CONVERGENCE PROPERTIES OF BLIND ALGORITHMS FOR BASE STATION CDMA RECEIVERS

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ABSTRACT

In this paper, the blind estimation of wireless CDMA receiver coefficients from the second order statistics of the signals is considered. Although many algorithms have been proposed so far, their performance analysis has always been carried out assuming perfect receiver coefficients estimation and/or under time-invariant conditions. In this article, we present some decision-directed blind algorithms and use a time-varying vector channel simulator to compare their performance with those of many recently proposed algorithms. It is shown that decision-directed chip-level algorithms can operate without the use of training sequences to avoid catastrophic error propagation, and that one should not expect an increase in performance from using least squares instead of least-mean-square. Furthermore the unpracticability of the bit-level algorithm [1] and of the least significant algorithm [4] under time varying environment is outlined. The performance of the principal component (Stanford) algorithm [7] is also studied.

1. INTRODUCTION

To increase the capacity of CDMA wireless systems which is interference limited, the negative effects of interchip-interference (ICI) and/or multiuser interference (MUI) must be reduced. Recently, 2-D RAKE receivers have been proposed to reduce these interferences by exploiting both the spatial and temporal diversity of the channel [3]. The effectiveness of using such an approach rely heavily on the estimation procedure used to obtain the coefficients of the 2-D RAKE receivers. In this paper, blind estimation of the receiver coefficients is considered.

Blind estimation exploiting the second order cyclostationarity property of the oversampled received signals [11] is not considered because of the increase in the problem dimensionality resulting from oversampling. Algorithms based on higher order statistics (e.g. [5]) are known to exhibit a relatively slow convergence [11] and are therefore not analyzed here. Only algorithms based on the second order statistics (SOS) of the signals are being considered in this study.

Many estimation procedures based on the SOS have been proposed so far (e.g. [1, 4, 7]). The performance analysis of such algorithms has always been carried out assuming perfect estimation of the receiver coefficients and/or assuming a time-invariant channel. In this article we use the time-varying (mixed-phase) channel simulator for the uplink transmission presented in [9] to evaluate and compare the performance of many blind estimation algorithms based on SOS. We consider different values of signal to interference plus noise ratio (SINR) and mobile speed.

2. DATA MODEL

We consider an asynchronous CDMA system with multiple receivers at the base station only. Under the narrowband array assumption (the array size must be $\ll$ the speed of light divided by the bandwidth of the incoming signal), the multipath transmission between the mobile of interest and the N_e-element antenna array can be represented by a baseband FIR vector channel.

The transmission system is modeled as a single-input multi-output (SIMO) discrete system, where the signals are sampled once every chip. The discrete-time version of the baseband N_e-dimensional vector channel impulse response is denoted by $\mathbf{h}_k(j)$, where the chip index j correspond to the source excitation time and k is the time-differentiable path index (in order for two paths to be time-differentiable, their relative delay of arrival must be greater then the inverse of the bandwidth of the transmitted signal). The baseband model for the N_e-dimensional sampled received signals at the base station receivers is

$$\mathbf{s}(j) \triangleq \sum_{k=0}^{M-1} \mathbf{h}_k(j-k)z(j-k) + \mathbf{e}(j) \tag{1}$$

where M is the number of time-differentiable paths, $z(j)$ is the chip sequence, and $\mathbf{e}(j)$ is the noise vector (including MUI). Assuming that each transmitted bit is spread into L chips, the aperiodic spreading code for the nth bit, $I(n)$, is denoted by $\mathbf{m}(n) = [m(nL) \; \ldots \; m(nL+L-1)]^T$, where $|m(nL+j)|$ is normalized to one, so that

$$z(j) = I(n)m(nL+j), \qquad n = \left\lfloor \frac{j}{L} \right\rfloor. \tag{2}$$

Note that if QPSK is used, the spreading code is complex.

The following assumptions are made: the information symbols $I(n)$ are binary and independent; the spreading codes are complex binary; the noise vector $\mathbf{e}(j)$ in (1) is zero-mean and white in time and space.

