COOPERATIVE MIMO-BEAMFORMING FOR MULTIUSER RELAY NETWORKS

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ABSTRACT

In this paper, we develop a beamforming algorithm for multiuser MIMO-relaying wireless systems. We consider a relaying scenario with multiple sources transmitting to one or more destination nodes through several relay terminals. Each relay is equipped with multiple antennas. We jointly design the beamforming matrices of the cooperating relays by minimizing both the noise received at each destination node and the interference caused by the sources not targeting this node. We impose additional constraints that preserve the received signal from each source at its targeted destination node. The relay beamforming problem is shown to be a convex optimization problem and is formulated as a second-order cone program that can be efficiently solved using interior point methods. Numerical simulations are presented showing the superior performance of our beamforming technique compared to previously proposed zero forcing relay beamforming.

Index terms– Array signal processing, cooperative relaying, second-order cone programming.

1. INTRODUCTION

Cooperative relaying systems have received considerable attention in the last decade, see [1] and the references therein. The basic idea of cooperative relaying is to introduce intermediate nodes (relays) that forward the received data from the source to the destination. Cooperative relaying brings a large number of advantages to wireless communication systems. For example, it provides spatial diversity since the relay terminals form a distributed antenna array [2]. This diversity can be further exploited by applying distributed space-time coding [3]. Another advantage of cooperative relaying is the increase in the range of communication which can be further extended via beamforming [4].

Cooperative relaying can also be used to provide spatial multiplexing in multiuser communication scenarios where multiple signal sources are targeting one or more destination nodes. A multiuser relaying scheme called multiuser zero forcing relaying was proposed in [5]. This relaying technique uses beamforming to eliminate the interference between different source and destination pairs. Zero forcing relaying requires full knowledge of the channels from the sources to the relays and from the relays to the destination nodes. This channel information can be obtained using orthogonal pilot sequences broadcasted from the source and destination nodes to all the relay terminals. The channel estimates are then transmitted to a processing center that computes the beamforming coefficients and feeds them back to the relay terminals. It was shown in [4] that zero forcing relaying can greatly increase the average sum rate of the system (measured in bits/channel use) compared to the single source, single relay, single destination case. However, zero forcing beamforming is known to be suboptimal when the signal-to-noise ratio (SNR) of the sources is relatively low as it results in increased noise power at the destination nodes. Also, the zero forcing algorithm proposed in [5] was derived under the assumption that each source signal is targeting a distinct destination node, and hence, it cannot handle the more complicated multiplexing case where multiple sources are targeting the same destination node. Finally, this algorithm is only applicable to relaying schemes with single antenna relays and cannot exploit the performance gains due to multiple antennas at the relay terminals.

In this paper, we develop a multiuser beamforming algorithm for relay networks with multiple antennas at the relays. Our algorithm is derived under the same assumptions as those in [5], i.e., the channels between the relay terminals and different source and destination nodes are known with enough accuracy. We design the beamforming matrix such that both the noise received at each destination node and the interference caused by the sources not targeting this node are minimized. We also impose linear constraints that preserve each source signal at its targeted destination. The resulting optimization problem is shown to be convex and is formulated as a second order cone program (SOCP) that can be efficiently solved with polynomial complexity using interior point methods [6], [7]. We provide numerical simulations showing the superior performance of our beamforming technique compared to zero forcing beamforming in terms of the received signal-to-interference-plus-noise ratio (SINR) and symbol error rate (SER) when each relay is equipped with a single antenna. Simulation results also indicate that the use of multiple antennas at the relay terminals can significantly improve the system performance compared to single antenna relaying. This can be attributed to the additional degrees of freedom available for beamforming due to the block diagonal structure of the stacked beamforming matrix.

2. SIGNAL MODEL

We consider K relay terminals that linearly process the signals received from I statistically-independent narrowband sources and forward each signal to its destination. We assume that the kth relay terminal is equipped with an m_k-element antenna array that is used for receiving from the I sources and another m_k-element array transmitting to the destination nodes as shown in Fig. 1 [1]. Let $\boldsymbol{h}_i^{(k)}$ denote the $m_k \times 1$ vector containing the channel coefficients from the ith source to the kth relay terminal. The $m_k \times 1$ received signal vector at the kth relay terminal can be written as

$$\boldsymbol{x}_k = \sum_{i=1}^{I} \sqrt{P_i} \boldsymbol{h}_i^{(k)} s_i + \boldsymbol{n}_r^{(k)} \qquad (1)$$

This work was supported in parts by InterDigital Canada Ltee, by the Natural Sciences and Engineering Research Council (NSERC) of Canada, and by the Partnerships for Research on Microelectronics, Photonics, and Telecommunications (PROMPT) of Québec.

