ACHIEVING FULLY-DIGITAL PERFORMANCE BY HYBRID ANALOG/DIGITAL BEAMFORMING IN WIDE-BAND MASSIVE-MIMO SYSTEMS

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ABSTRACT

In this paper, we study the realization of any given fully-digital precoder (FDP) by hybrid analog/digital precoding (HADP) in wide-band mmWave systems. We first formulate the massive-MIMO OFDM-based HADP system design and then, introduce the notion of perfect reconstruction at sampling point (PRSP) for FDP realization. Furthermore, the minimum number of required RF chains and its trade-off with the bandwidth of the transmitted signal is discussed. Next, we present a novel FDP realization with two RF chains. Finally, simulation results are presented showing the superiority of the proposed hybrid design to recently published works.

***Index Terms*—** Hybrid beamforming, hybrid analog digital beamforming, massive-MIMO, mmWave, OFDM, wide-band.

1. INTRODUCTION

Massive-multiple-input multiple-output (MIMO) and millimeter wave (mmWave) communications are the top candidates of many future wireless standards [1]. However, both mmWave communication and massive-MIMO face some fundamental challenges [2–4]. Surprisingly, combination of the both, not only accentuates their capabilities but also covers the shortcomings of each method. Massive-MIMO offers a fine resolution directional communication which can combat the open air path loss of mmWave. In turn, operation in millimeter wavelengths reduces the dimension of antenna elements which facilitates implementation of massive-MIMO [4].

Conventional MIMO systems are implemented fully-digital, i.e., each antenna element is driven by its dedicated radio-frequency (RF) chain [5, 6]. However, fully-digital precoding (FDP) is not practical in massive-MIMO systems due to the production costs and huge power consumption of the large number of required RF chains [7, 8]. Hybrid analog/digital precoding (HADP) facilitates the implementation of massive-MIMO systems by reducing the number of RF chains. In fact, by adding a layer of analog precoding which is performed by the means of RF circuitry after base-band digital processing, the number of RF chains is drastically reduced [7–13].

Many prior works in HADP literature focused on designing the hybrid beamformer directly [7–9]. However, since HADP system cannot outperform FDP and ultimately can achieve the performance of FDP [10, 11], FDP realization by HADP is an interesting technique which is studied in [10–13]. Narrow-band FDP realization was discussed in [10–12]. Particularly in [11, 14], we showed only a single RF chain is enough for realization of any given FDP. An alternative single RF chain architecture was also introduced in [15] based on outphasing principle. A wide-band FDP realization is presented in [13] where number of RF chain is equal to the rank of the combined digital precoder matrices.

In this paper, we use time domain signal reconstruction analysis for realizing any given FDP in wide-band HADP. To this end, we first present the system formulation and required conditions. Next, we introduce the notion of perfect reconstruction at sampling point (PRSP). A generalized technique for FDP realization with arbitrary number of RF chains is presented. Then, the trade-off between number of RF chains and bandwidth of the output is discussed. Particularly, we propose a novel simple design with only two RF chains and finally simulation results are presented.

2. SYSTEM MODEL

We consider an OFDM based mmWave massive-MIMO transmitter with N_T antennas and N_{RF} RF chains. Each OFDM symbol has N_c sub-carriers with N_{cp} cyclic prefixes. At each sub-carrier N_s symbols are transmitted. The symbol vector of the kth sub-carrier is denoted by $\mathbf{s}[k] = [s_{k,1}, s_{k,2}, ..., s_{k,N_s}]^T$, where $s_{k,i}$ is taken from a discrete constellation $\mathscr{A}$ (such as M-QAM or M-PSK).

2.1. FDP and Conventional HADP Architectures

In conventional MIMO systems, fully-digital precoding precoding can be performed at each sub-carrier,

$$\mathbf{y}_{\text{FD}}[k] = \mathbf{P}_{\text{FD}}[k]\mathbf{s}[k], \tag{1}$$

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where $\mathbf{y}_{\mathrm{FD}}[k]$ and $\mathbf{P}_{\mathrm{FD}}[k]$ are transmitted signal and FDP matrix at kth sub-carrier. In this case, $\mathbf{P}_{\mathrm{FD}}[k]$s are $N_T \times N_s$ matrices which requires N_T RF chains.