3. 2-D RAKE RECEIVER

The standard RAKE receiver [8] is a scalar filter that combines the time-differentiable paths of a received signal. Recently, a space-time generalization of the RAKE filter has been proposed [3] to

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exploit not only the path diversity but also the spatial diversity of the channel. The 2-D RAKE receiver for CDMA is illustrated in Fig 1. In this figure, $s_i(j)$ is the signal received at the ith antenna and sampled at the chip rate; $g_{ki}(j)$ is the kth coefficient for the receiving filter of order N corresponding to the ith antenna; $I'(n)$ is the transmitted bit estimate which is equal to the sign of the real part of $y(n)$, the despreader output. This despreader simply multiplies the spreading sequence for the nth bit with the filtered received signal corresponding to that same bit and does a summation over the chips. The signals being feed into the despreader are therefore at the chip rate while those coming out are at the bit rate. In order for the 2-D RAKE receiver to

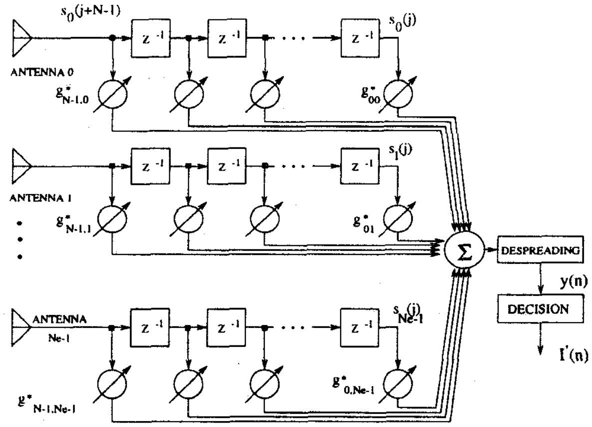

Figure 1: 2-D RAKE receiver for CDMA.

properly detect the nth transmitted bit $I(n)$, one must select the appropriate N_eN-dimensional channel-dependent weight vector $\mathbf{g}(j) = [g_{00}(j) \quad g_{01}(j) \quad \cdots \quad g_{N-1,N_e-1}(j)]^T$.

The optimum receivers directly obtain the estimate of the optimal equalizer coefficients from the recursive minimization of a given error criterion. The various optimum receivers differ by the type of error criteria or by the type of adaptive algorithms used to obtain the coefficients. Presented next are the three error criteria that we consider.

3.1. Chip-level mean-square error on the transmitted signal

One commonly used criterion to minimize is the chip-level mean-square error (MSE), defined as

$$E\left\{\mathbf{g}(j)^H \mathbf{s}_N(j) - I(n)m(nL+j)\right\}, \tag{3}$$

where

$$\mathbf{s}_N(j) \triangleq \left[\mathbf{s}^H(j) \quad \cdots \quad \mathbf{s}^H(j+N-1)\right]^H. \tag{4}$$

The optimal $\mathbf{g}(j)$ in that case is given by

$$\mathbf{g}(j) = \mathbf{R}^{-1}_{\mathbf{s}_N,\mathbf{s}_N}(j)\, \mathbf{r}_N(j), \tag{5}$$

where $\mathbf{R}_{\mathbf{s}_N,\mathbf{s}_N}(j) = E\left\{\mathbf{s}_N(j)\mathbf{s}_N^H(j)\right\}$ and $\mathbf{r}_N(j) = E\left\{\mathbf{s}_N(j)I(n)m^*(n; j-nL)\right\}$. From (1), (2) and (4), it can be shown that (assuming here that $N > M$)

$$\mathbf{r}_N(j) = \left[\mathbf{h}_0^H(j) \quad \cdots \quad \mathbf{h}_{M-1}^H(j) \quad \mathbf{0} \quad \cdots \quad \mathbf{0}\right]^H. \tag{6}$$

One can show that $\mathbf{R}_{\mathbf{s}_N,\mathbf{s}_N}(j)$ tends towards a scaled identity matrix as the number of interferes surrounding the base increases. That means that in such a case the optimal receiver coefficients for the kth path are well approximated by $\mathbf{g}_k(j) = \mathbf{h}_k(j)$, $k = 0, \ldots, N-1$ (channel matched filter).

