

Sensor Localization in NLOS Environments with Anchor Uncertainty and Unknown Clock Parameters

Siamak Yousefi*, Reza Monir Vaghefi[†], Xiao-Wen Chang[‡], Benoit Champagne*, and R. Michael Buehrer[†]

*Department of Electrical and Computer Engineering, McGill University, Montreal, Quebec, H3A 0E9, Canada

[†]Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA 24061, USA

[‡]School of Computer Science, McGill University, Montreal, Quebec, H3A 0E9, Canada

Email: siamak.yousefi@mail.mcgill.ca; vaghefi@vt.edu; chang@cs.mcgill.ca; benoit.champagne@mcgill.ca; buehrer@vt.edu

Abstract—In this paper, joint sensor localization and synchronization in non-cooperative wireless sensor networks (WSNs) using time-of-arrival (TOA) measurements is studied. In addition to zero-mean errors in TOA measurements we consider other sources of error such as non-line-of-sight (NLOS) propagation and anchor uncertainty to make our technique more useful in practice, where the presence of these errors is inevitable. The proposed technique is based on semi-definite programming (SDP) relaxation which can be solved in polynomial time and guarantees convergence to the global minimum. It is shown that the optimal accuracy is obtained by discarding the NLOS measurements and applying the maximum likelihood (ML) technique to jointly estimate the positions of anchor nodes, and the position and clock parameters of the sensor node. The results show that the proposed SDP technique, which does not require prior identification of NLOS links, is robust against NLOS errors and its performance is close to that of optimal accuracy.

Index Terms—Convex relaxation, joint localization and synchronization, non-line of sight, semidefinite programming.

I. INTRODUCTION

Sensor localization has recently received much attention due to the tremendous number of applications, e.g., in surveillance, healthcare, security, etc [1], [2]. The signal of Global Positioning System (GPS) is typically either severely degraded or unavailable in indoor places and dense urban areas. Furthermore, the battery consumption of a GPS receiver is high and thus using GPS may not be suitable for a wireless sensor network (WSN) with power constraints. Due to the mentioned limitations of the GPS, a sensor-based localization system needs to be employed to offer a reliable localization service for different applications. In sensor-based localization, fixed anchor nodes with known locations are used instead of satellites and a sensor with an unknown position is localized by using the measurements obtained within the network.

Different measurements can be exploited between the nodes for the aim of localization, among which time-of-arrival (TOA) measurements in ultra wide-band (UWB) systems have received great attention due to the high resolution of the timing pulses [3]. Although a two-way ranging (TWR) protocol can remove the sensor's clock offset and provide relatively accurate TOA measurements, the effect of clock skew can lead to large ranging errors in some applications. Therefore, synchronization is an essential part of TOA-based localization.

In some systems, the clocks at the sensor nodes are first synchronized, e.g., see in [4], and then the localization can

be performed by various techniques as summarized in [5]. However, the two-step approaches might yield poor localization performance due to inaccurate synchronization. Due to the close relationship between range and clock parameters, several studies have focused on joint approaches, where synchronization and localization are performed simultaneously. Different techniques in the literature for joint localization and synchronization show that the performance can be improved over the two-step approaches [6]–[9].

The cost function that needs to be minimized for the localization problem or joint localization and synchronization is nonlinear and non-convex; hence the maximum likelihood (ML) method needs an accurate initial point such that it converges to the global minimum. To avoid this problem, the cost function can be relaxed to a convex one such that the global optimum can be obtained in polynomial time by iterative techniques. For localization problems, relaxations to semi-definite programming (SDP) and second order cone programming (SOCP) problems have been considered in [10] and [11], respectively, where it is assumed that no clock parameters exist in the cost function. When the clock parameters are also unknown, an SDP relaxation of the optimization problem has been considered in [12].

Although these estimators perform well in line-of-sight (LOS) situations, in practice this assumption is not satisfied due to multipaths and blockage of the LOS signal. In fact, one of the main sources of error in WSN localization is the non-line-of-sight (NLOS) problem which is usually inevitable in indoor places and urban canyons, and results in positively biased TOA measurements. TOA-based localization techniques in NLOS situations have been widely studied in the literature. A summary of different NLOS mitigation techniques can be found in [13]. In scenarios where the NLOS measurements can not be identified, robust techniques against NLOS errors have been recently proposed in [14], [15]. However, all previous studies assume that perfect synchronization is maintained between the sensor and the anchor nodes, thus localization performances will degrade in the presence of synchronization errors.

In this work, we consider joint localization and synchronization of a sensor in a non-cooperative WSN with anchor uncertainty and NLOS errors, where we assume that the NLOS links have not been identified. We first consider the

optimal result, i.e., the ML estimate, which can be obtained by identifying the NLOS measurements and discarding them, and only minimizing the 2-norms of the errors due to the LOS measurements [16]. In this case we assume that we have a sufficient number of links. We then relax the ML problem into an SDP problem such that it is robust against NLOS errors in the TOA measurements. The simulation results show that the proposed technique can obtain a performance close to the optimal solution when the number of NLOS measurements is low compared to LOS ones. Furthermore, its performance is also robust in severe NLOS contamination and outperforms state-of-the-art techniques.