In existing hybrid structures, the transmitted signal in OFDM HADP can be written as:

$$\mathbf{y}_{\mathrm{EH}}[k] = \mathbf{P}_{\mathrm{A}}\mathbf{P}_{\mathrm{D}}[k]\mathbf{s}[k], \tag{2}$$

where $\mathbf{y}_{\mathrm{EH}}[k]$ and $\mathbf{P}_{\mathrm{D}}[k]$ are the transmitted signal and digital precoder of kth sub-carrier, respectively, and $\mathbf{P}_{\mathrm{A}}$ is the analog precoder for all sub-carriers.

Fig. 1. depicts the transmitter of a conventional OFDM HADP [16–21]. Symbols are first precoded at each sub-carrier, then N_{RF} OFDM symbols are constructed by performing inverse Fourier transform and appending cyclic prefixes. After being converted to RF signals, the OFDM symbols are precoded in time domain by RF precoder, hence the same phase shift for all sub-carriers.

3. REALIZING ANY FDP IN OFDM HADP ARCHITECTURE

In most prior works [16–21], frequency domain analysis is used for designing hybrid precoders. In the paper, we perform time domain signal processing for HADP design.

3.1. FDP Realization in HADP

Our goal is to realize any given FD precoder in hybrid analog/digital architecture. The OFDM signal for each antenna can be calculated in frequency domain according to the chosen FDP using (1). Thus, the desired time domain transmitted signal of i^{th} antenna, i.e., $x^i_{\mathrm{FD}}(t)$ can be easily obtained by inverse Fourier transform. Let $x^i_{\mathrm{HY}}(t)$ denote the time domain transmitted signal in HADP from ith antenna during $t \in [0, NT_s]$ with $T_s = 1/F_s$, where F_s is the sampling frequency and $N = N_c + N_{cp}$. Thus, we have

$$\mathbf{x}^i_{\mathrm{HY}}(n) \triangleq x^i_{\mathrm{HY}}(nT_s) \tag{3}$$

where $\mathbf{x}^i_{\mathrm{HY}}(n)$ for $n = 0, 2, ..., N_c - 1$ is the sampled time domain signal and can be also obtained by

$$\mathbf{x}^i_{\mathrm{HY}}(n) = \sum_{k=0}^{N_c} \mathbf{y}_{\mathrm{HY}}[k](i) e^{-j2\pi n/N_c} \tag{4}$$

where $\mathbf{y}_{\mathrm{HY}}[k]$ is the precoded frequency domain signal and $\mathbf{y}_{\mathrm{HY}}[k](i)$ denotes its the ith entry. In practice, $\mathbf{x}^i_{\mathrm{HY}}$ is converted to $x^i_{\mathrm{HY}}(t)$ by the means of RF chains. However, since we want to reconstruct $x^i_{\mathrm{HY}}(t)$, it is helpful to have the reverse relationships. Assuming the same hardware constrictions as conventional OFDM HADP [16–21], we write the problem formulation for the realization of any FDP in HADP.

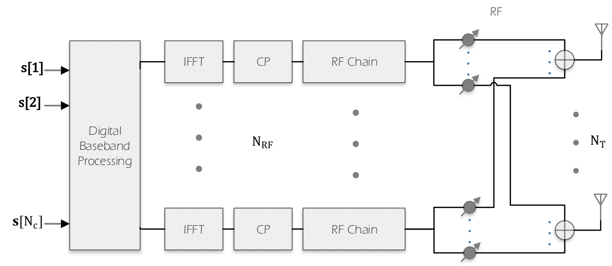

Fig. 1: Conventional OFDM HADP.

Proposition 1. *Any FDP $\mathbf{P}_{FD}[k]$ can be realized in hybrid architecture with N_{RF} RF chains if for $i = 1, 2, ..., N_T$, $m = 0, 1, 2, ..., M-1$ and $p = 0, 1, 2, ..., N_{RF} - 1$ there exist non-overlapping functions $x_{m,p}(t)$, i.e.,*

$$supp(x_{\tilde{m},p}(t)) \neq supp(x_{\hat{m},p}(t)) \tag{5}$$

where $supp(x) = \{t \in \mathbb{R} | x(t) \neq 0\}$ for $\tilde{m} \neq \hat{m}$ and phases $\phi^i_{m,p} \in [0, 2\pi]$ such that for each i and for $t \in [0, NT_s]$:

$$x^i_{FD}(t) = x^i_{HY}(t) \tag{6}$$

with

$$x^i_{HY}(t) = \sum_{p=0}^{N_{RF}-1} \sum_{m=0}^{M-1} e^{j\phi^i_{m,p}}\, x_{m,p}(t) \tag{7}$$

