SOURCE DETECTION AND DOA ESTIMATION FROM TWO OBSERVATIONS OF A FINITE LINE APERTURE

Benoit Champagne

INRS-Télécommunications, Université du Québec
16 place du Commerce, Verdun, Québec, Canada H3E 1H6

ABSTRACT

This paper deals with the joint problem of source detection and direction of arrival (DOA) estimation from the observation of a line aperture of finite length at two distinct times. The emphasis is given to the practical case of small *KL* product, where *K* is the bandwidth of the spatial signal and *L* is the aperture length. The optimum (log-likelihood) processor for detection and DOA estimation is derived. A suboptimum processor with reduced computational complexity is also obtained based on a high signal-to-noise ratio (SNR) approximation. Computer simulations indicate that significant improvements in performance are possible with these processors when compared with more conventional correlation-based processing techniques. Moreover, at high SNR, the proposed DOA estimators are nearly efficient.

I. INTRODUCTION

In several applications of array processing, the time interval available to detect a plane wave signal or to estimate its direction of arrival (DOA) is limited. This occurs for instance when monitoring transient signals of short duration. In this paper, we investigate the joint problem of source detection and DOA estimation for a plane wave signal observed in the presence of noise over a line aperture of finite length at only two consecutive times.

This problem has a well known dual in the time domain: detection and DOA estimation of a plane wave observed over a finite time aperture (window) at two spatially separated locations. The corresponding estimation problem is better known as time delay estimation (TDE). Several TDE processors have been developed in the past under the asymptotic condition of long observation window [1]. However, these processors perform generally poorly when the observation window is too short. Recently, an exact version of the maximum likelihood TDE processor which incorporates the effects of finite observation window has been proposed to overcome this difficulty [2].

In the present context, the condition of long observation window used in TDE is equivalent to that of long spatial aperture. However, due to the assumed transient nature of the signal, to physical limitations in the size of the line aperture and to the behavior of the observed spatial signals as a function of the DOA, this condition is not very practical. Based on the results of [2] for the TDE problem, it appears necessary to take into account the effects of finite aperture length for a proper treatment of the problem under consideration here.

This paper focuses on the derivation and the performance evaluation of the exact optimum processor for source detection and DOA estimation using the observations of a finite line aperture at two consecutive times. The emphasis is given to the practical case of small *KL* product, where *K* is the bandwidth of the spatial signal and *L* is the aperture length. This study provides and extension to the results of [3] where only the estimation problem is considered.

II. SIGNAL MODEL; CONNECTION TO TDE

Consider a plane wave incident on a continuous line aperture of length L, with extremities located at $x=0$ and $x=L$ along the x-axis, as shown in Fig. 1. Assuming that the propagation medium is non-dispersive and that the aperture operates as an ideal transducer, the response $r(x,t)$ of the aperture at position x and time t can be expressed as

$$r(x,t) = s(t + \frac{x}{c}\cos\theta) + n(x,t) \tag{1}$$

where $s(t)$ is the plane wave signal, θ is the corresponding DOA measured from endfire, c is the wave propagation velocity and $n(x,t)$ is an additive noise term. Sources of noise include the electronic circuitry used for the measurement as well as unwanted signals propagating in the environment.

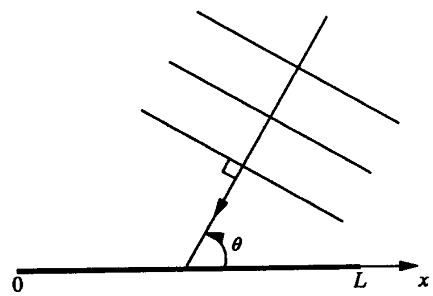

Fig. 1. Plane wave incident on a continuous line aperture of length L.

The response of the aperture along the x-axis is observed (sampled) at two distinct times, say $t=0$ and $t=T_s$, resulting in the following spatial signals:

$$\begin{aligned} r_1(x) &= a(x;\theta) + n_1(x), \qquad 0 \le x \le L, \\ r_2(x) &= a(x+\Delta;\theta) + n_2(x), \end{aligned} \tag{2}$$

where

$$a(x;\theta) = s(\frac{x}{c}\cos\theta) \tag{3}$$

is the observed signal component at time $t=0$,

$$\Delta = c\,T_s/\cos\theta \qquad (4)$$

is the space shift between the two signal components and $r_i(x)=r(x,t_i)$, $n_i(x)=n(x,t_i)$ for $i=1,2$.