The remainder of this paper is organized as follows. In Section II, the system model of joint synchronization and localization in NLOS environments is introduced. The optimal performance of the system is discussed in Section III. The proposed NLOS mitigation technique based on SDP is described in Section IV. The simulation results are illustrated in Section V. Finally Section VI concludes the paper.

II. SYSTEM MODEL

The following notation is used throughout the paper. Lowercase and uppercase letters denote scalar values. Bold uppercase and bold lowercase letters denote matrices and vectors, respectively. \mathbf{I}_M and $\mathbf{0}_M$ denote the $M \times M$ identity and zero matrices, respectively. $\mathbf{1}_M$ is a length- M column vector of ones. For arbitrary symmetric matrices \mathbf{A} and \mathbf{B} , $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

We consider a two-dimensional (2D) network with M fixed anchor nodes located at positions $\mathbf{x}_j = [x_j, y_j]^T \in \mathbb{R}^2$ for $j = 1, \dots, M$, and a sensor located at $\mathbf{x} = [x, y]^T \in \mathbb{R}^2$. The position of the sensor is unknown while the position of the anchor nodes are estimated through another system, e.g., GPS. Therefore, estimates of \mathbf{x}_j 's are available as \mathbf{a}_j , which are modeled herein as

$$\mathbf{a}_j = \mathbf{x}_j + \mathbf{u}_j, \quad j = 1, \dots, M \quad (1)$$

where $\mathbf{u}_j \in \mathbb{R}^2$ is a zero-mean Gaussian error with covariance matrix Φ_j , which is assumed to be known.

The anchor nodes are synchronized with a reference clock, while the sensor is not synchronized with the network and its local clock reading can be modelled as

$$C(t) = \alpha t + \theta_0 \quad (2)$$

where t is the true time, and θ_0 and α are the relative clock offset and clock skew parameters between the clock of the sensor and those of the anchor nodes, respectively. Since the anchor nodes can be synchronized with an accurate local clock, we can assume that the clock skews of the anchor nodes are equal to 1 and their clock offsets are 0. The scheme of the local times of a sensor and an anchor are illustrated in Fig. 1.

The links between the sensor and the anchor nodes are divided into two sets: \mathcal{L} and \mathcal{N} contain the indices of the anchor nodes which have LOS and NLOS links with the sensor, respectively. In exchanging the timing signals, the j -th anchor node transmits at time $t_{j,s,0}$ and if the sensor clock

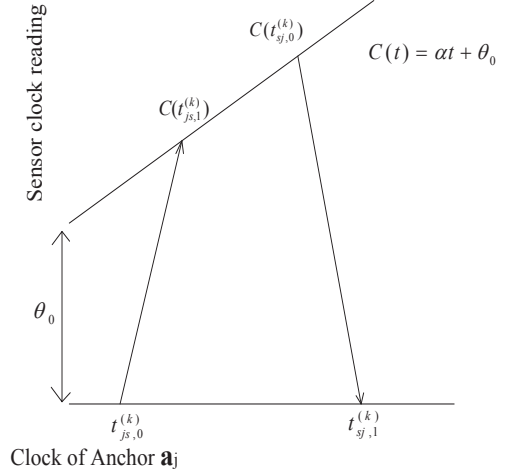


Fig. 1. Illustration of the local times of a sensor and an anchor.

is synchronized with the anchor clocks, the timing signal is received at the sensor at $t_{j,s,1}$ where

$$t_{j,s,1} = \begin{cases} t_{j,s,0} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,0}, & j \in \mathcal{L} \\ t_{j,s,0} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,0}}{c} + n_{j,0}, & j \in \mathcal{N} \end{cases} \quad (3)$$

where $n_{j,0}$ is a zero-mean Gaussian measurement noise with a variance of σ_j^2 . The variance of the measurement noise depends on SNR, bandwidth, signal duration, and carrier frequency [17], and can be modelled as

$$\sigma_j^2 = \frac{\xi}{c^2} \|\mathbf{x} - \mathbf{x}_j\|^{\gamma_j} \quad (4)$$

where ξ is a positive scaling factor whose value depends on the propagation environment and hardware implementation aspects. γ_j is the path-loss exponent whose value depends on the propagation environment as

$$\gamma_j = \gamma_{\mathcal{L}}, \quad j \in \mathcal{L}, \quad \gamma_j = \gamma_{\mathcal{N}}, \quad j \in \mathcal{N}. \quad (5)$$

where usually $\gamma_{\mathcal{L}} = 2$ (free space) and $\gamma_{\mathcal{N}} = 3$ (harsh environments) are considered [18]. Finally, $b_{j,0}$ in (3) is the NLOS bias which is a large positive random variable, and has been modeled as uniform [19], or exponential [20], to name a few. Note that we do not assume there is any *a-priori* information about the NLOS distribution or which link is in NLOS. Due to the erroneous behavior of sensor's clock, the clock reading at the sensor is

$$C(t_{j,s,1}) = \alpha t_{j,s,1} + \theta_0. \quad (6)$$

Therefore, we can model the clock reading as

$$C(t_{j,s,1}) = \begin{cases} \alpha(t_{j,s,0} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,0}) + \theta_0 \\ \alpha(t_{j,s,0} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,0}}{c} + n_{j,0}) + \theta_0 \end{cases} \quad (7)$$