Proof. Since (6) guarantees FDP realization, we just need to show (7) can be the output of HADP. If the condition in (5) is satisfied, then all $x_{m,p}(t)$s for $m = 1, 2, ..., M$ can be generated by the p^{th} RF chain:

$$x_p(t) = \sum_{m=0}^{M-1} x_{m,p}(t) \tag{8}$$

Assuming the phase-shifters can be updated M times during $t \in [0, NT_s]$ period, the transmitted signal of ith antenna is given by (5). □

Thus, in order to design HADP which can realize any FDP, a set of MN_TN_{RF} angles ($\phi^i_{m,p}$ for $i = 1, 2, ..., N_T$, $m = 1, 2, ..., M$ and $p = 1, 2, ..., N_{RF}$) must be designed as well as MN_{RF} functions: $x_{m,p}(t)$. This design problem can be tackled by different techniques. Particularly, since $x^i_{\mathrm{HY}}(t)$s are continuous and complex valued functions, wavelets and time-frequency analysis are proper ways to solve this problem. In this paper, however, based on the Shannon-Nyquist sampling theorem, we focus on the sampled signal for designing the HADP for FDP realization.

3.2. FDP realization by Perfect Reconstruction at Sampling Point

In this section, we first present the general conditions for realizing FDP with HADP in the sense of PRSP by arbitrary

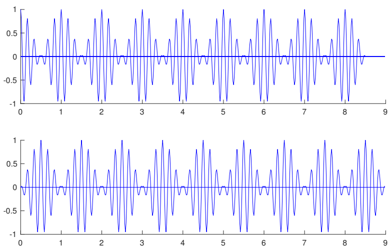

Fig. 2: Output signal of RF chains.

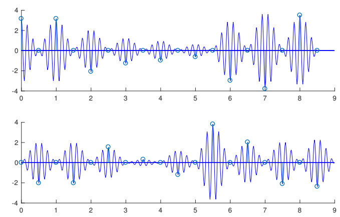

Fig. 3: Output signal of phase shifters for one antenna element.

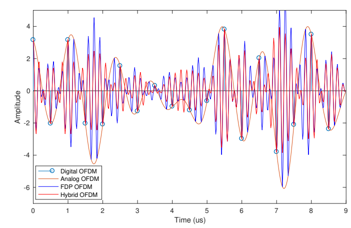

Fig. 4: Comparison of FDP and HADP waveforms.

number of RF chains N_{RF}. We then discuss the fundamental trade-off between the number of RF chains the the bandwidth of the signal. Finally, we present a hybrid design with two RF chains using raised cosine pulses.

3.2.1. Perfect Reconstruction at Sampling Point

Instead of the strict condition (6), we only need to make sure the output signal of the hybrid system is equal to the FDP signal the sampling points

$$x^i_{\text{FD}}(nT_s) = x^i_{\text{HY}}(nT_s), \tag{9}$$

Indeed, according to Shannon-Nyquist sampling theorem, (9) and (6) are equivalent if the bandwidth of $x^i_{\text{FD}}(t)$ and $x^i_{\text{HY}}(t)$ are the same. However, since we are not bound to this restriction, in general, it is possible to have (9) whilst (6) being violated. Clearly, (6) is the ideal case but requires more RF chains as will be discussed in Remark 1. Consequently, under perfect time offset synchronization the performance is the same and the design is simplified.

The generated signal of p^{th} RF chain was given in (8) for the M non-overlapping functions $x_{1,p}(t), x_{2,p}(t), ..., x_{M,p}(t)$. On way of reducing the design parameters is to use shifted version of a windowed function to build $x_{m,p}(t)$s. Let us consider an energy signal $p_p(t)$ such that $p_p(t) = 0$ for $t \notin [0, \tau]$. Thus, if $\frac{M}{N}$ is an integer and we have

$$\tau = \frac{N}{M} T_s \tag{10}$$

generated signal of pth RF chain(8) can be written as

$$x_p(t) = \sum_{m=0}^{M-1} p_p(t - m\tau). \tag{11}$$

Note that $p_p(t)$ must be an energy signal so that it can be generated by an RF chain. Now, we can present the PRSP HADP realization of any given FDP.

