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OPTIMUM SPACE-TIME PROCESSING FOR SEMI-STATIONARY SIGNALS IN SPATIALLY CORRELATED NOISE

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ABSTRACT

This paper addresses the problem of optimum space-time processing for multiple Gaussian source signals transmitted through a slowly-varying linear channel and monitored with a passive array of sensors in the presence of spatially correlated noise. To solve this problem, a new class of linear systems (LS) referred to as *semi-stationary* is introduced. These LS are characterized by time-frequency representations whose variations in time occur over intervals much larger than the corresponding system correlation time. The general conditions under which semi-stationary LS can be used in array processing are investigated and shown to be satisfied in many applications. By modeling the slowly-varying linear channel as a semi-stationary LS and using the factorization properties of the optimum processor, closed form expressions are obtained for the log-likelihood function of the array output and for the associated Cramér-Rao lower bound on estimator variance.

I. INTRODUCTION

When a passive array of sensors is used to monitor a radiating source, it may be unrealistic to model the sensor outputs as a stationary vector random process. For instance, in the presence of source or receiver motion, the differential delays between the signal components received at the various sensors are time-varying and, as a result, the array output vector process exhibits a slowly-varying non-stationary behavior. Even though these non-stationarities pose a serious difficulty from a mathematical point of view, it is of primary importance to incorporate any a priori knowledge about them in the array processing algorithm for at least two reasons. First, if not compensated, they can seriously degrade the performance of the processing scheme [1]. Secondly, by properly modeling and processing the sensor outputs, it is actually possible to exploit the non-stationary nature of the received signals to improve the accuracy of the track parameter estimates of the moving source [2].

Based on these considerations, different aspects of the problem of optimum space-time processing for a slowly moving source radiating a wideband random signal and monitored with a passive array of sensors in the presence of noise have been addressed in the literature. In [3], the log-likelihood processor (LLP) is derived under the realistic assumption that $v/c \ll 1$, where v is the speed of the source and c is the wave propagation velocity. In [4], the Cramér-Rao lower bound (CRLB) on the error covariance matrix of differential Doppler shift estimates is obtained under similar conditions. However, these studies are limited in scope since they apply only in the case of a *single* source monitored in the presence of *spatially uncorrelated noise*, conditions which are rarely satisfied in applications. Besides these practical limitations, the analysis in [3]-[4] does not seem to take full advantage of the slowly-varying nature of the non-stationarities present in the received signals. For instance, the derivation of the CRLB in [4] is considerably tedious and it is not clear how it could be extended to more general situations.

In this paper, we address the problem of optimum space-time processing for *multiple* Gaussian source signals transmitted through a *slowly varying* linear channel and monitored in the presence of *spatially correlated* noise. To this end, we introduce a new class of slowly-varying linear systems (LS) referred to as semi-stationary. These LS are characterized by time-varying frequency responses whose variations in time occur over intervals much longer than the corresponding system correlation time. The general conditions of applicability of semi-stationary LS in the context of array processing are obtained and expressed in terms of important physical parameters. The properties of semi-stationary LS are then used in connection with the factorization properties of the LLP [5] to derive closed form expressions for the log-likelihood function (LLF) of the sensor outputs and for the associated CRLB on estimator variance.

II. SEMI-STATIONARY LINEAR SYSTEMS

Consider a LS L with impulse response $L(t,u)$. By the principle of superposition, the response of L to an arbitrary input function $f(t)$ is given by

$$[Lf](t) = \int L(t,u) f(u)\, du, \quad t,u \in (-\infty,\infty). \tag{1}$$

The *system function* (SF) associated with L is defined by [6]

$$C(t,\omega) = e^{-j\omega t} L\{e^{j\omega t}\} = \int L(t,u)\, e^{-j\omega(t-u)}\, du. \tag{2}$$

When L is time-invariant, $L(t,u)=L(t-u)$ and (2) reduces to the conventional definition of a transfer function.

The limitations of the SF $C(t,\omega)$ become apparent when we try to cascade two or more time-varying LS. Indeed, let L_1 and L_2 be arbitrary LS with SF $C_1(t,\omega)$ and $C_2(t,\omega)$, respectively, and let $h = L_2 L_1 f$, as shown in Fig. 1.