By dividing (7) by α we have

$$-t_{j,s,0} = \begin{cases} \frac{C(t_{j,s,1}) - \theta_0}{\alpha} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,0} \\ \frac{C(t_{j,s,1}) - \theta_0}{\alpha} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,0}}{c} + n_{j,0} \end{cases} \quad (8)$$

Then after a certain delay the sensor transmits at time $t_{sj,0}$, the emitted signal travels the required distance and arrives at the j -th anchor at time $t_{sj,1}$:

$$t_{sj,1} = \begin{cases} t_{sj,0} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,1}, & j \in \mathcal{L} \\ t_{sj,0} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,1}}{c} + n_{j,1}, & j \in \mathcal{N} \end{cases} \quad (9)$$

However, the sensor does not know $t_{sj,0}$ accurately due to its clock error; hence, its clock reading is $C(t_{sj,0})$ which is related to $t_{sj,0}$ as

$$C(t_{sj,0}) = \alpha t_{sj,0} + \theta_0. \quad (10)$$

Therefore, the received time stamp at an anchor node is

$$t_{sj,1} = \begin{cases} \frac{C(t_{sj,0}) - \theta_0}{\alpha} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,1}, & j \in \mathcal{L} \\ \frac{C(t_{sj,0}) - \theta_0}{\alpha} + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,1}}{c} + n_{j,1}, & j \in \mathcal{N} \end{cases} \quad (11)$$

To simplify the expressions, we define two auxiliary variables as $\theta_1 = 1/\alpha$ and $\theta_2 = \theta_0/\alpha$. Therefore, for each anchor node with a LOS link, i.e., $j \in \mathcal{L}$, we have

$$-t_{js,0} = -C(t_{js,1})\theta_1 + \theta_2 + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,0} \quad (12)$$

$$t_{sj,1} = C(t_{sj,0})\theta_1 - \theta_2 + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + n_{j,1}. \quad (13)$$

Similarly, for each NLOS links, i.e., $j \in \mathcal{N}$, we have

$$-t_{js,0} = -C(t_{js,1})\theta_1 + \theta_2 + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,0}}{c} + n_{j,0} \quad (14)$$

$$t_{sj,1} = C(t_{sj,0})\theta_1 - \theta_2 + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + \frac{b_{j,1}}{c} + n_{j,1}. \quad (15)$$

In the following sections, the optimal performance expected from the above system model is provided.

III. PERFORMANCE EVALUATION

The performance analysis of NLOS localization for synchronous TOA-based networks is given in [15], [16] where it is shown that the Cramér-Rao lower bound (CRLB) of the sensor location is only dependent on the LOS links, if no statistical information about NLOS distribution is available [16]. Moreover, the optimal accuracy can be achieved by identifying the NLOS connections, discarding them, and employing an ML estimation technique to maximize the likelihood of the LOS measurements given the unknown parameters. Therefore, the ML estimate obtained by using only LOS connections can be used as a benchmark if a sufficient number of LOS measurements are available. The ML estimates of $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_M, \mathbf{x}]$ and $\boldsymbol{\theta} = [\theta_1, \theta_2]^T$ are obtained by solving the following minimization problem [21]:

$$\begin{aligned} \min_{\mathbf{X}, \boldsymbol{\theta}} \sum_{j \in \mathcal{L}} & \left[w_j \left(t_{js,0} - C(t_{js,1})\theta_1 + \theta_2 + \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} \right)^2 \right. \\ & \left. + w_j \left(t_{sj,1} - C(t_{sj,0})\theta_1 - \theta_2 - \frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} \right)^2 \right] \\ & + \sum_{j=1}^M (\mathbf{x}_j - \mathbf{a}_j)^T \boldsymbol{\Phi}_j^{-1} (\mathbf{x}_j - \mathbf{a}_j) \end{aligned} \quad (16)$$

where $w_j = (\sigma_j^2)^{-1}$ are the weighting elements equal to the inverse of the measurement noise variances. In this work, we have assumed that NLOS connections cannot be identified. Hence, finding the ML estimate, the solution of (16) is rather idealistic, but it will provide a lower bound on the localization error in order to compare the accuracy of our proposed technique under certain conditions. The ML estimate can be obtained only if there are at least four LOS connections available. When the sensor does not have a sufficient number of LOS connections, we can no longer obtain the ML estimate by (16), as no unique solution can be found in this case [22].