3.2. Bit-level mean-square error on the transmitted signal

An alternate way to look at the optimum receiver is to consider the following bit-level mean-square error:

$$E\left\{|y(n) - I(n)|^2\right\} = E\left\{|\mathbf{g}^H \mathbf{S}_N(n)\mathbf{m}^*(n) - I(n)|^2\right\} \tag{7}$$

$$= E\left\{|\mathbf{g}^H \mathbf{x}_N(n) - I(n)|^2\right\} \tag{8}$$

where

$$\mathbf{S}_N(n) = \left[\mathbf{s}_N(nL) \quad \cdots \quad \mathbf{s}_N(nL+L-1)\right], \tag{9}$$

$$\mathbf{x}_N(n) = \mathbf{S}_N(n)\mathbf{m}^*(n), \tag{10}$$

and we temporarily suppose that the receiver coefficients (the elements of $\mathbf{g}$) are time-invariant over the observation interval. The optimum receiver coefficients are then given by

$$\mathbf{g}(n) = \mathbf{R}^{-1}_{\mathbf{x}_N,\mathbf{x}_N}(n)\, \mathbf{r}_N(n), \tag{11}$$

where $\mathbf{R}_{\mathbf{x}_N,\mathbf{x}_N}(n) \triangleq E\left\{\mathbf{x}_N(n)\mathbf{x}_N^H(n)\right\}$. One can show that the equalizer coefficients given by (5) and (11) both minimize (3) *and* (8) [4], i.e. the optimal chip-level and bit-level equalizers give identical SINR after convergence.

3.3. Chip-level mean-square error on the received signal

From (5) and (6) one can see that the optimal receiver can be easily obtained from the channel estimate. The channel estimation procedure can be done independently for each antenna i and generally involves the minimization of a mean-square-error performance measure such as ($i = 0, 1, \ldots, N_e - 1$)

$$E\left\{\|\mathbf{s}_i(n) - \mathbf{G}(n)\mathfrak{h}_i\|^2\right\} \tag{12}$$

where

$$\mathbf{s}_i(n) = \left[s_i(nL) \quad \cdots \quad s_i(nL+L-1)\right]^T \tag{13}$$

$$\mathbf{G}(n) = \begin{bmatrix} z(nL) & \cdots & z(nL-N+1) \\ z(nL+1) & \cdots & z(nL-N+2) \\ \vdots & \cdots & \vdots \\ z(nL+L-1) & \cdots & z(nL+L-N) \end{bmatrix} \tag{14}$$

$$\mathfrak{h}_i = \left[h_{0i} \quad h_{1i} \quad \cdots \quad h_{N-1,i}\right]^T \tag{15}$$

$s_i(nL+j)$ is the $(i+1)$th element of the vector $\mathbf{s}(nL+j)$ in (1), and h_{ki} is the $(i+1)$th element of the vector $\mathbf{h}_k$ in (1) that we want to estimate.

4. BLIND ALGORITHMS FOR RAKE RECEIVERS

In this section, the various blind algorithms considered are introduced. The three subsections correspond to the three error criteria presented in the previous section.

Note that decision-directed blind equalization schemes exploiting the known spreading code in CDMA give and ambiguity in the sign of the detected bit sequence. Differential encoding/decoding of the bit sequences must therefore be used at the transmitter/receiver to ensure proper detection. Other blind equalization schemes which are based on the computation of an eigenvector exhibit a phase ambiguity in the post-decision variable $y(n)$ sequence, differential decoding is then done on this sequence rather than ulteriorly on the bit sequence.

4.1. Chip-level equalizer

4.1.1. Decision-directed chip-level algorithms (MMSE-C)

We propose to use a vectorial decision-directed version of the error criterion (3):

$$E\left\{||\mathbf{g}^H \mathbf{S}_N(n) - I'(n)\mathbf{m}^T(n)/L||^2\right\} \quad (16)$$

Note that by using a vectorial form the adaptation is done once a bit even if a chip-level error criterion is used. The least-mean-square (LMS), the direct matrix inversion (DMI) and the recursive least squares (RLS) algorithms [6] can be used to recursively update the channel coefficients. Depending on which adaptation algorithm is used, we will refer to the equalizer algorithm as the LMS-, the DMI- or the RLS-MMSE-C (where the -C stands for chip-level).

4.1.2. Least significant equalizer (LSE) [4]

The least significant equalizer is in fact the zero-forcing equalizer for an underloaded CDMA system where the number of effective antennas is greater than the number of time-indifferentiable multipath reflections in any time-differentiable path. In this case it is possible to perfectly recover the transmitted signals with a single-user receiver in the absence of ambient noise. The least significant equalizer is optimum in a deterministic sense in interference suppression even if the number of users is too high to permit zero-forcing conditions [4].