[1]Physically, each relay terminal can have only one antenna array operating in full duplex mode.

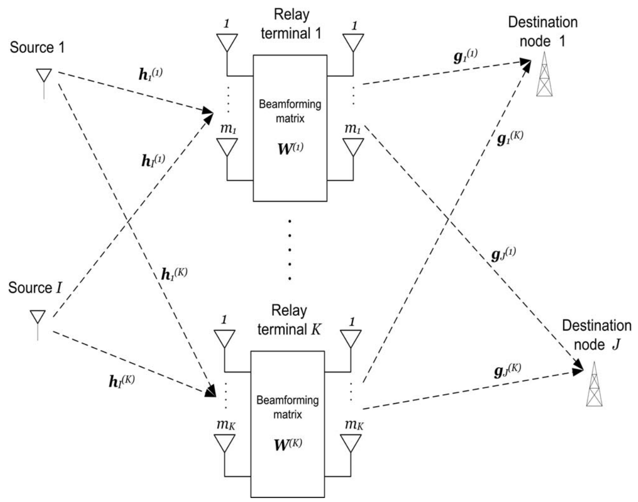

Fig. 1. System Model.

where s_i is the unit-power signal transmitted by the ith source, P_i is the transmission power of the ith source, $\boldsymbol{n}_r^{(k)}$ is the $m_k \times 1$ vector of white Gaussian noise with zero mean and covariance matrix $\sigma_{r,k}^2 \boldsymbol{I}$, and the subscript $(\cdot)_r$ refers to the relay terminals. The received signal vector by the kth relay terminal is linearly processed by the $m_k \times m_k$ beamforming matrix $\boldsymbol{W}^{(k)}$ before transmission to the destination nodes. The relay beamforming matrices at the K terminals are jointly designed such that they focus each of the I sources at its targeted destination node while reducing the received noise power and the interference caused by the sources that are not targeting this node.

In this work, we assume that each of the J destination nodes is equipped only with one antenna. Let $\boldsymbol{g}_j^{(k)}$ be the $m_k \times 1$ vector containing the complex conjugate of the channel coefficients from the kth relay terminal to the jth destination node. Therefore, we can write the received signal at the jth destination as

$$y_j = \sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \left(\sum_{i=1}^{I} \sqrt{P_i} \boldsymbol{h}_i^{(k)} s_i + \boldsymbol{n}_r^{(k)} \right) + n_{d,j} \quad (2)$$

where $\{\boldsymbol{W}^{(k)}\}_{k=1}^{K}$ is the set of beamforming matrices employed at the K relay terminals, $n_{d,j}$ is the white Gaussian noise with zero mean and variance $\sigma_{d,j}^2$ induced at the jth destination node, the subscript $(\cdot)_d$ refers to the destination node, and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively.

The received signal at the jth destination consists of three different components; the desired signals, i.e., the signals from the sources targeting the jth destination node, multiuser interference from the other sources, and noise. Let q_j be the number of sources targeting the jth destination. We define the $q_j \times 1$ vector $\boldsymbol{s}_j$ such that it contains the signals transmitted by these sources. We also define the diagonal $q_j \times q_j$ matrix $\boldsymbol{P}_j$ that contains the transmission power of the q_j sources, and the $m_k \times q_j$ matrix $\boldsymbol{H}_j^{(k)}$ whose columns are the corresponding channel vectors from these sources to the kth relay terminal. Therefore, the desired signal received at the jth destination node is given by

$$y_j^{(\mathrm{D})} = \sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{H}_j^{(k)} \boldsymbol{P}_j^{\frac{1}{2}} \boldsymbol{s}_j. \quad (3)$$