IV. SEMIDEFINITE PROGRAMMING

In this section, we develop an SDP relaxation technique for NLOS localization. Finding the ML estimate by solving (16) has three main problems. First, we need to identify the NLOS connections perfectly which is a difficult task. Second, access to a sufficient number of LOS anchors is not always possible due to limited network connectivity or NLOS situation, thus finding the ML estimate will be intractable. Third, the optimization problem in (16) is not convex. Since the cost function is nonlinear, an iterative method must be applied which requires an initial point. Also since the cost function is non-convex, the iterative solver might be stuck in a local minimum and produce large estimation errors. To alleviate these issues, an SDP relaxation technique is developed, which uses all the measurements and requires no NLOS identification. As a result of this relaxation, the optimization problem becomes convex and convergence to the global minimum is guaranteed.

The expressions in (12) can be written in matrix form as

$$\mathbf{t}^a = \mathbf{T}^s \boldsymbol{\theta} + \mathbf{d} + \mathbf{n} \quad (17)$$

where

$$\mathbf{t}^a = \begin{bmatrix} -t_{1s,0} \\ t_{s1,1} \\ \vdots \\ -t_{Ms,0} \\ t_{sM,1} \end{bmatrix}, \quad \mathbf{T}^s = \begin{bmatrix} -C(t_{1s,1}) & 1 \\ C(t_{s1,0}) & -1 \\ \vdots & \vdots \\ -C(t_{Ms,1}) & 1 \\ C(t_{sM,0}) & -1 \end{bmatrix}$$

$$\mathbf{d} = \frac{1}{c} \begin{bmatrix} d_1 \\ d_1 \\ \vdots \\ d_M \\ d_M \end{bmatrix}, \quad \mathbf{n}_k = \begin{bmatrix} n_{1,0} \\ n_{1,1} \\ \vdots \\ n_{M,0} \\ n_{M,1} \end{bmatrix},$$

with $d_j = \|\mathbf{x} - \mathbf{x}_j\|$. Since we do not know if a connection is NLOS or not, we assume all connections are NLOS. As the NLOS biases are positive and much larger than the noise terms it follows that with a high probability

$$\frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} - C(t_{js,1})\theta_1 + \theta_2 \leq -t_{js,0}, \quad j = 1, \dots, M \quad (18)$$

$$\frac{\|\mathbf{x} - \mathbf{x}_j\|}{c} + C(t_{sj,0})\theta_1 - \theta_2 \leq t_{sj,1}, \quad j = 1, \dots, M \quad (19)$$

Since (18) and (19) might not often hold true for LOS connections as the measurement noise has a zero-mean, it may be better to add a portion of the measurement noise standard deviation to the right side of them, to make them relaxed, similar to what done in [23]. However, in our simulations we observed better results in situations with large ratios of NLOS to LOS links, and only slightly worse results in low ratios of NLOS to LOS links, when (18) and (19) are used without modification. We then formulate the following nonlinear least squares problem

$$\min_{\mathbf{X}, \boldsymbol{\theta}} \left\| \boldsymbol{\Sigma}^{-\frac{1}{2}} (\mathbf{t}^a - \mathbf{d} - \mathbf{T}^s \boldsymbol{\theta}) \right\|^2 + \sum_{j=1}^M (\mathbf{x}_j - \mathbf{a}_j)^T \boldsymbol{\Phi}_j^{-1} (\mathbf{x}_j - \mathbf{a}_j) \quad (20)$$

s.t. (18), (19),

where $\boldsymbol{\Sigma} = \text{diag}\{\sigma_1^2, \sigma_1^2, \sigma_2^2, \sigma_2^2, \dots, \sigma_M^2, \sigma_M^2\}$ and we assume that this matrix is known in the algorithm, which can be achieved approximately by obtaining a set of consecutive TOA measurements and empirically calculating the sample variance for each link. Although the constraints in (18) and (19) are convex, the problem in (20) is still non-convex. In the following, we relax this optimization problem to a convex one. The first term in (20) can be expressed as

$$\text{Trace}\left\{ \boldsymbol{\Sigma}^{-1} (\mathbf{t}^a - \mathbf{d} - \mathbf{T}^s \boldsymbol{\theta}) (\mathbf{t}^a - \mathbf{d} - \mathbf{T}^s \boldsymbol{\theta})^T \right\} \quad (21)$$

which can also be rewritten as

$$\text{Trace}\left\{ \boldsymbol{\Sigma}^{-1} (\mathbf{t}^a - \mathbf{U}\tilde{\mathbf{d}}) (\mathbf{t}^a - \mathbf{U}\tilde{\mathbf{d}})^T \right\}$$

where

$$\mathbf{U} = \begin{bmatrix} 1 & \dots & 0 & -C(t_{1s,1}) & 1 \\ 1 & \dots & 0 & C(t_{s1,1}) & -1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \dots & 1 & -C(t_{Ms,1}) & 1 \\ 0 & \dots & 1 & C(t_{sM,1}) & -1 \end{bmatrix}, \quad \tilde{\mathbf{d}} = \begin{bmatrix} \frac{\|\mathbf{x} - \mathbf{x}_1\|}{c} \\ \frac{\|\mathbf{x} - \mathbf{x}_2\|}{c} \\ \vdots \\ \frac{\|\mathbf{x} - \mathbf{x}_M\|}{c} \\ \theta_1 \\ \theta_2 \end{bmatrix}.$$

