to be semi-stationary.

For the observation model (1), the channel impulse response $L(t,u)$ between the moving-source signal $a(t)$ and the vector of received signal components $[a(t-d_1(t)),\dots,a(t-d_M(t))]^T$ is given by

$$L(t,u) = [\delta(t-d_1(t)-u),\dots,\delta(t-d_M(t)-u)]^T \tag{3}$$

where $\delta(t)$ represents the Dirac delta function. The conditions under which $L(t,u)$ (3) is semi-stationary have been investigated in [4], using appropriate window functions to model the finite ($<\infty$) time-bandwith product of any practical system. These conditions are summarized below. Let B denote the system bandwidth and suppose that $|d_i(t)| \le D_0$ and $|d_i'(t)| \le D_1$, where D_0 and D_1 are the maximum delay and delay rate, respectively. Then, for $L(t,u)$ to be semi-stationary, the following asymptotic conditions must be satisfied: $BT \gg 1$, $D_0/T \ll 1$, $D_1 \ll 1$ and $BD_0D_1 \ll 1$. These conditions, whose interpretations can be found in [4], are satisfied in most applications because of the usually small source speed when compared to the signal transmission velocity.

III. ML ESTIMATOR

The ML estimator of the parameter vector θ, denoted $\hat{\theta}_{ML}(x)$, is obtained by maximizing the log-likelihood function (LLF) of the sensor outputs, denoted $\ln\Lambda(x;\theta)$, over a set of *a priori* parameter values θ. For the Gaussian observation model under consideration here, the LLF can be written in the form

$$\ln\Lambda(x;\theta) = \tfrac{1}{2}\,\{\int y^T(t)\,\hat{a}(t)\,dt \;-\; l_b\;\} \tag{4}$$

where the signals $y(t)$ and $\hat{a}(t)$ are obtained from the observed sensor output signals $x_i(t)$ by means of linear filtering operations and where l_b is a bias term independent of the observed signals. Note however, that the filtering operations and l_b depend on the unknown parameter vector θ to be estimated.

Under the assumption of semi-stationarity, closed form expressions for the signals $y(t)$ and $\hat{a}(t)$ and the bias term l_b can be obtained quite simply by specializing the general results presented in [4] to the observation model (1). Beginning with $y(t)$, we find

$$y(t) = \frac{1}{2\pi}\int Y(t,\omega)\, e^{j\omega t}\, d\omega \tag{5}$$

with

$$Y(t,\omega) = \frac{1}{N_B(\omega)}\,[C^H(t,\omega)X(\omega) - g(\omega)\rho(t,\omega)D^H(\omega)X(\omega)], \tag{6}$$

where

$$X(\omega) = \int x(t)\, e^{-j\omega t}\, dt \tag{7}$$

$$C(t,\omega) = [e^{j\omega d_1(t)},\dots,e^{j\omega d_M(t)}]^H \tag{8}$$

$$D(\omega) = [e^{j\omega\delta_1},\dots,e^{j\omega\delta_M}]^H \tag{9}$$

$$\mathrm{INR_o} = M\,\frac{N_I(\omega)}{N_B(\omega)} \tag{10}$$

$$g(\omega) = \frac{\mathrm{INR_o}}{1+\mathrm{INR_o}} \tag{11}$$

$$\rho(t,\omega) = \frac{1}{M}\,C^H(t,\omega)D(\omega). \tag{12}$$

In (7), $X(\omega)$ represents the Fourier transform of the observed sensor output signal $x(t)$, taken over the entire observation interval. The quantity $C(t,\omega)$ (8) is the system function associated with the impulse response (3). Note that $C(t,\omega)$ can be interpreted as a time-varying steering vector. In a similar way, $D(\omega)$ (9) is the steering vector of the interference source. In (10), $\mathrm{INR_o}$ represents the interference-to-noise ratio (in the absence of source signal) at the output of a conventional beamformer steered at the interference. The quantity $g(\omega)$ (11) is a Wiener-type frequency response satisfying $0 \le g(\omega) \le 1$, with $g(\omega)=1$ only in the limit $\mathrm{INR_o}=\infty$. For a fixed value of $t=t_o$, $\rho(t_o,\omega)$ (12) is analogous to the complex response of a conventional beamformer steered along $C(t_o,\omega)$, to a directional wavefront with steering vector $D(\omega)$. Observe that $0 \le |\rho(t,\omega)| \le 1$, with $|\rho(t,\omega)|=1$ when $d_\mu(t)-\delta_\mu$ is constant for all values of μ.