Similarly, we define the $(I - q_j) \times 1$ vector $\bar{\boldsymbol{s}}_j$ such that it contains the signals transmitted by the interfering sources, i.e., the sources that are not targeting the jth destination node. We also define the diagonal $(I-q_j) \times (I-q_j)$ matrix $\bar{\boldsymbol{P}}_j$ that contains the corresponding transmission powers of these sources, and the $m_k \times (I - q_j)$ matrix $\bar{\boldsymbol{H}}_j^{(k)}$ whose columns are the channel vectors from these sources to the kth relay terminal. Thus, the received signal at the jth destination node due to the interference is given by

$$y_j^{(\mathrm{I})} = \sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \bar{\boldsymbol{H}}_j^{(k)} \bar{\boldsymbol{P}}_j^{\frac{1}{2}} \bar{\boldsymbol{s}}_j. \quad (4)$$

whose power is equal to

$$\sigma_{I,j}^2 = \Big(\sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \bar{\boldsymbol{H}}_j^{(k)} \Big) \bar{\boldsymbol{P}}_j \Big(\sum_{k=1}^{K} \bar{\boldsymbol{H}}_j^{(k)H} \boldsymbol{W}^{(k)} \boldsymbol{g}_j^{(k)} \Big). \quad (5)$$

The received noise component is given by

$$y_j^{(\mathrm{N})} = \sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{n}_r^{(k)} + n_{d,j}, \quad (6)$$

and the received noise power is equal to

$$\sigma_{N,j}^2 = \sum_{k=1}^{K} \sigma_{r,k}^2 \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{W}^{(k)} \boldsymbol{g}_j^{(k)} + \sigma_{d,j}^2. \quad (7)$$

3. RELAY BEAMFORMING

In this section, we discuss the joint design of the relay beamforming matrices $\{\boldsymbol{W}^{(k)}\}_{k=1}^{K}$. The function of the relay terminals is to retransmit the signals received from the I sources to their targeted destination nodes. In order for the destination nodes to be able to efficiently detect their designated sources, there has to be minimum noise and interference from the other signal sources. In [4], Wittneben et. al. developed a multiuser zero forcing relaying scheme that eliminates the interference between different source/destination pairs by distributed beamforming. However, their approach discards the received noise power at the destination nodes which degrades the received SINR. Moreover, the zero forcing relay beamforming algorithm presented in [4] is only applicable to single antenna relay terminals. Also, it can not handle the case when multiple sources are targeting the same destination node.

Instead of completely suppressing the interference at the destination nodes, we design the beamforming matrices by minimizing both the received noise power at each destination node and the received interference due to the sources not targeting this node. Hence, we can write the cost function of the relay beamforming problem as

$$\sum_{j=1}^{J} \sigma_{I,j}^2 + \sum_{j=1}^{J} \sigma_{N,j}^2 = \sum_{j=1}^{J} \Big\| \sum_{k=1}^{K} \bar{\boldsymbol{P}}_j^{\frac{1}{2}} \bar{\boldsymbol{H}}_j^{(k)H} \boldsymbol{W}^{(k)} \boldsymbol{g}_j^{(k)} \Big\|^2 + \sum_{j=1}^{J} \sum_{k=1}^{K} \sigma_{r,k}^2 \Big\| \boldsymbol{W}^{(k)} \boldsymbol{g}_j^{(k)} \Big\|^2. \quad (8)$$

In order to avoid the trivial solution $\{\boldsymbol{W}^{(k)}\}_{k=1}^{K} = \boldsymbol{0}$ and to control the received power of each source at its targeted destination, we impose additional constraints that control the amplitude and phase of the received signal due to each source at its destination node. We can write these constraints as

$$\sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{H}_j^{(k)} \boldsymbol{P}_j^{\frac{1}{2}} = \boldsymbol{f}^{(j)T} \quad \forall j = 1, \ldots, J \quad (9)$$

where the $q_j \times 1$ vector $\boldsymbol{f}^{(j)} = [f_1^{(j)}, \ldots, f_{q_j}^{(j)}]^T$. Hence, the received signal at the jth destination node due to the q_j sources targeting this node is given by

$$y_j^{(\mathrm{D})} = \boldsymbol{f}^{(j)T} \boldsymbol{s}_j. \tag{10}$$

Note that if multiple sources are targeting the same destination, i.e., if $q_j > 1$, we can use time, frequency, or code division multiplexing such that these sources can be separated at the destination node.