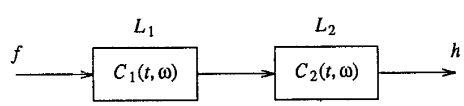

Fig. 1. Cascade of two arbitrary LS L_1 and L_2.

Then, it is not true in general that

$$h(t) = \frac{1}{2\pi} \int C_2(t,\omega)\, C_1(t,\omega)\, F(\omega)\, e^{j\omega t}\, d\omega. \tag{3}$$

Of course, (3) is satisfied when L_1 and L_2 are time-invariant, i.e. when $C_i(t,\omega) = C_i(\omega)$, $(i=1,2)$. Hence, contrarily to the conventional transfer function for time-invariant LS, the SF $C(t,\omega)$ does

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not provide a simple mean for analyzing a cascade of arbitrary time-varying LS.

There seems to be, however, one particular case of considerable practical interest where (3) could be used without introducing any significant error. Indeed, suppose that the following conditions are satisfied: (i) there exists a positive constant τ such that $L_2(t,u) \approx 0$ whenever $|t-u| > \tau$; and (ii) over time intervals on the order of τ, $C_1(t,\omega)$ does not vary significantly, i.e. $C_1(t,\omega) \approx C_1(u,\omega)$ whenever $|t-u| < \tau$. Then, it is not difficult to argue that the right hand side of (3) should provide a "good" approximation to $h(t)$. First, we note that

$$h(t) = \frac{1}{2\pi} \int \{ \int L_2(t,u)\, C_1(u,\omega)\, e^{j\omega u}\, du \}\, F(\omega)\, d\omega \tag{4}$$

where $F(\omega)$ is the Fourier transform of $f(t)$. Now consider the bracketted quantity in (4). Invoking (i), (ii) and (2), it is easy to see that

$$\int L_2(t,u)\, C_1(u,\omega)\, e^{j\omega u}\, du \approx C_1(t,\omega)\, C_2(t,\omega)\, e^{j\omega t}. \tag{5}$$

Equation (3) follows by substituting (5) into (4). While the above argument is informative of the mathematical principles that we want to emphasize in this study, it is rather intuitive in nature and does not provide any measure of the approximation error made in using (3). We now introduce a new class of slowly-varying LS for which this approximation error can be quantified.

Consider a LS L, with impulse response $L(t,u)$ and SF $C(t,\omega)$. Let

$$\tau = \sup_t \tau(t), \quad \tau(t) = \frac{\int |L(t,u)|\, |t-u|\, du}{\sup_\omega |C(t,\omega)|}, \tag{6}$$

$$\beta = \sup_t \beta(t), \quad \beta(t) = \frac{\int |\partial_t C(t,\omega)|\, d\omega}{\sup_t \int |C(t,\omega)|\, d\omega}, \tag{7}$$

where ∂_t indicates a partial derivative with respect to t and sup denotes the least upper bound (the denominators in (6)-(7) are assumed finite). τ (6) provides a measure of the duration of $L(t,u)$ and is referred to as the *correlation time* of L. β (7) provides a measure of the bandwidth of $\int C(t,\omega) d\omega$ (when interpreted as a function of t) and is referred to as the *spectral-fluctuation bandwidth* of L. We shall say that L is *semi-stationary* if

$$\beta\tau \ll 1. \tag{8}$$

We immediately note that time-invariant linear systems are semi-stationary for if $L(t,u) = L(t-u)$, then $\partial_t C(t,\omega) = 0$.

Asymptotic convolution theorem. Let L_i $(i=1,2)$ be semi-stationary LS with maximum spectral-fluctuation bandwidth β and maximum correlation time τ, and let $h = L_2 L_1 f$. Then

$$h(t) = \frac{1}{2\pi} \int C_2(t,\omega)\, C_1(t,\omega)\, F(\omega)\, e^{j\omega t}\, d\omega + O(\beta\tau), \tag{9}$$

where the notation $y = O(x)$, i.e. y is of the order of x, is used to indicate that y/x remains bounded as x tends to 0.

Other important asymptotic properties of semi-stationary LS are stated and proved in [7].

