Diffusion LMS Algorithms for Sensor Networks over Non-ideal Inter-sensor Wireless Channels

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***Abstract*—In this paper, we propose diffusion-based least mean square (LMS) algorithms that are robust against fading phenomena in wireless channels. The proposed algorithms, developed by combining diffusion LMS and classical estimation approaches, are able to estimate and update the underlying system parameters at each node by exploiting the sensor measurements and the fused data obtained from the neighboring nodes. The fusion of the information at each node takes place based on a convex combination strategy whose coefficients are determined according to the channel state information, the noise statistics and the output error of the local adaptive filter. In this work, we assume the broadcast data from the sensors experience Rayleigh fading and are further contaminated by the additive noise. Numerical results demonstrate the efficiency of the proposed algorithms and show their satisfactory performance compared with the costly centralized adaptive techniques.**

***Index Terms*—Distributed adaptive algorithms; wireless sensor networks; diffusion cooperative strategy; non-ideal inter-sensor channels.**

I. Introduction

Modern wireless sensor networks (WSN), aimed to monitor various physical phenomena over a given geographical area, have to operate over a wide range of time-varying conditions, due to e.g. fading channels, deviations in network energy profile, changes in network topology, and other factors that exhibit random behavior over time and space. From a signal processing perspective, there is consequently a strong need to develop robust *distributed* and *adaptive* parameter estimation algorithms that can perform efficiently under such time-varying conditions. Moreover, these algorithms must exhibit low computational complexity and small data rates to fulfil the network survivability and energy requirements.

Work in distributed adaptive algorithms traces back to distributed computation and optimization which are mature research topics in computer science. In optimization theory, distributed iterative algorithms based on incremental gradient schemes have been proposed to solve distributed least square problems [1]. Later, distributed iterative algorithms have been investigated for in-network processing to reduce the overall communication bandwidth and energy requirements of sensor networks [2]. Recently, *incremental* adaptive algorithms based on LMS and recursive least square (RLS) have been reported [3]–[5] to further minimize the use of network resources. In these algorithms, the exchange of information between sensors (also called nodes here) is achieved through a Hamiltonian cycle; while they function well for low-energy profile networks, setting such a cycle may not be trivial for a large size network topology.

In [6] and [7], *diffusion* based adaptive algorithms have been introduced to improve the network scalability. Since then, different versions and improvements of these algorithms have been proposed, among them are the diffusion Kalman filtering [8] and diffusion adaptive algorithms with adaptive combiner [9]. The latter improves the overall performance of diffusion-based adaptive algorithms by upgrading the local node combiner to an adaptive one.

In a diffusion strategy, each node communicates with its immediate neighbors in its range (i.e, reachable via single hop link), thereby avoiding the need to establish a Hamiltonian loop in the network. However, these algorithms require higher communication rate than the incremental-based algorithms. To rectify this problem, the authors in [10] have proposed distributed consensus LMS and RLS algorithms that have a lower communication rate as compared with the approaches in [6] and [7]. In these algorithms, the exchange of information between sensors follows a hierarchical communication structure that substantially decreases the overall communication rate of the network.

In spite of the significant advances offered by distributed adaptive algorithms above, they may function improperly in real-world applications due to neglecting the effect of wireless channels impairments in the process. To address this concern, the authors in [10], [11] have incorporated additive Gaussian noise in the underlying model of the inter-sensor communication which lead to improved distributed adaptive algorithms under such conditions. Yet, in this work, important impairments of wireless channels, such as multipath fading and phase distortion are not taken into consideration.

In this work, we present new diffusion-based LMS algorithms that can operate under the detrimental effects of both fading and noise in inter-sensor communication links. The proposed algorithms are obtained as a combination of diffusion LMS and linear estimation approaches, including the best linear unbiased estimation (BLUE). The developed distributed adaptive algorithms estimate and track the underlying system

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parameters at each node by exploiting the sensor current measurement and the available data obtained from the neighboring nodes. These data are fused according to a convex combination whose coefficients depend on the channel state information, the noise statistics and the output error of the local adaptive filter. Numerical experiments confirm the efficiency of the proposed algorithms and show their satisfactory performance compared with their centralized adaptive counterparts.