By expanding the second term in the objective function in (20), and ignoring the constant terms, in line with [24], we can rewrite the optimization problem in (20) as

$$\min_{\mathbf{X}, \boldsymbol{\theta}} \text{Trace}\left\{ \boldsymbol{\Sigma}^{-1} (\mathbf{t}^a (\mathbf{t}^a)^T - 2\mathbf{U}\tilde{\mathbf{d}} (\mathbf{t}^a)^T + \mathbf{U}\mathbf{D}\mathbf{U}^T) \right\} + \sum_{j=1}^M \left(\text{Trace}(\boldsymbol{\Phi}_j^{-1} \boldsymbol{\Xi}_j) - 2\mathbf{a}_j^T \boldsymbol{\Phi}_j^{-1} \mathbf{x}_j \right) \quad (22)$$

s.t. (18), (19).

where

$$\mathbf{D} = \tilde{\mathbf{d}}\tilde{\mathbf{d}}^T \quad (23)$$

$$\boldsymbol{\Xi}_j = \mathbf{x}_j \mathbf{x}_j^T, \quad j = 1, \dots, M. \quad (24)$$

Note that the first M diagonal elements of the matrix \mathbf{D} are

$$[\mathbf{D}]_{jj} = \frac{d_j^2}{c^2} = \frac{1}{c^2} [\mathbf{e}_j^T \quad -1] \mathbf{Z} \begin{bmatrix} \mathbf{e}_j \\ -1 \end{bmatrix}, \quad j = 1, \dots, M, \quad (25)$$

where

$$\mathbf{Z} = \mathbf{X}^T \mathbf{X} \quad (26)$$

and \mathbf{e}_j is an $M \times 1$ vector in which the j th element is one and the other elements are zero. Now we regard \mathbf{D} , $\boldsymbol{\Xi}_j$, $\tilde{\mathbf{d}}$, and \mathbf{Z} as variables and by including all the equalities in (23), (24), (25), and (26) into the optimization problem in (22), similar to [18], the problem becomes

$$\min_{\mathbf{X}, \{\boldsymbol{\Xi}_j\}, \mathbf{D}, \mathbf{Z}, \tilde{\mathbf{d}}} \text{Trace}\left\{ \boldsymbol{\Sigma}^{-1} (\mathbf{t}^a (\mathbf{t}^a)^T - 2\mathbf{U}\tilde{\mathbf{d}} (\mathbf{t}^a)^T + \mathbf{U}\mathbf{D}\mathbf{U}^T) \right\} + \sum_{j=1}^M \left(\text{Trace}(\boldsymbol{\Phi}_j^{-1} \boldsymbol{\Xi}_j) - 2\mathbf{a}_j^T \boldsymbol{\Phi}_j^{-1} \mathbf{x}_j \right) \quad (27)$$

s.t. $\mathbf{D} = \tilde{\mathbf{d}}\tilde{\mathbf{d}}^T$, $\mathbf{Z} = \mathbf{X}^T \mathbf{X}$,

$$[\mathbf{D}]_{jj} = \frac{1}{c^2} [\mathbf{e}_j^T \quad -1] \mathbf{Z} \begin{bmatrix} \mathbf{e}_j \\ -1 \end{bmatrix},$$

$$\boldsymbol{\Xi}_j = \mathbf{x}_j \mathbf{x}_j^T, \quad \tilde{d}_j \geq 0, \quad j = 1, \dots, M,$$

(18), (19).

where we have added the obvious constraint that $\tilde{d}_j \geq 0$ for $j = \{1, \dots, M\}$ since $\tilde{d}_j = \|\mathbf{x} - \mathbf{x}_j\|/c$. Now the cost function of the problem in (27) is linear in terms of individual elements of matrices \mathbf{D} and $\boldsymbol{\Xi}_j$. The constraints in (25) are affine with respect to \mathbf{X} and \mathbf{Z} hence they are convex. However, the constraints in (23), (24), (26) are non-convex, which make the optimization problem in (27) non-convex. Relaxing the aforementioned non-convex constraints in (27) is done by replacing the equalities with linear matrix inequalities, hence we obtain

$$\min_{\mathbf{X}, \{\boldsymbol{\Xi}_j\}, \mathbf{D}, \mathbf{Z}, \tilde{\mathbf{d}}} \text{Trace}\left\{ \boldsymbol{\Sigma}^{-1} (\mathbf{t}^a (\mathbf{t}^a)^T - 2\mathbf{U}\tilde{\mathbf{d}} (\mathbf{t}^a)^T + \mathbf{U}\mathbf{D}\mathbf{U}^T) \right\} + \sum_{j=1}^M \left(\text{Trace}(\boldsymbol{\Phi}_j^{-1} \boldsymbol{\Xi}_j) - 2\mathbf{a}_j^T \boldsymbol{\Phi}_j^{-1} \mathbf{x}_j \right) \quad (28)$$