Theorem 1. *Any FDP can be realized in the sense of PRSP by hybrid architecture with N_{RF} RF chains and $2N_T N_{RF}$ phase shifters with the parameter $M = N/N_{RF}$.*

Proof. From (7), for ${N_{RF}}' = 2N_{RF}$ we can write

$$x^i_{\text{HY}}(t) = \sum_{p'=0}^{{N_{RF}}'-1} \sum_{m=0}^{M-1} e^{j\varphi^i_{m,p'}} \, x_{m,p'}(t). \tag{12}$$

In case, $x_{m,p}(t) = x_{m,p+{N_{RF}}'/2}(t)$ for $p = 0, 1, ..., \frac{{N_{RF}}'}{2} - 1$, (i.e., the second half of the RF chains generate the same signal as the first half) the above equation can be written as:

$$x^i_{\text{HY}}(t) = \sum_{p=0}^{N_{RF}-1} \sum_{m=0}^{M-1} (e^{j\phi^i_{m,p,1}} + e^{j\phi^i_{m,p,2}}) \, x_{m,p}(t), \tag{13}$$

where $\phi^i_{m,p,1} = \varphi^i_{m,p}$ and $\phi^i_{m,p,2} = \varphi^i_{m,2p-1}$ for $p = 0, 1, ..., N_{RF} - 1$, moreover, the number of RF chains is reduced by half. According to Theorem 1 of [13], for any given $a^i_{m,p} \in \mathbb{U}$ where $\mathbb{U} = \{z \in \mathbb{C}\colon |z| \le 2\}$ there exist $\phi^i_{m,p,1}$ and $\phi^i_{m,p,2}$ such that $a^i_{m,p} = \phi^i_{m,p,1} + \phi^i_{m,p,2}$. Thus, from (11) and (13) we arrive at

$$x^i_{\text{HY}}(t) = \sum_{p=0}^{N_{RF}-1} \sum_{m=0}^{M-1} c_0 a^i_{m,p} \, p_p(t - m\tau), \tag{14}$$

where $c_0 = \frac{1}{2} \max_i |x^i_{\text{HY}}(t)|$. We further take $p_p(t)$ as

$$p_p(t) = c_0 p(t - pT_s) \tag{15}$$

where $p(t)$ satisfies the Nyquist filters criterion, i.e.,

$$p(nT_s) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}, \tag{16}$$

hence,

$$x^i_{\text{HY}}(t) = \sum_{p=0}^{N_{RF}-1} \sum_{m=0}^{M-1} a^i_{m,p} \, p(t - pT_s - m\tau). \tag{17}$$

Now, from (17), (9) and (10), and using the definition in (3), for PRSP realization it is sufficient to have:

$$x^i_{\text{FD}}[n] = \sum_{p=0}^{N_{RF}-1} \sum_{m=0}^{M-1} a^i_{m,p} \, p[n - p - mN_{RF}]. \tag{18}$$

Now, for $n = p + mN_{RF}$, we can further have $a^i_{m,p} = x^i_{\text{FD}}[p + mN_{RF}]$ which concludes the proof. □

Remark 1. *The minimum number of required RF chains for realizing any given FDP is $N_{RF} = 1$. However, reducing the number of RF chains comes with the price of extra bandwidth. On the other hand, for $N_{RF} = N$ RF chains, sinc function can be generated by $p(t)$ which satisfies* (6) *and results in the most favorable spectral characteristics.*

Proposition 2. *There is a trade-off between number of RF chains and bandwidth of the output signal, where decreasing N_{RF} results in an increase in bandwidth.*

Proof. Based on the OFDM parameters, N is given, and from (10) we have:

$$\tau = N_{RF} T_s. \tag{19}$$

Therefore, decreasing N_{RF} also decreases τ which is the length of $p(t)$. Reducing τ will increase the bandwidth of $p(t)$ which in turn increase the bandwidth of the output signal. □