The paper is organized as follows: In Section II, we present the system model and problem formulation. The detail descriptions and derivations of the proposed algorithms are given in Section III. In Section IV, we present the numerical results to support the proposed idea. This is followed by a brief conclusion in Section V.

II. System model and problem formulation

We consider a network of N sensors, randomly distributed over a geographic region to monitor a physical phenomenon characterized by a parameter vector $\mathbf{w} \in \mathbb{C}^M$. The network aims to estimate this vector by processing the set of sensor measurements obtained over space and time, by means of a distributed adaptive LMS algorithm. We assume a diffusion strategy, in which each node exchange information via wireless communication with all immediate neighbors in its range, i.e. reachable via a single hop link. The parameter estimation model for this type of in-network adaptive scheme can be expressed by the following set of equations:

$$d_k(i) = \mathbf{w}^H \mathbf{u}_k(i) + v_k(i) \tag{1}$$

$$\mathbf{r}_{k,l}(i) = h_{k,l}(i)\boldsymbol{\psi}_l(i-1) + \mathbf{n}_{k,l}(i), \quad l \in \mathcal{N}_k \tag{2}$$

$$\boldsymbol{\psi}_k(i) = f(\{\mathbf{r}_{k,l}(i)\}_{l\in\mathcal{N}_k}, d_k(i)), \tag{3}$$

where the superscript H represents the conjugate transpose operation, $k \in \{1, 2, \ldots, N\}$ is the sensor index, and $i \in \mathbb{N}$ indicates the discrete-time index of the adaptation cycle. Equation (1) is a descriptive model of the physical phenomenon under measurement. Specifically, it shows the relationship between the reference signal at the k^{th} sensor, denoted by $d_k(i)$, the system parameter vector $\mathbf{w}$, the local regressor vector $\mathbf{u}_k(i) \in \mathbb{C}^M$ and the measurement noise $v_k(i)$. For instance, $\mathbf{w}$ may represent the coefficients of discretized partial differential equations (PDE) that describes the physical phenomenon under consideration [12]. The measurement noise, $v_k(i)$ is a zero mean Gaussian process and uncorrelated across both space and time domains, i.e, $E[v_k(i)v_m(j)] = \sigma^2_{v(k)}\delta_{ij}\delta_{km}$, where $\sigma^2_{v(k)}$ denotes the noise variance.

Equation (2) characterizes the inter-sensor communications, incorporating the effect of noise and fading in the wireless links. In this equation, $\boldsymbol{\psi}_l(i)$ denotes the local estimate of $\mathbf{w}$, as maintained by sensor l at time i, while $\mathbf{r}_{k,l}(i)$ denotes the received data vector at the k^{th} sensor from the l^{th} sensor through the wireless diffusion process, where $l \in \mathcal{N}_k$ and $\mathcal{N}_k = \{\nu_{k,1}, \nu_{k,2}, \ldots \nu_{k,L_k}\}$ is the set of neighboring sensors in the range of node k, and $L_k = |\mathcal{N}_k|$ is the number of neighbors. The diffusion process can be sequential and governed by a time division multiplexing access (TDMA) scheme. The implementation aspects and time scheduling of sequential diffusion process are important issues in WSN and need extensive discussions that however fall beyond the scope of this article. According to (2), $\mathbf{r}_{k,l}(i)$ is the distorted version of the estimated system parameter $\boldsymbol{\psi}_l(i-1)$ corrupted by the Rayleigh fading channel, which is represented here by the time-varying complex gain $h_{k,l}(i)$, and additive white noise $\mathbf{n}_{k,l}(i) \in \mathbb{C}^{M\times 1}$. The noise sequence $\mathbf{n}_{k,l}(i)$ is a zero-mean additive white Gaussian (AWGN) process, uncorrelated across the sensors and time with diagonal covariance matrix $E[\mathbf{n}(k,l)\mathbf{n}(k,l)^H] = \sigma^2_{n(k,l)}\mathbf{I}_M$, where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix.