III. SEMI-STATIONARITY IN ARRAY PROCESSING

A. General discussion

For simplicity, consider a source-array configuration consisting of one moving source and two fixed sensors. Assuming that the signal transmission is ideal, the vector process $s(t)$ of received signal components at the sensor outputs is given by

$$s(t) = [La](t), \tag{10}$$

where L is a linear system defined by

$$[La](t) = \begin{bmatrix} a(t) \\ a(t-d(t)) \end{bmatrix}, \tag{11}$$

$a(t)$ is the reference signal at the output of the first sensor, and $d(t)$ is the intersensor delay.

Suppose that $a(t)$ is a zero-mean stationary random process with autocorrelation function $R_a(\tau)$. Then, according to (10)-(11), the autocorrelation function $R_s(t,u)$ of $s(t)$ is given by

$$R_s(t,u) = \begin{bmatrix} R_a(t-u) & R_a(t-u+d(u)) \\ R_a(t-d(t)-u) & R_a(t-d(t)-u+d(u)) \end{bmatrix}. \tag{12}$$

Unless $d(t)$ is constant, $R_s(t,u)$ is not a function of $t-u$ and therefore, the process $s(t)$ is not stationary.

A simple way of interpreting the non-stationarity in (12) is in terms of a "time-varying" spectral representation for $R_s(t,u)$. Indeed, it follows from (12) that

$$R_s(t,u) = \frac{1}{2\pi} \int C(t,\omega)\, G_a(\omega)\, C^H(u,\omega)\, e^{j\omega(t-u)}\, d\omega, \tag{13}$$

where $G_a(\omega)$ is the power spectral density (PSD) of the process $a(t)$ and

$$C(t,\omega) = \begin{bmatrix} 1 \\ e^{-j\omega d(t)} \end{bmatrix}. \tag{14}$$

Observe that $C(t,\omega)$ (14) is precisely the SF (2) of L (11). The quantity $C(t,\omega) G_a(\omega) C^H(u,\omega)$ appearing under the integral sign in (13) can be interpreted as a "time-varying spectral density matrix" responsible for the non-stationarity of the process $s(t)$.

Unless additional assumptions are made, neither of the above representations (12) and (13) for $R_s(t,u)$ can provide more information about the LLP for the non-stationary signal model (10)-(11) than what is already known from the analysis in [5]. As we now explain, however, $C(t,\omega)$ is a "slowly" varying function of time in most cases of practical interest and it should be possible somehow to exploit this fact in the study of the LLP.

Let $C_2(t,\omega) = e^{-j\omega d(t)}$ denote the second component of $C(t,\omega)$ (14) and observe that

$$|\partial_t C_2(t,\omega)| = |\omega|\, |d'(t)|\, |C_2(t,\omega)|, \tag{15}$$

where $d'(t)$ is the time derivative of $d(t)$. In most applications, $v \ll c$ and it follows that

$$|d'(t)| \ll 1. \tag{16}$$

Let B denote the bandwidth of $G_a(\omega)$. From (15) and (16), it follows that for all ω with $|\omega| < B$ and for all time intervals $\Delta t < B^{-1}$, we have

$$\Delta t\, |\partial_t C_2(t,\omega)| \ll |C_2(t,\omega)|. \tag{17}$$

This inequality means that over time intervals of the order of the (statistical) correlation time B^{-1} of the process $a(t)$, the spectral function $C(t,\omega)$ appearing in the integral representation (13) of $R_s(t,u)$ does not change significantly.

Random processes $s(t)$ with second order spectral representations of the type (13), where $C(t,\omega)$ is a slowly varying function of time in the above sense, have been studied extensively by Priestley [8]-[9]. However, his approach is not appropriate for the present application because it is too restrictive [10]. Besides, the structure of the optimum array processor, with its various components specified in terms of compositions and inversions of linear operators, is already known from [5]. Thus, we are actually more interested in the properties of time-varying LS with slowly varying SF of the type (14) than in the associated random process $s(t)$ (10). These considerations are indeed the main justification for the introduction and study of semi-stationary LS.