The above constraints in (9) consume $2\sum_j q_j$ real degrees of freedom. When the total number of relay antennas is small compared to the number of sources, the above constraints might degrade the noise and interference reduction capability of the beamformer. Moreover, when each destination node is targeted by only one source, i.e., $q_j = 1 \ \forall j = 1, \ldots, J$, there is no need to control both the amplitude and phase of each received signal. In this case it is sufficient to constrain the beamformer such that the received signal from each of the I sources is preserved at its targeted destination. Hence, we can replace the constraints in (9) by the following constraints that consume only $\sum_j q_j$ degrees of freedom

$$\mathrm{Re}\Big\{ \sum_{k=1}^{K} \boldsymbol{g}_j^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{H}_j^{(k)} \boldsymbol{P}_j^{\frac{1}{2}} \Big\} = \mathbf{1}_{q_j}^T \quad \forall j = 1, \ldots, J. \tag{11}$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part of a complex vector and $\mathbf{1}_{q_j}$ is a $q_j \times 1$ vector containing ones.

Let us define the optimization variables $\boldsymbol{w}^{(k)} = \mathrm{vec}\{\boldsymbol{W}^{(k)}\}$ where $\mathrm{vec}\{\cdot\}$ denotes the vectorization operator. Using the identity $\mathrm{vec}\{\boldsymbol{ABC}\} = (\boldsymbol{C}^T \otimes A)\mathrm{vec}\{\boldsymbol{B}\}$ where $\otimes$ denotes the Kronecker product of two matrices, we can write the relay beamforming problem with the constraints in (11)[2] as

$$\begin{aligned}
&\min_{\{t_{(\mathrm{I}),j}, t_{(\mathrm{N}),j}\}_{j=1}^{J}, \{\boldsymbol{w}^{(k)}\}_{k=1}^{K}} \quad \sum_{j=1}^{J} t_{(\mathrm{I}),j} + t_{(\mathrm{N}),j} \\
&\text{s.t.} \ \Big\| \sum_{k=1}^{K} \Big(\boldsymbol{g}_j^{(k)T} \otimes \big(\bar{\boldsymbol{P}}_j^{\frac{1}{2}} \bar{\boldsymbol{H}}_j^{(k)H} \big) \Big) \boldsymbol{w}^{(k)} \Big\| \le t_{(\mathrm{I}),j} \qquad j = 1:J \\
&\sum_{k=1}^{K} \sigma_{r,k}^2 \Big\| \big(\boldsymbol{g}_j^{(k)T} \otimes \boldsymbol{I}_{m_k} \big) \boldsymbol{w}^{(k)} \Big\|^2 \le t_{(\mathrm{N}),j}^2 \qquad j = 1:J \\
&\mathrm{Re}\Big\{ \sum_{k=1}^{K} \Big(\boldsymbol{g}_j^{(k)T} \otimes \big(\boldsymbol{P}_j^{\frac{1}{2}} \boldsymbol{H}_j^{(k)H} \big) \Big) \boldsymbol{w}^{(k)} \Big\} = \mathbf{1}_{q_j} \qquad j = 1:J.
\end{aligned} \tag{12}$$

The number of real optimization variables in the above problem is $n_v = 2J + 2\sum_k m_k^2$ and the number of complex degrees of freedom available for beamforming is $\sum_k m_k^2$. The first group of constraints in (12) is a set of J standard second-order cone constraints, where the jth constraint has $2(I - q_j) + 1$ real dimensions. The second constraint set can also be expressed as J second-order cone constraints each of dimension $1 + 2\sum_{k=1}^{K} m_k$. The last constraint set is a linear constraint of $\sum_j q_j$ real dimensions. The problem is thus a convex optimization problem that can be efficiently solved with polynomial complexity using interior point methods [6]. The computational complexity associated with solving an SOCP can be calculated as follows [7]. The number of iterations required to solve an SOCP problem using interior point methods is bounded by the square root of the number of constraints. The computational complexity associated with each iteration is of $\mathcal{O}(n_v^2 \sum_i v_i)$, where v_i is the dimension of the ith constraint. Therefore, the worst-case computational load of (12) is of $\mathcal{O}\left(J^{\frac{3}{2}} (M + I)(M^2 + J)^2 \right)$ where $M = \sum_k m_k$ is the total number of relay antennas.