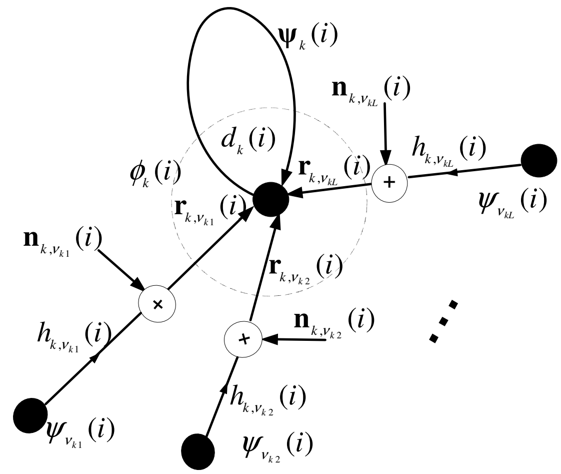

Fig. 1. Data communication at node k ($L \equiv L_k$)

Equation (3) is the update equation of the k^{th} sensor in which the local estimated system parameter, $\boldsymbol{\psi}_k(i)$, is updated by exploiting its own current local measurement, $d_k(i)$, and the received data from the neighboring sensors, i.e. $\mathbf{r}_{k,l}(i)$, $l \in \mathcal{N}_k$. The signal processing task and data communication model for node k, with $L \equiv L_k$ active neighbors is illustrated in Fig. 1.

The update equation (3) of node k can be designed based on different constraints and criteria, such as mode of cooperation between nodes, adaptive filtering scheme and wireless channel conditions. Assuming a diffusion strategy, LMS adaptive algorithm and an ideal wireless channel (i.e. $h_{k,l} = 1, n_{k,l}(i) = 0$) then (3) can be realized via the following two equations, which form the core of diffusion LMS algorithms developed in [6]:

$$\boldsymbol{\phi}_k(i) = \sum_{l\in\mathcal{N}_k} c_{k,l}(i)\boldsymbol{\psi}_l(i-1) \tag{4}$$

$$\boldsymbol{\psi}_k(i) = \boldsymbol{\phi}_k(i) + \mu\mathbf{u}^*_k(i)[d_k(i) - \boldsymbol{\phi}^T_k(i)\mathbf{u}_k(i)] \tag{5}$$

In (4), $c_{k,l}(i)$ denotes the $(k,l)^{th}$ entry of the network adjacency matrix $C(i) \in \mathbb{C}^{N\times N}$ in which $c_{k,l}(i) = c_{l,k}(i) \neq 0$ when $l \in \mathcal{N}_k$ and $c_{l,k}(i) = 0$ otherwise. Technically, $c_{k,l}(i)$ can be interpreted as the level of participation of node $l \in \mathcal{N}_k$ in the update of the system parameter at node k.

The diffusion algorithm, described by (4)-(5), may function improperly and diverge when used in a real-world wireless

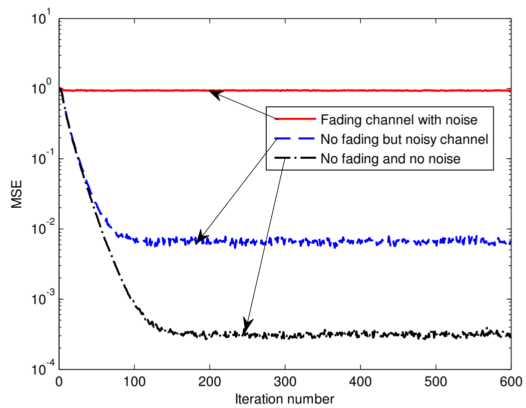

Fig. 2. Performance of conventional diffusion algorithms in wireless channels

environment. In practice, the undistorted value of the estimated system parameter vector, represented by $\boldsymbol{\psi}_l(i)$ in (4), is unavailable at node k; instead, only the distorted version of this estimated system parameter, i.e. $\mathbf{r}_{k,l}(i)$ as per (2), is available.