Assume that $\mathbf{g}$ corresponds to a zero-forcing equalizer for (16), we have $\mathbf{g}^H \mathbf{S}(n) = I'(n)\mathbf{m}^T(n)/L$. The only unknowns are $\mathbf{g}$ and the transmitted bits, $I'(n)$. A minimization problem with respect to these unknowns can therefore be formulated and the resulting optimal receiver coefficient vector is given by the least significant eigenvector of $\mathbf{R}_{\mathbf{S}_N,\mathbf{S}_N}(n) - \mathbf{R}_{\mathbf{x}_N,\mathbf{x}_N}(n)/L$ [4]. Note that this algorithm is not decision-directed.

4.2. Bit-level equalizer (MMSE-B) [1]

The cost function to minimize is a decision-directed version of (8): $E\left\{[I'(n) - y(n)]^2\right\}$. The LMS, DMI and RLS algorithms are used to recursively update the channel coefficients. The LMS algorithm was used in [1]. Depending on which adaptation algorithm is used we will refer to the equalizer algorithm as the LMS-, the DMI- or the RLS-MMSE-B (where the -B stands for bit-level).

4.3. Via channel estimation (CE)

4.3.1. Decision-directed algorithms

We propose to use the cost function (12) with the matrix $\mathbf{G}(n)$ replaced by its estimate obtained from the known code and the past detected bits (decision-directed). The channel estimation is done independently for each antenna $k = 0, \ldots, N-1$. Once again the LMS, DMI and RLS algorithms can be used and depending on which is chosen we will refer to the equalizer algorithm as the LMS-, the DMI- or the RLS-CE. Note that since $z(j)$ has a null autocorrelation for non-zero lags, the matrix to invert in DMI-CE will be close to a scaled identity matrix. Its computation might therefore be unnecessary.

	$i=0$	$i=1$	$i=2$	$i=3$
θ_i (degrees)	90	150	270	30
$2\Delta_i$ (degrees)	5	10	2	5

Table 1: Path angle of arrival parameters.

4.3.2. Principal Component algorithm (PC-MMSE)

This algorithm, commonly referred to as the Stanford algorithm, is based on the following observation [10]:

$$\mathbf{R}_{\mathbf{x}_N,\mathbf{x}_N}(n) - \mathbf{R}_{\mathbf{S}_N,\mathbf{S}_N}(n) = (L^2 - L)\,\mathbf{r}_N(n)\,\mathbf{r}_N^H(n), \quad (17)$$

where $\mathbf{R}_{\mathbf{S}_N,\mathbf{S}_N}(n) \triangleq E\left\{\mathbf{S}_N(n)\mathbf{S}_N^H(n)\right\}$. This indicates that $\mathbf{r}_N(n)$ can be determined as the principal eigenvector of the difference between the post- and pre-despreading correlation matrices. Once this eigenvector is calculated the optimal receiver coefficients can be obtained from (5) and the estimated pre-despreading correlation matrix. Note that in this paper, contrarily to what is done in [7], we take a composite channel approach (as in [4]) to deal simultaneously with all paths so that coherent combining is possible at the receiver.

5. NUMERICAL RESULTS AND CONCLUSIONS

We consider $M = N = 4$ time-differentiable paths and $N_e = 3$ receiving antennas uniformly distributed around a horizontal circular array and separated by half a wavelength. To simplify the analysis of the directivity of the channel we assume that all the propagation paths lie in the same plane as the array, and that they are uniformly distributed in azimuth angle in $\theta_i \pm \Delta_i$, where θ_i is the mean angle of arrival and $2\Delta_i$ is the angle spread for the i^{th} path (see Table 1). The carrier frequency f_c is $1GHz$ and the transmitted signal bandwidth B is $1.25MHz$. The spreading factor $L = 128$. The voice activity factor is 0.4.

Fig. 2 illustrates the convergence behavior of the BER versus time for $v = 15m/s$, $N_u = 200$; $v = 30m/s$, $N_u = 200$; and $v = 30m/s$, $N_u = 80$, where v is the mobile speed and N_u is the number of interferes. The number of realizations used to obtained the estimate is 4800. From the observation of these results and similar ones, we come to the following conclusions:

1. The convergence of the LSE equalizer is extremely slow and precludes its usage in a time-varying environment.
2. Bit-level equalizers (RLS-MMSE-B and LMS-MMSE-B) have poor BER performance in a time-varying environment once the initial conditions have been forgotten. An explication for this convergence problem comes from the high condition number of the cross-correlation matrix $\mathbf{R}_{\mathbf{x}_N,\mathbf{x}_N}(n)$ which result in a slow convergence of the LMS algorithm and in a high sensitivity of the LS solution in this nonzero residual problem [2].
3. With the considered value of L (processing gain), the decision-directed chip-level algorithms (MMSE-C and CE) do not need initial or periodically transmitted training sequences to avoid catastrophic error propagation.