The signals $s(t)$, $n_1(x)$ and $n_2(x)$ are modeled as uncorrelated stationary Gaussian random processes with zero-mean and known second-order statistics. The autocorrelation function and power spectral density of the process $s(t)$ are represented by $R_s(\tau)=E[s(t+\tau)s(t)]$, where $E[.]$ stands for statistical expectation, and $G_s(\omega)=\int R_s(\tau)\,e^{-j\omega\tau}\,d\tau$, respectively. At this point, no particular assumption is made on $R_s(\tau)$ and $G_s(\omega)$. The noise components $n_i(x)$, $i=1,2$, are modeled as normalized white noise processes with autocorrelation function and power spectral density given by

$$R_n(\xi)=\delta(\xi), \quad G_n(\kappa)=1. \qquad (5)$$

The following convention is used throughout: the variables τ and ω are used to denote time lag and temporal frequency, while the variables ξ and κ are used to denote space lag and spatial frequency, respectively.

In this paper, we consider the joint problem of source detection and DOA estimation from an observation of the spatial waveforms $r_i(x)$, $i=1,2$, on the interval $0\le x\le L$. In the first case, we have to decide between two hypotheses: source present or source absent (i.e. no signal component in (2)). In the second case, we know that a signal is present, but we do not know the true value of the DOA parameter, denoted θ^*. To simplify the discussion, it is assumed that $0\le\theta^*\le \arccos(c\,T_s/L)$, or equivalently $c\,T_s\le\Delta^*\le L$, where Δ^* is the true space shift corresponding to θ^*. When $\Delta^*>L$, there is no overlap between $a(x;\theta)$ and $a(x+\Delta;\theta)$ in (2) and coherent processing of $r_1(x)$ and $r_2(x)$ is of limited interest.

We now make further comments on the relation existing between the above problem and the TDE problem mentioned earlier. First, we note that there is a direct analogy between the model equation (2) and the conventional TDE model used in the literature [1], with the roles of space and time being exchanged. This means that conventional TDE processors can also be applied to the problem of interest here by making the appropriate modifications. Hence, in principle at least, θ^* can be estimated by first estimating the space shift Δ^* between $r_1(x)$ and $r_2(x)$ with a generalized cross-correlator and then using (4) to obtain the corresponding angle.

There are however fundamental differences between a conventional TDE model and the model equations (2)-(4). In a conventional (i.e. asymptotic) TDE model, it is assumed that the observation window is sufficiently long, so that $T/D\gg1$ and $TB\gg1$, where T/D represents the ratio of observation window to time delay and TB represents the time-bandwidth product. The spatial counterpart of T/D is the ratio of aperture length to space shift

$$L/\Delta=(L\cos\theta)/cT_s. \qquad (6)$$

The spatial counterpart of TB is the product KL, where K is the spatial bandwidth of the signal $a(x;\theta)$. If we denote by B the bandwidth of $G_s(\omega)$, it follows from (3) that

$$KL=(BL\cos\theta)/c. \qquad (7)$$

Hence, for a fixed L, both L/Δ and KL tend to zero as the DOA increases from endfire ($\theta=0^o$) to broadside ($\theta=90^o$).

In the present context, because of physical limitations on the aperture length L, the conditions $L/\Delta\gg1$ and $KL\gg1$ can only be satisfied over a limited range of values of θ. Moreover, there are situations where the product KL is intrinsically small, even for $\theta=0$. This occurs for instance when monitoring high-energy transient signals of short spatial duration. Consequently, when investigating the optimum processor associated with the signal model (2)-(4), it does not appear very practical to work under the asymptotic condition of long spatial aperture. In the rest of the paper, no such assumption is made; the emphasis is given to the more practical case of small KL product.