The term $C^H X$ in (6) can be interpreted as the output of a slowly varying beamformer continuously steered at the moving source (whose track is conditioned on the *a priori* value of θ), while the term $D^H X$ corresponds to the output of a conventional beamformer aimed at the fixed interference. The quantity $Y(t,\omega)$ can therefore be interpreted as the output of a narrow-band time-varying noise canceller, which differs from a conventional noise canceller by the dependence in t of $\rho(t,\omega)$ and $C(t,\omega)$. More generally, $y(t)$ (6) can be interpreted as the output of a wide-band noise canceller continuously "steered" at the moving source.

Specialization of the results of [4] to the observation model (1) yields the following expression for $\hat{a}(t)$ in (4):

$$\hat{a}(t) = \frac{1}{2\pi}\int G(t,\omega)\,Y(t,\omega)\,e^{-j\omega t}\,d\omega \tag{13}$$

where $Y(t,\omega)$ is given by (6) and

$$G(t,\omega) = \frac{A(\omega)}{1+\mathrm{SNR_o}[1-g(\omega)|\rho(t,\omega)|^2]}. \tag{14}$$

In (14), the quantity

$$\mathrm{SNR_o} = M\,\frac{A(\omega)}{N_B(\omega)} \tag{15}$$

represents the signal-to-noise ratio at the output of a conventional beamformer aimed at a fixed source with power spectral density $A(\omega)$. At low $\mathrm{SNR_o}$, $G(t,\omega)$ is approximately given by $A(\omega)$ and it is therefore independent of the unknown parameter vector θ. The situation is more complicated for intermediate values of $\mathrm{SNR_o}$ where in general, $G(t,\omega)$ further depends on the noise power spectral densities $N_B(\omega)$ and $N_I(\omega)$, and on the parameter vector θ through the complex response $\rho(t,\omega)$. We note however that at very high $\mathrm{SNR_o}$, $G(t,\omega)$ is approximately given by

$$G(t,\omega) = \frac{N_B(\omega)}{M\,[1-g(\omega)|\rho(t,\omega)|^2]} \tag{16}$$

which is independent of $A(\omega)$. These observations should be taken into consideration at the implementation level.

Under the assumption of semi-stationarity, multiplication of system functions in the time-frequency domain is equivalent to convolution in the time domain. Accordingly, we conclude from (13) that $\hat{a}(t)$ can be obtained directly from $y(t)$ by means of a filtering operation with system function $G(t,\omega)$. It can be shown that $\hat{a}(t)$ is actually the minimum mean square error estimate of the moving source signal $a(t)$ from the observation $x(t)$, $-T/2 < t < T/2$.

Finally, for the bias term l_b in (4), the application of the results of [4] yields the following result:

$$l_b = \frac{1}{2\pi} \int\int \ln\{1 + \mathrm{SNR_o}[1 - g(\omega)|\rho(t,\omega)|^2]\}\, d\omega dt. \tag{17}$$

We note that this term depends only on $\mathrm{SNR_o}$, $\mathrm{INR_o}$ and the complex response $\rho(t,\omega)$. Again, different cases occur in the evaluation of l_b depending on the value of $\mathrm{SNR_o}$.