s.t. $\begin{bmatrix} \mathbf{D} & \tilde{\mathbf{d}} \\ \tilde{\mathbf{d}}^T & 1 \end{bmatrix} \succeq \mathbf{0}_{M+3}$, $\begin{bmatrix} \mathbf{I}_2 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Z} \end{bmatrix} \succeq \mathbf{0}_{M+3}$,

$$[\mathbf{D}]_{jj} = \frac{1}{c^2} [\mathbf{e}_j^T \quad -1] \mathbf{Z} \begin{bmatrix} \mathbf{e}_j \\ -1 \end{bmatrix},$$

$$\begin{bmatrix} \boldsymbol{\Xi}_j & \mathbf{x}_j \\ \mathbf{x}_j^T & 1 \end{bmatrix} \succeq \mathbf{0}_3, \quad \tilde{d}_j \geq 0, \quad j = 1, \dots, M,$$

(18), (19).

which is an SDP problem and can be solved effectively with interior point methods [25]. Standard SDP solvers such as SeDuMi [26] can be employed to solve the SDP optimization problem in (28) in MATLAB.

V. SIMULATION RESULTS

In this section, computer simulations are conducted to evaluate the performance of the proposed SDP technique in NLOS environments. We consider a network with 8 anchor nodes and with a size of 100 m \times 100 m. Full connectivity is assumed, hence the sensor is connected to all anchor nodes. The sensor is randomly placed in the area. The value of

TABLE I
A SUMMARY OF THE CONSIDERED TECHNIQUES.

Estimator	Description
ML-LOS	The ML estimator using only LOS links, i.e., based on (16)
SDPM	The proposed SDP technique in (28)
SDP	An SDP technique considered in [12]
ML	The ML estimator using all connections
LLS	A linear estimator in [27]

$\gamma_{\mathcal{L}}$ and $\gamma_{\mathcal{N}}$ are set to 2 and 3, respectively. The value of ξ is set to 0.02 and the noise variance σ_j^2 is assumed to be known based on (4). The NLOS biases are generated from an exponential distribution whose mean is uniformly distributed between 10 and 30 m. The clock offset and skew of the sensor are drawn from a uniform distribution $\mathcal{U}[0, 10^{-1}]$ μs and $\mathcal{U}[0.995, 1.005]$, respectively. The positioning error covariance matrix of the anchor nodes is set to be $\Phi_j = 25\mathbf{I}_2$. In each experiment, 5000 realizations are generated. In each realization, the measurement noises, sensor location, and clock parameters are randomized.

The ML estimate is obtained by the MATLAB routine `lsqnonlin`, which uses the Levenberg-Marquardt method. The solver of the ML estimator is initialized with the true values of the sensor location to provide the actual benchmark on the error. The proposed SDP technique is implemented with the CVX toolbox [28] using SeDuMi as a solver [26]. Besides the proposed SDP technique, labeled as SDPM where M stands for "modified", four other estimators are considered for comparisons. The ML estimator using only LOS links (labeled as ML-LOS) is included as a benchmark. We also include the performance of the ML estimator using all measurements where no NLOS mitigation technique is used (labeled as ML). The performances of a linear estimator in [27] and an SDP technique in [12], which do not directly do any NLOS mitigation, are also provided in the simulations.

We have considered two NLOS scenarios in our simulations: mild and severe. In the mild scenario, 25% of the connections are NLOS, while in the severe scenario 75% of the connections are NLOS. In the simulated network, each sensor is connected to 8 anchor nodes. For the mild scenario, 2 out of the 8 connections are randomly selected as NLOS and randomly generated biases are added to them.

Fig. 2 shows the cumulative distribution function (CDF) of the obtained localization error of the sensor position. The localization error is defined as the Euclidean distance between the true position and estimated position of the sensor, i.e., $\|\mathbf{x} - \hat{\mathbf{x}}\|$, where $\hat{\mathbf{x}}$ is the estimated position. Fig. 2a demonstrates that as expected the optimal performance is provided by the ML estimator using only LOS connections. It should be noted that the performance of ML-LOS is only plotted as a benchmark for comparison. Since in this work, we have assumed that the estimator does not know which connections are NLOS, achieving this performance is very optimistic, as it requires to identify the NLOS connections perfectly and initialize the ML solver appropriately. The linear