Considering a simplified scenario where the source of distortion in (2) is only additive white Gaussian noise (i.e, no fading), the authors in [10] and [11] proposed diffusion-based LMS and RLS algorithms, aimed to suppress the effect of the link noise, $\mathbf{n}_{k,l}(i)$, from the received estimated system parameter $\mathbf{r}_{k,l}(i)$. These algorithms which were formulated as constrained distributed LMS and RLS problems, demand higher computational power than the unconstrained ones, since the Lagrange multipliers associated with each node need to be updated at every time iteration. In addition, despite their robustness to the noisy channel, they may diverge in real-world operation. This is because the complex gain of the wireless channel, $h_{k,l}(i)$, can mislead the search direction of the algorithm in reaching the optimal point. Fig. 2 supports our claim, where the solid red curve implies the divergence of diffusion-based adaptive algorithm under such conditions. In this work, we address this problem and develop two types of diffusion LMS algorithms that can be used in both AWGN and Rayleigh fading channels.

III. THE PROPOSED ALGORITHMS

Development of the proposed algorithms follows a four-step approach, corresponding to the main processing tasks of the algorithms at each node, namely, channel equalization, local estimation, data fusion and adaptive iteration.

As shown in Fig. 3, to retrieve the distorted estimated system parameters received through wireless channel, the first step is to remove the effect of the channel. This is simply done by multiplying the received data from the l^{th} sensor by the complex conjugate of the corresponding channel gain, i.e., $h^*_{k,l}(i)$, which amounts to a matched filtering operation. In the second step, the obtained filtered values from the first step are supplied to a linear estimator whose output, $\bar{\boldsymbol{\phi}}_k(i)$, is produced based on the available information about the statistical property of the channel and link noise. The third step is data fusion, where a convex combination of the linear

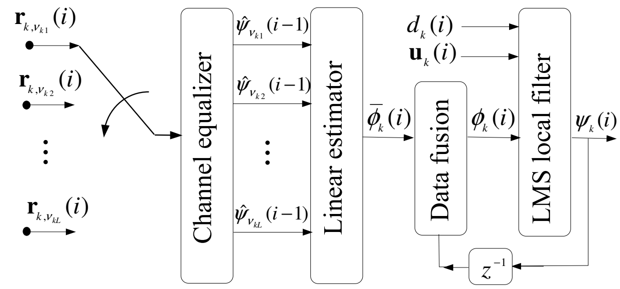

Fig. 3. Parameter estimation and tracking at node k

estimator's output ,$\bar{\boldsymbol{\phi}}_k(i)$, and the previous estimated system parameter, i.e. $\boldsymbol{\psi}_k(i-1)$, generates a refined estimate, $\boldsymbol{\phi}_k(i)$, to be used in the LMS update equation at node k. The convex coefficients are computed according to the variances of the measurement noise and other system parameters as explained below. The last step is to update or track the system parameter vector using a local LMS filter based on the refined estimate $\boldsymbol{\phi}_k(i)$, the regressor $\mathbf{u}_k(i)$ and the local reference $d_k(i)$. In the following subsections, we present the details of above four-step approach for two common wireless channel scenarios, i.e. AWGN and Rayleigh fading, leading to new forms of diffusion-based LMS algorithms.