The processor structure corresponding to the above equations for the LLF is shown in Fig. 2. This processor, referred to as the log-likelihood processor (LLP), consists of two functionally different subprocessors, namely: a slowly time-varying noise canceller (upper portion) followed by a quadratic post-processor (lower portion). The noise canceller performs the operation (5)-(12) on the sensor output signals $x_i(t)$. As indicated previously, it has one of its beams continuously steered toward the moving source while the other beam remains steered toward the fixed directional interference. The quadratic post-processor filters the output $y(t)$ of the noise canceller to form the minimum mean square error estimate of $a(t)$, denoted $\hat{a}(t)$, and then evaluates the scalar product of $y(t)$ and $\hat{a}(t)$. Finally, the bias term l_b is added. The ML estimator of θ is obtained by maximizing the output of this processor over the set of *a priori* values for θ. In theory, this maximization must be carried out in batch, i.e., once the observation of $x(t)$, $-T/2 < t < T/2$, is available, $\ln\Lambda(x;\theta)$ is evaluated for all possible values of θ in the parameter space and then maximized. In practice, though, because of computational limitations, the maximization can be achieved sequentially over time with the use of a recursive algorithm such as the Newton-Raphson algorithm.

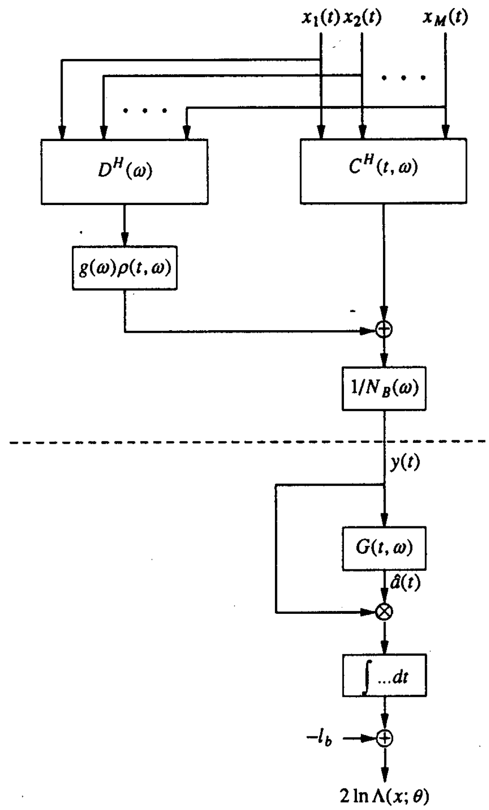

Fig. 2. LLP for ML estimation of time-varying delays in the presence of directional interference.

IV. CRLB

The CRLB sets a lower bound on the error covariance matrix of any unbiased estimator $\hat{\theta}(x)$ of θ. More precisely, it asserts that

$$E_\theta\{[\hat{\theta}(x)-\theta][\hat{\theta}(x)-\theta]^T\} \geq J(\theta)^{-1} \tag{18}$$

where E_θ is the expectation conditioned on θ and $J(\theta)$ is a square matrix known as the Fisher information matrix (FIM). The practical value of the CRLB lies mostly in the following property: in the small error regime (or equivalently, if the observation interval is sufficiently long) the error covariance matrix of the ML estimator of θ reaches the absolute minimum predicted by the CRLB. In this respect, the CRLB provides a realistic benchmark against which the performance of an arbitrary estimator of θ can be compared. In the following discussion, we use the general results presented in [4] to derive the FIM of the parameter vector θ associated with the observation model (1). We then use the resulting expressions to study the effects of directional interference on the estimation accuracy of time-varying delays, as predicted by the CRLB (18).

To simplify the evaluation of the FIM, it is assumed that

$$\sum_{\mu=1}^{M} d_\mu(t) = \sum_{\mu=1}^{M} \delta_\mu = 0. \tag{19}$$

In practice, this condition can be satisfied by taking the reference signals for the moving source and the directional interference at the proper locations. Making the appropriate specializations in the general results of [4] and using (19), a tedious but otherwise straightforward calculation reveals that for the observation model (1), the elements $J_{ij}(\theta)$ of the FIM are given by

$$J_{ij}(\theta) = \frac{1}{2\pi} \int_{-T/2}^{T/2}\int_{-B}^{B} \frac{\mathrm{SNR_o}^2}{1+\mathrm{SNR_o}(1-g|\rho|^2)} \left\{E_{ij} + \frac{F_{ij}}{1+\mathrm{SNR_o}(1-g|\rho|^2)}\right\} d\omega dt, \tag{20}$$