[2] The constraints in (9) can also be written as
$\sum_{k=1}^{K} \Big(\boldsymbol{g}_j^{(k)T} \otimes \big(\boldsymbol{P}_j^{\frac{1}{2}} \boldsymbol{H}_j^{(k)H} \big) \Big) \boldsymbol{w}^{(k)} = \mathrm{conj}\{\boldsymbol{f}^{(j)}\} \qquad j = 1:J$
where $\mathrm{conj}\{\cdot\}$ denotes the complex conjugate operator.

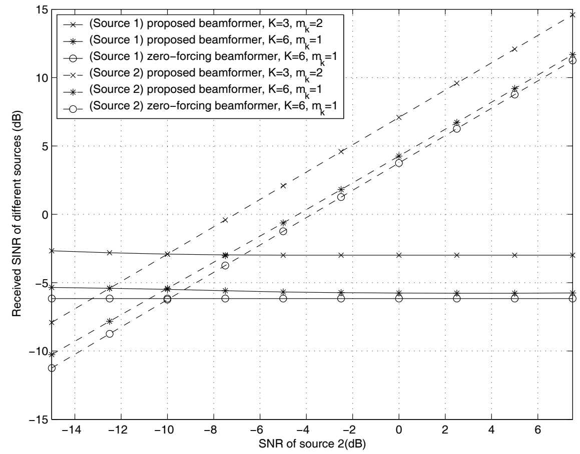

Fig. 2. Average received SINR versus the SNR of the second source.

4. SIMULATION RESULTS

We consider a wireless communication scenario with $I = 2$ sources each communicating with a distinct destination node ($J = 2$). The two sources are transmitting QPSK symbols that are received by the relay terminals and forwarded to the destination nodes after beamforming, i.e., we consider an amplify-and-forward relaying scheme. The channel from the sources to the relays and from the relays to the destinations are modeled as flat fading Rayleigh channels [8]. The SNR of the first source is fixed at -10 dB, and the SNR of the second source is varied between -20 dB and 7.5 dB. We compare the performance of the zero forcing relay beamformer proposed in [4] with $K = 6$ relay terminals each having a single antenna with the performance of our beamformer in (12) for two different relay configurations; $K = 6$ relay terminals each with a single antenna, and $K = 3$ relay terminals each having two antennas. Simulation results are averaged over 200 Monte Carlo runs.

We define the received SINR at the first destination node as

$$\mathrm{SINR}_1 = \frac{P_1 \left| \sum_k \boldsymbol{g}_1^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{h}_1^{(k)} \right|^2}{P_2 \left| \sum_k \boldsymbol{g}_1^{(k)H} \boldsymbol{W}^{(k)H} \boldsymbol{h}_2^{(k)} \right|^2 + \sum_k \left\| \boldsymbol{W}^{(k)} \boldsymbol{g}_1^{(k)} \right\|^2} \tag{13}$$

where the values of $\{\sigma_{r,k}^2\}_{k=1}^{K}$ are all equal to 1 and we have neglected the effect of the noise induced at the receivers of the destination nodes, i.e., $\{\sigma_{d,j}^2 = 0\}_{j=1}^{j=2}$, as they do not have any effect on the design of the relay beamforming matrices. Fig. 2 shows the received SINR at the two destination nodes. We can clearly see the performance gains achieved by our beamformers compared to zero forcing beamforming. In the case when each relay terminal is equipped with a single antenna, our beamformer offers a gain of 1 dB at low SNR. This can be attributed to its noise suppression capability which is absent in zero forcing beamforming. In the case of $K = 3$ relay terminals each with $m_k = 2$ antennas, our beamformer provides a performance gain of more than 3 dB compared to zero forcing beamforming with the same total number of relay antennas. This

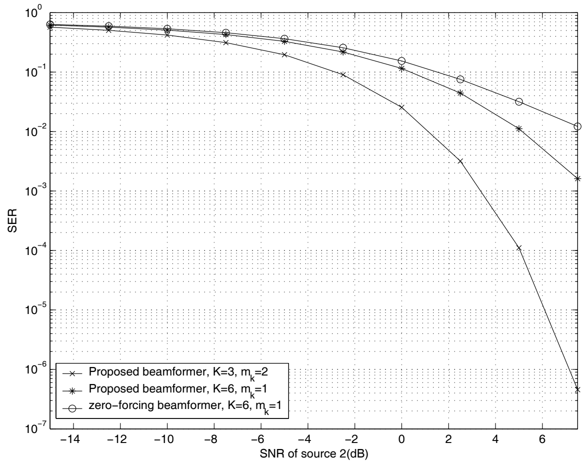

Fig. 3. Average SER versus the SNR for the second source.