A. Diffusion LMS algorithm over AWGN Channel

In this scenario, the received data by node k from the neighboring sensors can be expressed as

$$\mathbf{r}_{k,l}(i) = \boldsymbol{\psi}_l(i-1) + \mathbf{n}_{k,l}(i), \quad l \in \mathcal{N}_k. \tag{6}$$

Since the channel is assumed ideal, i.e, $h_{k,l}(i) = 1$, the first step of the proposed four-step approach, i.e. channel equalization, is unnecessary and we therefore proceed to the second step. Depending on the extent of the prior knowledge about the statistics of the noise in (6), the linear estimator at node k generates an estimate of the transmitted system parameters, i.e.:

$$\bar{\boldsymbol{\phi}}_k(i) = \Gamma_k(r_{k,l}(i) : l \in \mathcal{N}_k), \tag{7}$$

where $\Gamma_k()$ denotes the estimation operator. In this work, we assume that Γ_k is the BLUE estimator [13], but other choices are possible. In this case, we obtain

$$\bar{\boldsymbol{\phi}}_k(i) = \frac{\sum_{l \in \mathcal{N}_k} \mathbf{r}_{k,l}(i)\sigma^{-2}_{n(k,l)}}{\sum_{l \in \mathcal{N}_k} \sigma^{-2}_{n(k,l)}}, \tag{8}$$

Under the assumption that the estimated system parameter vectors at node l are error free, i.e. $\boldsymbol{\psi}_l(i) = \mathbf{w}$, the mean-square error of this estimator, computed as $E\|\mathbf{w} - \bar{\boldsymbol{\phi}}_k(i)\|^2 = (\sum_{l=1}^{\mathcal{N}_k} \sigma_l{}^{-2})^{-1}$, is minimum among all linear unbiased estimator. The third step, i.e. data fusion, is to express $\boldsymbol{\phi}_k(i)$ as a convex combination of $\bar{\boldsymbol{\phi}}_k(i)$ and $\boldsymbol{\psi}_k(i-1)$, as given by:

$$\boldsymbol{\phi}_k(i) = \beta_k \boldsymbol{\psi}_k(i-1) + (1-\beta_k)\bar{\boldsymbol{\phi}}_k(i) \tag{9}$$

where $\beta_k \in [0,1]$, can be computed based on the variances of the measurement noise at node k and its neighbors via

$$\beta_k = \frac{\sigma_{v(k)}^{-2}}{\sigma_{v(k)}^{-2} + \sigma_{v(\mathcal{N}_k)}^{-2}}, \tag{10}$$

where, $\sigma_{v(\mathcal{N}_k)}^2$, the average of the measurement noise variance, is given by

$$\sigma_{v(\mathcal{N}_k)}^2 = \frac{1}{L_k} \sum_{l \in \mathcal{N}_k} \sigma_{v(l)}^2. \tag{11}$$

In the last step, the local estimate of the system parameter vector at node k, i.e., $\boldsymbol{\psi}_k(i)$, is updated by substituting the value of the refined estimate $\boldsymbol{\phi}_k(i)$ from (9) into the distributed LMS update equation given in (5). This estimation process, describing the signal processing tasks performed over the network within one adaptation cycle, is summarized in Algorithm 1.

Algorithm 1 Diffusion LMS algorithm for AWGN channel

for $k = 1 : N$ **do**
  $\{$Diffusion process$\}$
  for $l \in \mathcal{N}_k$ **do**
    $\mathbf{r}_{k,l}(i) = \boldsymbol{\psi}_l(i-1) + \mathbf{n}_{k,l}(i)$
  end for
  $\{$Local parameters update$\}$
  $\bar{\boldsymbol{\phi}}_k(i) = \frac{\sum_{l\in\mathcal{N}_k} \mathbf{r}_{k,l}(i)\sigma_{n(k,l)}^{-2}}{\sum_{l\in\mathcal{N}_k} \sigma_{n(k,l)}^{-2}}$
  $\sigma_{v(\mathcal{N}_k)}^2 = \frac{1}{L_k}\sum_{l\in\mathcal{N}_k} \sigma_{v(l)}^2$
  $\beta_k = \frac{\sigma_{v(k)}^{-2}}{\sigma_{v(k)}^{-2}+\sigma_{v(\mathcal{N}_k)}^{-2}}$
  $\boldsymbol{\phi}_k(i) = \beta_k \boldsymbol{\psi}_k(i-1) + (1-\beta_k)\bar{\boldsymbol{\phi}}_k(i)$
  $e_k(i) = d_k(i) - \boldsymbol{\phi}_k^T(i)\mathbf{u}_k(i)$
  $\boldsymbol{\psi}_k(i) = \boldsymbol{\phi}_k(i) + \mu \mathbf{u}_k^*(i) e_k(i)$
end for