where

$$E_{ij} = \frac{\omega^2}{M}(1-g|\rho|^2)\sum_{\mu=1}^{M} \partial_i d_\mu \partial_j d_\mu \; - \; g\,\partial_i\rho\,\partial_j\rho^* \tag{21}$$

$$F_{ij} = g^2\rho^*\partial_i\rho\,\partial_j|\rho|^2 \tag{22}$$

In these expressions, B represents the source signal bandwidth (i.e., $A(\omega)=0$ for $|\omega| > B$), and $\partial_i = \partial/\partial\theta_i$ denotes a partial derivative with respect to θ_i, the i^{th} component of θ.

To obtain a better understanding of the effects of directional interference on the estimation accuracy of time-varying delays, as predicted by (20)-(22), it is preferable to focus our attention on a simple example involving only a few unknown parameters with direct physical interpretations. Consider a two-sensor array ($M=2$) and suppose that the delay functions $d_i(t)$ vary linearly with time. That is, let

$$-d_1(t) = d_2(t) = \tfrac{1}{2}(\theta_0 + \theta_1 \frac{t}{T}), \tag{23}$$

$$-\delta_1 = \delta_2 = \tfrac{1}{2}\delta_0, \tag{24}$$

where θ_0 and θ_1 are the unknown parameters to be estimated. Note that the condition (19) is satisfied by (23)-(24). The differential delay corresponding to $d_i(t)$ (23) is given by

$$\Delta(t) = d_2(t) - d_1(t) = \theta_0 + \theta_1 \frac{t}{T}. \qquad (25)$$

Hence, θ_0 represents the differential delay at time $t=0$, while θ_1 represents the total variation in differential delay over the observation interval. The normalized papameter θ_1/T is equal to the time derivative of $\Delta(t)$ at $t=0$, also referred to as the differential Doppler shift.

For $d_i(t)$ (23) and δ_i (24), the complex response (12) is given by

$$\rho(t,\omega) = \cos(\omega\gamma(t)/2). \qquad (26)$$

$$\gamma(t) = \Delta(t) - \delta_0. \qquad (27)$$

Substituting (23) and (26) in (20)-(22), we obtain

$$J_{ij}(\theta) = \frac{1}{8\pi T^{i+j}} \int_{-T/2}^{T/2} \int_0^B \omega^2 t^{i+j} \frac{\mathrm{SNR}_o^2}{1+\mathrm{SNR}_o(1-g\cos^2(\omega\gamma(t)/2))}$$

$$\{2(1-g) + \frac{g^2 \sin^2(\omega\gamma(t))}{1+\mathrm{SNR}_o(1-g\cos^2(\omega\gamma(t)/2))}\} d\omega dt \qquad (28)$$

Equation (28) clearly shows one of the effects of interference on the FIM: the introduction of "oscillations" in the integrand of $J_{ij}(\theta)$, which can be interpreted as the time-frequency distribution of mutual information. These oscillations, characterized by the trigonometric functions of $\omega\gamma(t)/2$, become more and more important as the level of interference $g(\omega)$ increases from 0 to 1.

To gain a deeper insight into the problem, the effects of directional interference on the CRLB were investigated numerically. For this purpose, the functions $\mathrm{INR}_o(\omega)$ (10) and $\mathrm{SNR}_o(\omega)$ (15) were modeled as flat low-pass spectra with bandwidth B and amplitude INR_o and SNR_o, respectively. The equation (28) was used to evaluate the components of the FIM $J(\theta)$. The CRLB was then obtained by formal inversion of $J(\theta)$ as in (18). The results of these computations appear in Fig. 3, which shows the components 00, 01 and 11 of the CRLB as a funtion of $B(\theta_0 - \delta_0)$ for fixed values of $B\theta_1 = 1$ and $\mathrm{SNR}_o = 0\mathrm{dB}$, and for five different values of the directional interference-to-noise ratio, namely INR_o = -10dB, 0dB, 3dB, 10dB and $+\infty$. Note that the quantity $B(\theta_0 - \delta_0)$ represents the time delay separation between the moving source signal and the directional interference at time $t=0$, normalized by the source signal correlation time $1/B$, while $B\theta_1$ represents the total variation in $\Delta(t)$ (25), again normalized by $1/B$.