III. OPTIMUM PROCESSOR

The optimum processor, also known as the log-likelihood processor, evaluates the log-likelihood function (LLF) of the observed data. For the problem under consideration here, the observed data consists of the vector process

$$\mathbf{r}(x)=[r_1(x),r_2(x)]^T, \quad 0\le x\le L, \qquad (8)$$

where $r_i(x)$, $i=1,2$, are given by (2)-(4) and the superscript T denotes transposition. We note that $\mathbf{r}(x)$ belongs to the general class of Gaussian signals in additive Gaussian white noise. Consequently, its LLF, denoted $\ln\Lambda(\mathbf{r}(.);\theta)$, can be expressed in the form

$$\ln\Lambda(\mathbf{r}(.);\theta)=\tfrac{1}{2}\,[l_1(\mathbf{r}(.);\theta)-l_2(\theta)], \qquad (9)$$

$$l_1(\mathbf{r}(.);\theta)=\sum_{i=1}^{\infty}\frac{\lambda_i}{1+\lambda_i}\,\{\int_0^L \phi_i^T(x)\,\mathbf{r}(x)\,dx\,\}^2, \qquad (10)$$

$$l_2(\theta)=\sum_{i=1}^{\infty}\ln(1+\lambda_i). \qquad (11)$$

In (9)-(11), λ_i and $\phi_i(x)$ are the eigenvalues and normalized eigenfunctions of the vector process

$$\mathbf{a}(x;\theta)=[a(x;\theta),a(x+\Delta;\theta)]^T. \qquad (12)$$

They are obtained by solving the integral equations

$$\int_0^L E[\mathbf{a}(x)\mathbf{a}(\xi)^T]\phi_i(\xi)d\xi=\lambda_i\phi_i(x), \quad 0\le x\le L, \qquad (13)$$

$$\int_0^L \phi_i^T(x)\phi_j(x)dx=\delta_{ij}. \qquad (14)$$

Although not specified explicitly, the eigenvalues λ_i and eigenfunctions $\phi_i(x)$ depend on the DOA parameter θ.

In a typical detection problem, the output of the optimum processor, as given by the LLF (9)-(11), is compared to a preset threshold. A decision is made in favor of a source being present only if the LLF exceeds the threshold. In a typical estimation problem where no a priori information is available about the true value of the DOA θ^*, the LLF is evaluated for different values of the hypothetical DOA parameter θ. The value of θ resulting in a maximum for the LLF is selected as the estimate of θ^*. This estimate is known as the maximum likelihood (ML) estimate.

To completely specify the optimum processor, we must solve the integral equations (13)-(14). Following the approach of [2], this can be achieved in two steps. In the first step, the vector integral equations are transformed into equivalent scalar integral equations by applying a generalized beamforming operation on the observed data $\mathbf{r}(x)$ (8). In the second step, the resulting "reduced" integral equations are solved, using either an analytical or a numerical approach.

When $0 \le \theta \le \arccos(cT_s/L)$, the results of the dimensionality reduction can be summarized as follows: Let λ_i denote the non-zero eigenvalues of (13) and $\phi_i(t)$ be the corresponding normalized eigenfunctions. Then,

$$\phi_i(x) = [\eta_i(x), \eta_i(x+\Delta)]^T. \quad (15)$$

The functions $\eta_i(x)$ and the eigenvalues λ_i satisfy the scalar integral equations:

$$\int_0^{L+\Delta} R_s(\frac{(x-\xi)}{c}\cos\theta)\eta_i(\xi)\rho(\xi)d\xi = \lambda_i\eta_i(x), \quad (16)$$

$$\int_0^{L+\Delta} \eta_i(x)\eta_j(x)\rho(x)dx = \delta_{ij}, \quad (17)$$

where

$$\rho(x) = \begin{cases} 1, & 0 < x < \Delta \ or \ L < x < L+\Delta. \\ 2, & \Delta < x < L. \end{cases} \quad (18)$$

is a spatial weighting function.

In general, it is possible to obtain a numerical solution to (16)-(17) by following these steps: sampling uniformly in the spatial domain, expressing (16) as a matrix eigenvalue problem and using a general computer routine for eigenvalue decomposition. However, when the source signal $s(t)$ has a rational power spectral density function, an exact analytical solution to (16)-(17) may be obtained by properly modifying the technique of [2].