3.2.2. FDP Realization with Two RF Chains

In this sub-section, we present an FDP realization with two RF chains as an example. First, let us take $p(t)$ as the windowed raised cosine filter with roll of factor 0.5 with duration of $2T_s$ which satisfies (11) and (16). Thereby, considering the polar representation $x^i_{\text{FD}}[n] = |x^i_{\text{FD}}[n]| \exp(j\angle x^i_{\text{FD}}[n])$, for $n = p + mN_{RF}$, the phase-shifter parameters are then calculated as

$$\phi^i_{m,p,1} = \angle x^i_{\text{FD}}[n] + \cos^{-1}\left(\frac{|x^i_{\text{FD}}[n]|}{2c_0}\right) \tag{20a}$$

$$\phi^i_{m,p,2} = \angle x^i_{\text{FD}}[n] - \cos^{-1}\left(\frac{|x^i_{\text{FD}}[n]|}{2c_0}\right). \tag{20b}$$

To clarify the concept of PRSP FDP realization, an example for the the real part of the signal (In-phase component) is presented below. We consider an OFDM system with $N_c = 16$, $N_{cp} = 2$, $BW = 2MHz$, carrier frequency $F_c = 6GHz$, and sampling frequency = $10F_c$. Without loss of generality, we only present the signal of one arbitrary antenna element.

Consequently, for $N_{RF} = 2$ and $N = 18$, we have $M = 9$; thus, the RF signal generated by two RF chains can be written follows which are also illustrated in Fig. 2:

$$x^1_{RF}(t) = \sum_{m=1}^{9} x^I_{m,1}(t) cos(\omega_c t) \tag{21a}$$

$$x^2_{RF}(t) = \sum_{m=1}^{9} x^I_{m,2}(t) cos(\omega_c t) \tag{21b}$$

Passing $x^1_{RF}(t)$ and $x^2_{RF}(t)$ through RF precoder, the result is shown in Fig. 3.

In Fig. 4. FDP and HADP are presented as well as the digital and baseband OFDM signal. It can be observed that in the sampling points all the signals have the same value as we also showed mathematically.

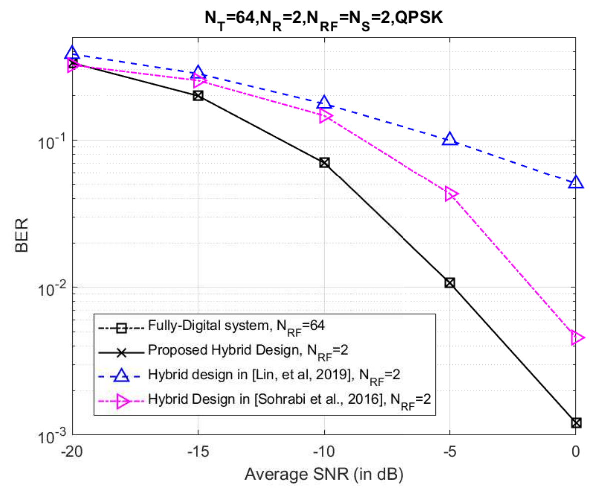

Fig. 5: BER versus SNR for different methods.

4. SIMULATION RESULTS

In this section, we present simulation results for FDP system our proposed HADP and the HADP design in recently published works. The same wide-band channel model presented in [22] is used for the simulations, where the number of clusters is set to 5 and the number of rays in each clusters is set to 10. It is assumed that the complex path gain are distributed as $\mathcal{CN}(0, 1)$ and the angles of arrival and departure are generated according to the Laplacian distribution where the mean cluster angles are independently and uniformly distributed in $[0, 2\pi]$. The angular spread is 10 degrees within each cluster.

Simulations are performed for single user massive-MIMO system with the following parameters: $N_c = 64$, $N_{cp} = 16$, $BW = 2MHz$, $N_T = 64$ transmit antennas, $N_R = 16$ receive antennas, $N_s = N_{RF} = 2$, $F_c = 6GHz$ and QPSK modulation. Fig. 5. shows the BER performance versus SNR of the fully digital system, our proposed HADP design HADP in [20], and [22]. It can be seen that while our design matches the FDP it could outperform the design in [20] by more than 5 dB.

5. CONCLUSION

In this paper, we formulated the problem and presented the conditions for realization of any given FDP in HADP systems. Then, we introduced the notion of PRSP for time domain signal reconstruction of OFDM signals. We presented the general solution for arbitrary number of RF chains and particularly, presented a design for two RF chains. Finally, simulations were performed confirming the superiority of our proposed method over recently published works.

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