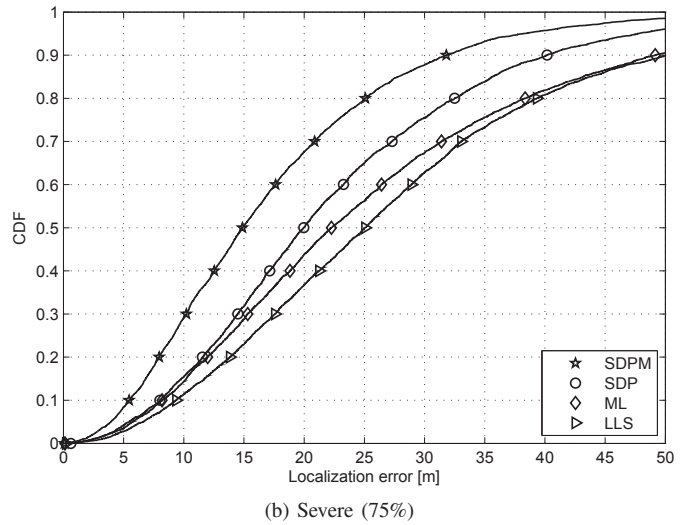
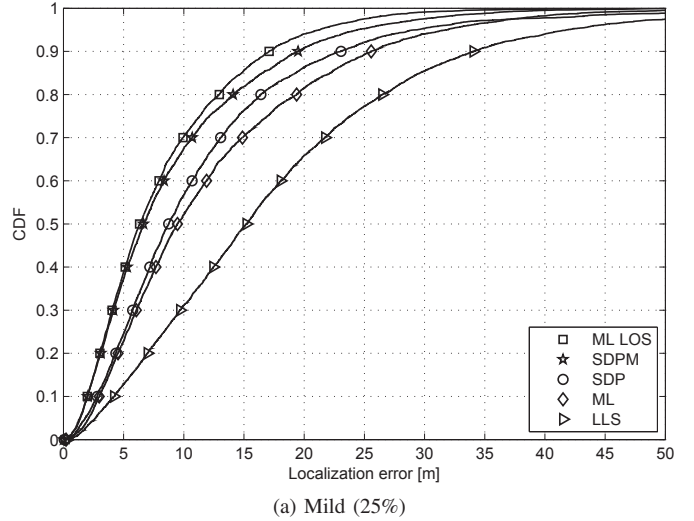


Fig. 2. The CDF of localization error in estimating the sensor position using the considered estimators, where 25% and 75% of the connections are NLOS in mild and severe scenarios, respectively. SDPM represents the performance of the proposed SDP technique.

estimator with no NLOS mitigation technique provides the worst performance. This is mainly due to the fact that NLOS connections are treated as LOS ones which can degrade the performance substantially, even in a mild NLOS environment where only one or two connections are NLOS. The ML estimator without NLOS mitigation also performs poorly. The SDP technique without NLOS mitigation performs better than LLS and ML, since SDP techniques are typically more robust against outliers and large measurement errors. The technique proposed in this paper, SDPM outperforms the other estimators and its performance is very close to that of ML-LOS. Note that unlike ML-LOS, the SDPM does not know which connections are NLOS, yet provides excellent performance.

Fig. 2b shows the performance of the considered estimators in a severe NLOS environment. In this case, 6 out of 8 connections are NLOS and the sensor has only 2 LOS connections. ML-LOS is not included in this simulation, since the sensor

TABLE II
THE LOCALIZATION AND SYNCHRONIZATION RMSE OF THE CONSIDERED
TECHNIQUES IN THE MILD NLOS SCENARIO.

Estimator	Location [m]	Clock Skew	Clock Offset [μ s]
ML-LOS	10.48	0.11	5.10
SDPM	10.71	0.12	5.21
SDP	12.07	0.14	5.88
ML	13.10	0.19	6.52
LLS	16.17	0.24	8.27

does not have access to a sufficient number of anchor nodes if NLOS connections are discarded. The sensor needs LOS connections to at least 4 anchor nodes to be localizable, hence ML-LOS is intractable in this case. The performances of all other estimators degrade in a severe NLOS environment, since the majority of connections are NLOS. However, Fig. 2b shows that the proposed estimator, SDPM still outperforms the other estimators significantly in a severe NLOS environment. More specifically, the proposed technique, SDPM with a NLOS mitigation ability can improve the localization error by 30% at 80% CDF in comparison with the SDP technique without NLOS mitigation.

In Table II, we compare different techniques in terms of the root-mean-square error (RMSE) performance. The RMSEs of location, clock skew, and clock offset estimates for the mild NLOS scenario are provided. Table II shows the close relationship between the localization and synchronization accuracy. The estimator with more precise synchronization provides a higher localization accuracy as well.

VI. CONCLUSION AND FUTURE WORK

In this paper, we have developed a technique for joint localization and synchronization for WSNs which mitigates the impacts of anchor position error and NLOS propagation. A novel SDP relaxation technique with an ability to mitigate NLOS propagation was developed such that the cost function becomes convex and easy to solve by standard convex optimization techniques. The simulation results show that the performance of the proposed technique is close to the optimal performance and is robust against NLOS errors. Extension of the proposed technique to a cooperative WSN and further validation of the results through experimental analysis would be considered in the future work.