B. Conditions of applicability

In order to relate the condition (8) defining a semi-stationary LS to important physical parameters characterizing the problem of array processing in the presence of moving sources, consider the representative SF

$$C(t,\omega) = \phi(t)\,\Psi(\omega)\,e^{-j\omega d(t)}, \tag{18}$$

where

$$\phi(t) = e^{-t^2/2T^2}, \tag{19}$$

$$\Psi(\omega) = e^{-\omega^2/2B^2}. \tag{20}$$

The function $\phi(t)$ (19) is a Gaussian weighting which is used to model the finite temporal extent of the processing taking place at the sensor outputs. In this respect, T provides a measure of the observation interval. In the same way, $\Psi(\omega)$ (20) is a Gaussian weighting which is used to model the finite frequency response of the channel or the processor, and B is a measure of the corresponding bandwidth. In (18), $d(t)$ is a time-varying delay function which represents the difference in time of arrival of the signal wavefront at two different locations. We assume that

$$|d(t)| \le D_0, \quad |d'(t)| \le D_1, \tag{21}$$

where D_0 and D_1 are the maximum delay and delay rate, respectively.

The calculation of the correlation time τ (6) and the spectral fluctuation bandwidth β (7) for $C(t,\omega)$ (18) is carried out in [7]. For τ, we find that

$$\tau < B^{-1} + D_0. \tag{22}$$

Hence, the correlation time of the LS specified by (18) is of the order of the inverse bandwidth of $\Psi(\omega)$ (20) plus the maximum delay D_0 (21). For β, the results indicate that

$$\beta < T^{-1} + BD_1. \tag{23}$$

The term T^{-1} in (23) is a measure of the bandwidth of the weighting function $\phi(t)$ used to model the finite temporal extent of $C(t,\omega)$ (18). In the limit $T\to\infty$, (23) reduces to $\beta < BD_1$. When the bandwidth of $d(t)$ is much smaller than B (which will usually be the case in applications), the quantity BD_1 provides a measure of the bandwidth of the frequency modulated signal $e^{-jBd(t)}$.

Equations (22) and (23) imply that

$$\beta\tau < (BT)^{-1} + D_0 T^{-1} + D_1 + BD_0 D_1. \tag{24}$$

Therefore, the LS specified by (18) will be semi-stationary (i.e. $\beta\tau \ll 1$) if the following conditions are satisfied:

$$BT \gg 1 \tag{25}$$

$$D_0/T \ll 1 \tag{26}$$

$$D_1 \ll 1 \tag{27}$$

$$BD_0 D_1 \ll 1 \tag{28}$$

According to (25), the product of observation time and processor bandwidth (known as the time-bandwidth product) must be large. Equation (26) means that the maximum delay is small in comparison to the observation time. Equation (27) imposes a constraint on the maximum delay rate. Finally, (28) can be interpreted as a requirement that the maximum variation in the phase of $e^{jBd(t)}$ during the time interval D_0 be negligible.

Recall that (25) and (26), together with $D_1 = 0$, are the conventional assumptions made in the study of time-invariant array processors [11]. Therefore, (27) and (28) can be regarded as a relaxation of the condition $D_1 = 0$, made possible by the use of semi-stationary LS instead of conventional time-invariant LS.

IV. SPACE-TIME LLP

Consider a source-array configuration consisting of N sources, M sensors, and a transmission medium, as shown in Fig. 2.

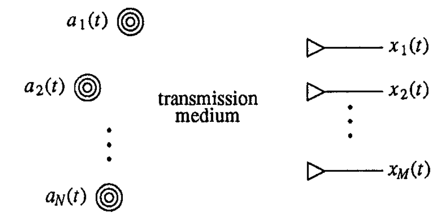

Fig. 2. Source-array configuration.

The following assumptions are made:

(i) The vector $x(t) = [x_1(t),\ldots,x_M(t)]^T$ of sensor outputs is given by

$$x(t) = [La](t) + n(t), \qquad -\infty < t < \infty, \tag{29}$$

where $a(t) = [a_1(t),\ldots,a_N(t)]^T$ is the source signal vector, $n(t) = [n_1(t),\ldots,n_M(t)]^T$ is the sensor noise signal vector, and L is a linear operator with impulse response $L(t,u)$ and SF $C(t,\omega)$ modeling the signal transmission from the sources to the sensors.

(ii) $a(t)$ and $n(t)$ are samples from zero-mean, uncorrelated Gaussian random processes with autocorrelations $R_a(t,u)$ and $R_n(t,u)$, respectively.

(iii) $L(t,u)$, $R_a(t,u)$ and $R_n(t,u)$ are *semi-stationary* with maximum correlation time τ and maximum spectral-fluctuation bandwidth β.