4. The increase in complexity resulting from the use of DMI or RLS over LMS does not translate in an increase in convergence speed or, generally, in tracking performance.
5. In all the studied cases, the PC-MMSE algorithm does not give an improvement in performance over what can be achieved with a more classical decision-directed approach. In fact, the PC-MMSE algorithm is noticeably worst than the chip-level decision-directed approaches for moderate mobile speed.
6. Overall the LMS-CE algorithm has the best performance.

Note that the numerical complexity of LMS-CE and DMI-CE (surprisingly RLS-CE is more computationaly complex) is dominated by the channnel-to-equalizer coefficients transformation. We observed (although not shown here) that for the considered values of N_u, this transformation is no longer required since taking the channel coefficients directly as equalizer coefficients gives similar BER performance (sometimes better, especially when the mobile speed is high and there is a high number of interferes). It is therefore possible to significantly reduce the complexity of the CE algorithms making them computationally competitive with the MMSE-C algorithms.

6. REFERENCES

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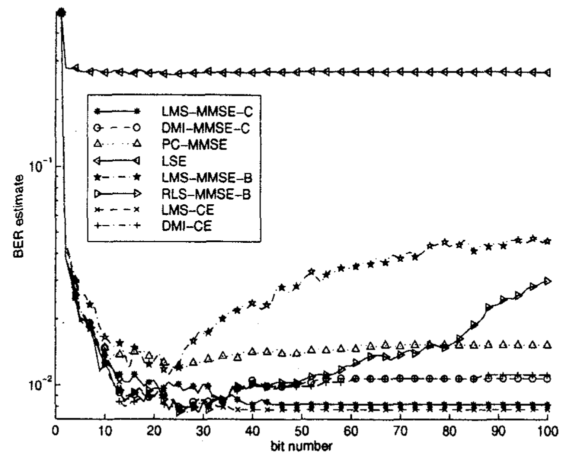
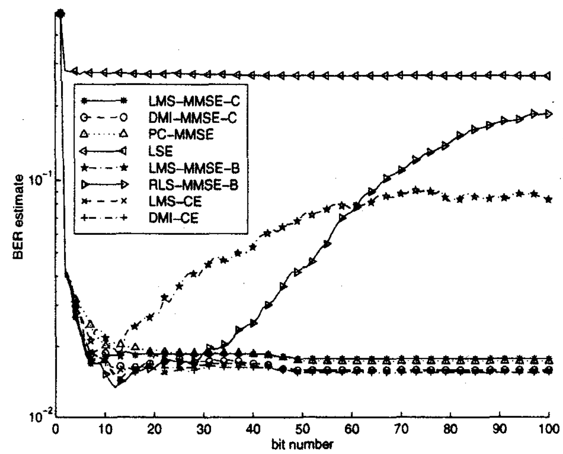
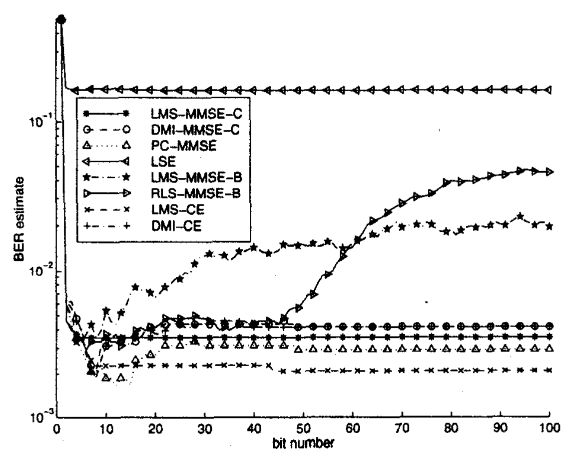


(a) $N_u = 200, v = 15m/s$



(b) $N_u = 200, v = 30m/s$



(c) $N_u = 80, v = 30m/s$

Figure 2: BER estimate vs bit index for different speed and number of interferes.