Upon substitution of (15) in (10), we obtain

$$l_1(\mathbf{r}(.);\theta) = \sum_{i=1}^{\infty}\frac{\lambda_i}{1+\lambda_i}\{\int_0^{L+\Delta}\eta_i(x)u(x)dx\}^2, \quad (19)$$

where

$$u(x) = r_1(x) + r_2(x-\Delta), \quad 0 < x < L+\Delta, \quad (20)$$

with $r_i(x)=0$ for $x<0$ and $x>L$. Hence, the optimum processor (9)-(11) can be decomposed into a cascade of two functionally distinct sub-processors. The first sub-processor, which performs the operation in (20), is the spatial counterpart of a delay-and-sum beamformer. It attempts to recombine $r_1(x)$ and $r_2(x)$ coherently based on the assumed signal model (2)-(4). The second sub-processor, defined by (19), (9) and (11), can be interpreted as a log-likelihood energy detector for the scalar signal $u(x)$ at the beamformer output.

At high signal-to-noise ratio (SNR), the following approximation can be obtained for $l_1(\mathbf{r}(.);\theta)$ in (19):

$$l_1(\mathbf{r}(.);\theta) \approx \int_0^{L+\Delta}\frac{u^2(x)}{\rho(x)}dx \quad (21)$$

The expression (21) has two important advantages over (19). From a computational viewpoint, (21) is considerably simpler since it does not require the calculation and the use of the eigenfunctions $\eta_i(x)$. From a statistical viewpoint, (21) is more robust since it does not require any knowledge of the source signal statistics.

Additional details concerning the digital implementation of the optimum processor can be found in [3].

IV. COMPUTER SIMULATIONS

Computer simulations were used to evaluate the performance of the optimum and sub-optimum processors derived in Section III. Conventional correlation-based processors adapted from the TDE literature were also evaluated for the purpose of comparison.

The following scenario was used for the simulations. The continuous-time signal $s(t)$ was modeled as a zero-mean stationary Gaussian process with a first-order Butterworth spectral density function of the form $G_s(\omega) = 2\alpha P/(\omega^2+\alpha^2)$, where P represents the mean-square value of $s(t)$ and α is a measure of its spectral width ($G_s(\alpha)$ = -3dB).

The continuous line aperture was realized as a uniform line array of $N_s=32$ sensors. To minimize the effects of spatial aliasing, the maximum angular frequency of interest was taken as $2\pi f_{max}=16\alpha$ (-24dB point of $G_s(\omega)$), and the sensor spacing was set to $L_s=\lambda_{min}/2$, where $\lambda_{min}=c/f_{max}$ is the wavelength corresponding to f_{max}. The resulting aperture length, i.e. $L=2\pi c/\alpha$, was used as the unit of distance in the simulations. The time interval T_s between successive observations of the array response was chosen as a multiple of $L_s/c=2f_{max}$, which is the Nyquist rate corresponding to f_{max}.

Synthetic random signals corresponding to specific values of SNR and true DOA θ^* were generated according to the above specifications. These signals were simultaneously applied to the input of four different processors, namely: optimum processor (OPT); high-SNR approximation to optimum processor (HOPT); asymptotic processor based on large *KL* product assumption (ASYM); and a processor calculating the tapered cross-correlation (TCC) [4]. The ASYM processor is an adaptation of the Hannan-Thompson processor used for TDE [1]; modifications have been included to account for the dependence of the spectrum of the process $a(x;\theta)$ in (3) on the DOA parameter θ.

The detection and DOA estimation performance of each processor were evaluated statistically by running a large number of independent experiments. For any given value of the detection threshold, 10^4 independent experiments were run, with and without source signals. The corresponding probabilities of detection p_d and false alarm p_f were then estimated as relative frequencies. To obtain the receiver operating characteristic (ROC) curve (i.e. p_d versus p_f), this procedure was repeated for several values of the threshold.

In the case of DOA estimation, an initial estimate of Δ was obtained by maximizing the processor output over a discrete set of values of Δ given by $\Delta=\Delta^*\pm k\,L_s$, $k=0,\ldots,5$. (By considering a search region of this type, we limit our attention to the small-error behavior of the estimators.) The initial estimate was then refined by means of a three point quadratic interpolation scheme. The resulting estimate $\hat{\Delta}$ was finally converted to a DOA estimate $\hat{\theta}$ by means of (4). The sample mean and variance of the estimates were evaluated by running 1000 independent experiments.