Contrarily to the conventional PSD matrix of a stationary process, the SF of an arbitrary autocorrelation kernel $R_a(t,u)$ will not, in general, be Hermitian and non-negative definite. However, it can be verified that this will "nearly" be the case if $R_a(t,u)$ is semi-stationary, as assumed in (iii). This leads us naturally to make an additional assumption about $R_a(t,u)$ and $R_n(t,u)$, namely:

(iv) There exists a $N\times N$ non-negative Hermitian matrix $A(t,\omega) = A^*(t,-\omega)$ such that

$$R_a(t,u) = \frac{1}{2\pi}\int A(t,\omega)\,e^{j\omega(t-u)}\,d\omega + O(\beta\tau). \tag{30}$$

Similarly, there exists a $M\times M$ non-negative Hermitian matrix $N(t,\omega) = N^*(t,-\omega)$ in terms of which $R_n(t,u)$ can be expressed as in (30). It is further assumed that $N(t,\omega) \ge \eta I_M$, where $\eta > 0$ is independent of t and ω and where I_M is the $M\times M$ identity matrix. (This ensures that the noise-prewhitening operation is well-defined.)

When $a(t)$ is stationary, one can (and should) use for $A(t,\omega)$ the PSD matrix of $a(t)$, in which case (30) is exact, i.e. $O(\beta\tau) = 0$.

By definition, the LLP for the above model evaluates the log-likelihood function (LLF), $\ln\Lambda(x)$, of the array output vector $x(t)$ (29). Using the properties of semi-stationary LS derived in [7] and the factorization properties of the LLP [5], the following expressions can be obtained for the LLF:

$$\ln\Lambda(x) = \tfrac{1}{2}\{l_1(x) - l_2\}, \tag{31}$$

$$l_1(x) = \int y^T(t)\,\hat{a}(t)\,dt, \tag{32}$$

$$y(t) = \frac{1}{2\pi}\int Y(t,\omega)\,e^{j\omega t}\,d\omega \tag{33}$$

$$Y(t,\omega) = C^H(t,\omega)N^{-1}(t,\omega)X(\omega), \tag{34}$$

$$X(\omega) = \int x(t)\,e^{-j\omega t}\,dt. \tag{35}$$

$$\hat{a}(t) = \frac{1}{2\pi}\int G(t,\omega)\,Y(t,\omega)\,e^{j\omega t}\,dt, \tag{36}$$

$$G(t,\omega) = [I + A(t,\omega)\,\Omega(t,\omega)]^{-1}A(t,\omega), \tag{37}$$

$$\Omega(t,\omega) = C^H(t,\omega)N^{-1}(t,\omega)C(t,\omega). \quad (38)$$

$$l_2 = \frac{1}{2\pi}\int\int \ln\det\{I + A(t,\omega)\Omega(t,\omega)\}\, d\omega\, dt, \quad (39)$$

where I is the $N\times N$ identy matrix. $l_1(x)$ (32) simply evaluates the scalar product (or correlation integral) of the N-component vector signals $y(t)$ and $\hat{a}(t)$. $y(t)$ (33)-(35) is obtained by passing the sensor output vector $x(t)$ through a space-time whitening filter followed by a generalized beamformer "steered" at the N individual sources present in the signal model (29). $\hat{a}(t)$ (36)-(38) is the noncausal minimum mean square error (MMSE) estimate of the source signal $a(t)$ from the sensor outputs $x(t)$, $-\infty < t < \infty$. Finally, l_2 (39) is a bias term independent of the observed data. The above expressions are asymptotic in nature and can be used with great accuracy whenever $\beta\tau \ll 1$. We note that the results of [3] can be obtained as a particular case of these expressions.

V. CRLB

The general expressions derived in the previous Section for the LLF can be used to find the ML estimator of any unknown parameter present in the observation model (29) for $x(t)$. It is well known that when the observation interval is sufficiently long, which is the case under the assumption of semi-stationarity, this estimator achieves the best performance among all unbiased estimators, with its error covariance matrix reaching the absolute minimum predicted by the Cramér-Rao lower bound (CRLB) [2]. We now present general expressions for the CRLB that apply to any source-array configuration satisfying the basic assumptions of (i)-(iv) stated at the beginning of Section IV.