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REFERENCES

- [1] R. Zekavat and R. M. Buehrer, *Handbook of Position Location: Theory, Practice and Advances*. Wiley-IEEE Press, 2011.
- [2] A. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Process. Magazine*, vol. 22, no. 4, pp. 24–40, Jul. 2005.
- [3] S. Gezici, Z. Tian, G. Giannakis, H. Kobayashi, A. Molisch, H. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios: a look at positioning aspects for future sensor networks," *IEEE Signal Process. Mag.*, vol. 22, pp. 70–84, Jul. 2005.

- [4] Y.-C. Wu, Q. Chaudhari, and E. Serpedin, "Clock synchronization of wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 28, no. 1, pp. 124–138, Jan. 2011.
- [5] K. Yu, I. Sharp, and G. Y. Jay, *Ground-Based Wireless Positioning*. John Wiley and Sons Ltd., 2009.
- [6] M. Sun and L. Yang, "On the joint time synchronization and source localization using TOA measurements," *Int. J. Distrib. Sens. N.*, pp. 1–11, Jan. 2013.
- [7] S. Yousefi, X. Chang, and B. Champagne, "A joint localization and synchronization technique using TOA at multiple antenna receivers," in *Proc. IEEE ASILOMAR*, Nov. 2013, pp. 2017–2021.
- [8] S. Zhu and Z. Ding, "Joint synchronization and localization using TOAs: A linearization based WLS solution," *IEEE J. Sel. Areas Commun.*, vol. 28, pp. 1017–1025, Sep. 2010.
- [9] R. M. Vaghefi and R. M. Buehrer, "A linear estimator for joint synchronization and localization in wireless sensor networks," in *Proc. IEEE GLOBECOM*, Dec. 2014.
- [10] A. M.-C. So and Y. Ye, "Theory of semidefinite programming for sensor network localization," *Math. Program., Series B*, vol. 109, no. 2–3, pp. 367–384, Oct. 2007.
- [11] P. Tseng, "Second-order cone programming relaxation of sensor network localization," *SIAM J. on Optimization*, vol. 18, no. 1, pp. 156–185, Feb. 2007.
- [12] R. M. Vaghefi and R. M. Buehrer, "Joint TOA-based sensor synchronization and localization using semidefinite programming," in *Proc. IEEE ICC*, Jun. 2014, pp. 520–525.
- [13] I. Guvenc and C.-C. Chong, "A survey on TOA based wireless localization and NLOS mitigation techniques," *IEEE Commun. Surveys Tuts.*, vol. 11, no. 3, pp. 107–124, Aug. 2009.
- [14] S. Yousefi, X. Chang, and B. Champagne, "Distributed cooperative localization without NLOS identification," in *Proc. IEEE WPNC*, Mar. 2014, pp. 1–6.
- [15] R. M. Vaghefi, J. Schloemann, and R. M. Buehrer, "NLOS mitigation in TOA-based localization using semidefinite programming," in *Proc. IEEE WPNC*, Mar. 2013, pp. 1–6.
- [16] Y. Qi, H. Kobayashi, and H. Suda, "Analysis of wireless geolocation in a non-line-of-sight environment," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 672–681, Mar. 2006.
- [17] A. Quazi, "An overview on the time delay estimate in active and passive systems for target localization," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 29, no. 3, pp. 527–533, Jun. 1981.
- [18] R. M. Vaghefi and R. M. Buehrer, "Asynchronous time-of-arrival-based source localization," in *Proc. IEEE ICASSP*, May 2013, pp. 4086–4090.
- [19] S. Venkatesh and R. Buehrer, "Non-line-of-sight identification in ultra-wideband systems based on received signal statistics," *IET Microwaves, Antennas Propagation*, vol. 1, no. 6, pp. 1120–1130, Dec. 2007.
- [20] A. Molisch, "Ultrawideband propagation channels-theory, measurement, and modeling," *IEEE Trans. on Vehicular Tech.*, vol. 54, no. 5, pp. 1528–1545, Sep. 2005.
- [21] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ: Prentice-Hall, 1993.
- [22] R. M. Vaghefi and R. M. Buehrer, "Cooperative sensor localization with NLOS mitigation using semidefinite programming," in *Proc. IEEE WPNC*, Mar. 2012, pp. 13–18.
- [23] S. Yousefi, X. Chang, and B. Champagne, "Mobile localization in non-line-of-sight using constrained square-root unscented Kalman filter," *IEEE Trans. on Vehicular Tech.*, vol. 55, no. 4, pp. 1291–1301, Jul. 2014.
- [24] K. W.-K. Lui, W.-K. Ma, H.-C. So, and F. K. Chan, "Semi-definite programming algorithms for sensor network node localization with uncertainties in anchor positions and/or propagation speed," *IEEE Trans. on Signal Process.*, vol. 57, no. 2, pp. 752–763, 2009.
- [25] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Rev.*, vol. 38, no. 1, pp. 49–95, Mar. 1996.
- [26] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optim. Method Softw.*, vol. 11, no. 1–4, pp. 625–653, Mar. 1999.
- [27] J. Zheng and Y.-C. Wu, "Joint time synchronization and localization of an unknown node in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1309–1320, Mar. 2010.
- [28] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," <http://cvxr.com/cvx>, May 2010.