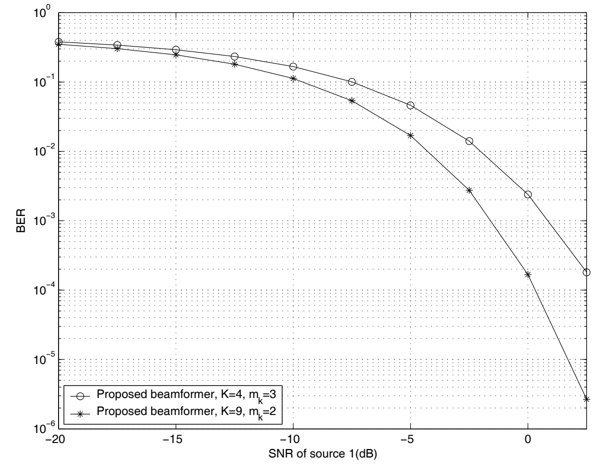

Fig. 4. Average BER versus the SNR.

performance gain is due to the additional degrees of freedom available for beamforming with the use of multiple antennas at the relay terminals, i.e., $\sum_k m_k^2$ degrees of freedom compared to $\sum_k m_k$ degrees of freedom for single antenna relays. Fig. 3 shows the SER for the second source at its targeted destination versus its SNR for different relay beamformers. We can clearly see that the performance improvements achieved by our beamformers in terms of the received SINR is translated into a corresponding improvement in the SER.

Next, we consider a multiplexing scenario with $I = 4$ sources transmitting BPSK symbols. The first two sources are targeting the first destination node and the third and fourth sources are targeting the second destination node. Note that the zero forcing relaying algorithm of [5] cannot be applied to this relaying scenario where multiple sources are targeting the same base station. We control the magnitude and phase of each source signal at its destination node using the constraint in (9) with $\boldsymbol{f}^{(j)} = [1, \sqrt{-1}]^T$ for $j = 1$ and 2. Hence, the first and second sources can be detected at the first destination node from the real and imaginary parts of the received signal, respectively. The SNRs of the third and fourth sources are fixed at 10 dB whereas the SNRs of the first two sources are kept equal and are varied between -20 and 2.5 dB. Simulation results are averaged over 200 Monte Carlo runs.

Fig. 4 shows the average bit error rate (BER) of the first source versus its SNR for two relaying scenarios; one with $K = 9$ relays each having $m_k = 2$ antennas and the second with $K = 4$ relays each having $m_k = 3$ antennas. We can clearly see that the first destination node can separate the data of the first source from that of the second and efficiently detect it. We also notice the superior performance of the $K = 9$, $m_k = 2$ relaying scenario to that of the $K = 3$, $m_k = 4$ relaying scenario even though the relay beamforming matrices have the same number of degrees of freedom in both cases, i.e., $\sum_k m_k^2 = 36$. This is can be attributed to the higher number of antennas in the $K = 9$, $m_k = 2$ case which leads to a higher spatial diversity order and hence a better performance in terms of the BER.

5. CONCLUSION

We have presented a beamforming algorithm for multiuser cooperative MIMO-relaying wireless systems. The beamforming matrices at different cooperating relay terminals are jointly designed such that both the noise received at each destination node and the interference caused by the sources not targeting this node are minimized. The received desired signal at each destination node is preserved through linear constraints. The relay beamforming problem is formulated as an SOCP that can be efficiently solved with polynomial complexity using interior point methods. We have presented numerical simulations showing the superior performance of our beamforming technique in terms of the SINR and BER compared to the recently developed zero forcing relay beamforming algorithm. Furthermore, our beamforming technique can handle more complicated relaying scenarios such as those with multiple sources targeting the same destination and multiple input multiple output relaying which cannot be handled by the zero forcing beamforming algorithm.

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