Many important observations can be made from Fig. 3. At very low interference-to-noise ratio, i.e. INR_o = -10dB, the effect of interference on the CRLB is negligible, that is, the CRLB is nearly diagonal with diagonal elements almost equal to those that would be obtained by setting $g=0$ in (28). As expected, there is a performance degradation as INR_o increases, and for INR_o = 0dB, the diagonal elements of the CRLB increase by a factor of approximatly 1.7 (2.3dB). It is interesting to note that this degradation occurs regardless of the time delay separation between the moving source and the directional interference, i.e. $B(\theta_0 - \delta_0)$. In practical terms, this means that the degradation does not disappear as the angular separation between the source and the interference increases. We note however that for such values of INR_o, the off-diagonal elements of the CRLB remain close to zero, so that the estimation errors in estimating θ_0 and θ_1 still remain uncorrelated.

At higher values of INR_o, besides an overall deterioration relatively independent of $B(\theta_0 - \delta_0)$, the elements of the CRLB become dominated by oscillations, as can be predicted from (28). The exact shapes of these oscillations, as well as the locations of their relative extremum, depend on the values of SNR_o and $B\theta_1$. The local maxima of the oscillations in the diagonal elements of the

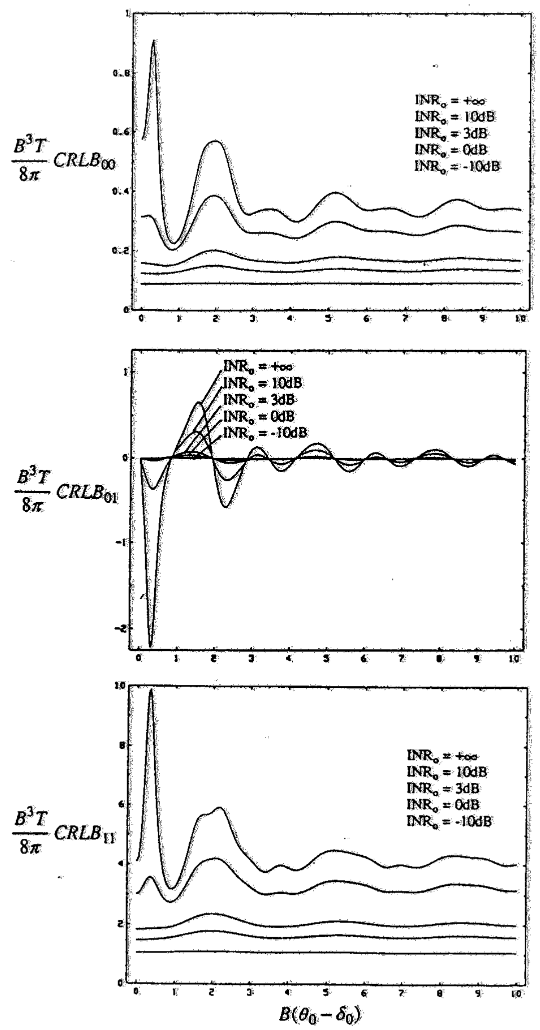

Fig. 3. Elements of CRLB $= J(\theta)^{-1}$ as a function of $B(\theta_0 - \delta_0)$ for $B\theta_1 = 1$, SNR_o = 0dB and INR_o = -10dB, 0dB, 3dB, 10dB and $+\infty$.

CRLB generally correspond to major increases in estimator variance for θ_0 and θ_1. Moreover, the oscillations occurring in the off-diagonal elements of the CRLB can now introduce a large degree of correlation between the estimation errors of θ_0 and θ_1. This constitutes an important observation since in applications of differential Doppler shift estimation to source localization problems, it is generally assumed that these estimation errors are uncorrelated.

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