First, we recall the definitions of the Fisher information matrix (FIM) and the CRLB. Let θ be the vector of unknown parameters in the observation model (29). The FIM, denoted $J(\theta)$, is a square matrix of dimension equal to the number of parameters in θ, whose i,j^{th} element is given by

$$J_{ij}(\theta) = -E_\theta\{\partial_i\partial_j \ln\Lambda(x;\theta)\}, \quad (40)$$

where E_θ is the expectation conditioned on θ and $\partial_i \equiv \partial/\partial\theta_i$ indicates a partial derivative with respect to θ_i, the i^{th} component of θ. The CRLB sets a lower bound on the error covariance matrix of any unbiased estimator $\hat{\theta}(x)$ of θ. More precisely, it asserts that

$$E_\theta\{[\hat{\theta}(x)-\theta][\hat{\theta}(x)-\theta]^T\} \geq J(\theta)^{-1} \quad (41)$$

where $J(\theta)^{-1}$ is the inverse of the FIM.

Under the assumptions (i)-(iv) stated in Section IV, the following expression can be obtained for the elements of $J(\theta)$,

$$J_{ij}(\theta) = \frac{1}{4\pi}\int\int \mathrm{Tr}\{\partial_i C_R(t,\omega;\theta)\partial_j C_H(t,\omega;\theta)\}\, d\omega\, dt, \quad (42)$$

$$C_R(t,\omega;\theta) = N^{-1/2}CAC^H N^{-1/2}, \quad (43)$$

$$C_H(t,\omega;\theta) = N^{-1/2}C(I + A\Omega)^{-1}AC^H N^{-1/2}. \quad (44)$$

In (43)-(44), the dependence of A, C, N and Ω (38) on t, ω and θ has been omitted for convenience. Some important specializations of (42) are now presented.

Let θ_a and θ_l be the vectors of unknown parameters in terms of which the source signal autocorrelation $R_a(t,u;\theta_a)$ and the channel impulse response $L(t,u;\theta_l)$ are specified. θ_a and θ_l will be referred to as the *source signal* and *transmission* parameter vectors, respectively. Depending whether θ_i and θ_j in (42) are source signal or transmission parameters, four distinct cases occur in the evaluation of $J_{ij}(\theta)$. However, since the FIM is symmetric, only three of them need to be considered. Substituting (43)-(44) in (42) and using the dependence relations $A \equiv A(t,\omega;\theta_a)$, $C \equiv C(t,\omega;\theta_l)$, $N \equiv N(t,\omega)$, and $\Omega \equiv \Omega(t,\omega;\theta_l)$, it can be verified that *when θ_i and θ_j are both source signal parameters,*

$$J_{ij}(\theta) = \frac{1}{4\pi}\int\int \mathrm{Tr}\{\partial_i A\,\Omega(I+A\Omega)^{-1}\,\partial_j A\,\Omega(I+A\Omega)^{-1}\}\, d\omega\, dt. \quad (45)$$

It is interesting to note that in this case, the dependence of $J_{ij}(\theta)$ on the channel SF $C(t,\omega)$ is only through $\Omega = C^H N^{-1} C$. In a similar way, it can be verified that *when θ_i is a transmission parameter and θ_j is a source signal parameter,*

$$J_{ij}(\theta) = \frac{1}{2\pi}\int\int \mathrm{Tr}\{A\,\partial_i C^H\,N^{-1}\,C\,\Omega\,(I+A\Omega)^{-1}$$
$$\partial_j A\,\Omega\,(I+A\Omega)^{-1}\}\, d\omega\, dt. \quad (46)$$

In this case, $J_{ij}(\theta)$ depends explicitely on C (except in the case $N=1$ where further simplifications are possible; see [7]). Equation (46) can be used to draw general conclusions on how a lack of a priori knowledge of the source signal parameters affects the minimum variance achievable in estimating transmission parameters. Finally, *when both θ_i and θ_j are transmission parameters,*

$$J_{ij}(\theta) = \frac{1}{2\pi}\int\int \mathrm{Tr}\{A\,\partial_i C^H\,N^{-1}\,[\partial_j C\,G\,C^H + C\,G\,\partial_j C^H$$
$$- C\,G\,\partial_j\Omega\,G\,C^H]\,N^{-1}\,C\}\, d\omega\, dt \quad (47)$$

where G is given by (37). We note that the expression of the FIM for the Taylor coefficients of the differential delays given in [4] in the case of a single source and spatially uncorrelated noise follows trivially as a particular case of (47).

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