The following results correspond to a sampling interval of $T_s = 4L_s/c$ and a true DOA of $\theta^* = 48.2°$, i.e. $\Delta^* = 6L_s$. Fig. 2 shows the ROC curves of the various processors for SNR = 5dB. The results clearly indicate that better detection performance can be achieved with the OPT and HOPT processors. We note the particularly poor performance of the ASYM processor. Because it is based on the assumption of large aperture length, the ASYM processor does not properly handle edge effects, which are responsible for the performance deterioration. The TCC processor, which incorporates corrections for the finite aperture length, has better detection characteristics than the ASYM processor, although not as good as the OPT and HOPT processors.

Fig. 3 shows the sample bias (in degrees) of the corresponding estimators as a function of SNR. We note that for SNR ≥ 20dB, both OPT and HOPT estimators are practically unbiased. Fig. 4 shows the sample standard deviation (in degrees) of the four estimators as a function of SNR. The Cramer-Rao lower bound (CRLB) is also shown for the sake of comparison. The standard deviation of both OPT and HOPT estimators is comparable to the CRLB for SNR ≥ 20dB. We therefore conclude that these estimators are almost efficient at high SNR and that any significant improvement in performance is not feasible.

Finally, we note that the performance of the various processors is not uniform as a function of the true DOA parameter θ^*. For instance, it can be shown easily that the CRLB tends to ∞ in the limit $\theta \to 0$. This is due to the non-linear relation (4) between θ and Δ: for θ close to zero, a small error in Δ will induce a very large error in θ. It can also be argued that the performance of the estimators will deteriorate severely as $\Delta \to L$. In this case, there is no overlap between the observed signal components and the estimation process should be dominated by large errors. These observations have been confirmed by simulations. Fig. 5 shows the standard deviations of the OPT and HOPT estimators as a function of θ^o for SNR = 20dB. We clearly note the deviation from the CRLB as θ increases.

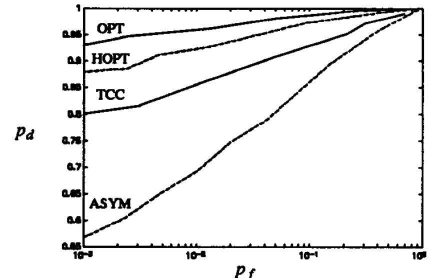

Fig. 2. ROC curves of the four processors for SNR = 5dB and true DOA $\theta^* = 48.2°$.

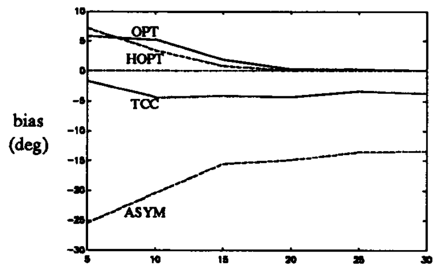

Fig. 3. Bias of the four DOA estimators as a function of SNR for true DOA $\theta^* = 48.2°$.

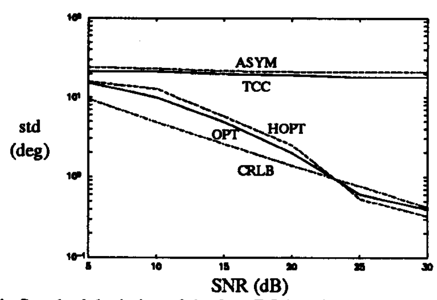

Fig. 4. Standard deviation of the four DOA estimators as a function of SNR for true DOA $\theta^* = 48.2°$.

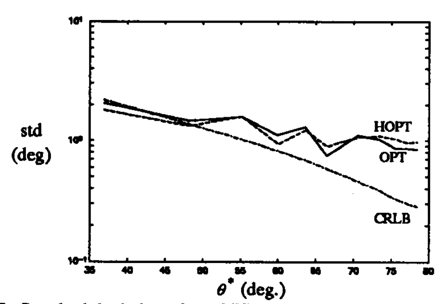

Fig. 5. Standard deviation of the OPT and HOPT estimators as a function of true DOA θ^* for SNR = 20dB.

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