B. Diffusion LMS algorithm over Rayleigh fading channel

We extend the above approach by further considering the fading effects of the inter-sensor wireless channels. In particular, we develop two different diffusion-based LMS algorithms by following the aforementioned four-step approach. The two algorithms differ with respect to the assumed available *a-priori* knowledge of the measurement noise power. When the inter-sensor wireless channels undergo Rayleigh fading, the received data vectors $\mathbf{r}_{k,l}(i)$ at node k are expressed by (2). In this equation, the channel gains $h_{k,l}(i)$, which have zero mean and variance $\sigma_{h(k,l)}^2$, can be constant or slowly time-varying during the adaptation process. In network modeling, the gains can be interpreted as random weights applied to the corresponding entries of the network adjacency matrix, i.e. $c_{k,l}(i)$.

In the first step, if we assume that the channel gains of neighboring nodes, i.e. $h_{k,l}(i)$ for $l \in \mathcal{N}$, are known, their effects can be removed by the following operation:

$$\hat{\boldsymbol{\psi}}_l(i-1) = h_{k,l}^*(i)\mathbf{r}_{k,l}(i)/|h_{k,l}(i)|^2, \tag{12}$$

which results in

$$\hat{\boldsymbol{\psi}}_l(i-1) = \boldsymbol{\psi}_l(i-1) + \frac{h_{k,l}^*(i)}{|h_{k,l}(i)|^2}\mathbf{n}_{k,l}(i). \tag{13}$$

Equation (13) has a similar form to that of the received data in the AWGN channel, i.e (6), except that the noise power is now changed to $\sigma_{n(k,l)}^2/\sigma_{h(k,l)}^2$. As a result, in the second step, by applying the BLUE estimator, we obtain

$$\bar{\boldsymbol{\phi}}_k(i) = \frac{\sum_{l\in\mathcal{N}_k} \hat{\boldsymbol{\psi}}_l(i-1)\sigma_{h(k,l)}^2\sigma_{n(k,l)}^{-2}}{\sum_{l\in\mathcal{N}_k} \sigma_{h(k,l)}^2\sigma_{n(k,l)}^{-2}}. \tag{14}$$

The third and fourth steps of the procedure, respectively data fusion and updating the local estimate of the system parameter vector $\boldsymbol{\psi}_k(i)$, is identical to that in the AWGN channel case. The resulting procedure for this scenario with Rayleigh fading is summarized in Algorithm 2, and identified as Type I.

Algorithm 2 Diffusion LMS for Rayleigh channel Type I

for $k = 1 : N$ **do**
  $\{$Diffusion process$\}$
  for $l \in \mathcal{N}_k$ **do**
    $\mathbf{r}_{k,l}(i) = h_{k,l}(i)\boldsymbol{\psi}_l(i-1) + \mathbf{n}_{k,l}(i)$
    $\hat{\boldsymbol{\psi}}_l(i-1) = h_{k,l}^*(i)\mathbf{r}_{k,l}(i)/|h_{k,l}(i)|^2$
  end for
  $\{$Local parameters update$\}$
  $\bar{\boldsymbol{\phi}}_k(i) = \frac{\sum_{l\in\mathcal{N}_k} \hat{\boldsymbol{\psi}}(i-1)\sigma_{h(k,l)}^2\sigma_{n(k,l)}^{-2}}{\sum_{l\in\mathcal{N}_k} \sigma_{h(k,l)}^2\sigma_{n(k,l)}^{-2}}$
  $\sigma_{v(\mathcal{N}_k)}^2 = \frac{1}{L_k}\sum_{l\in\mathcal{N}_k} \sigma_{v(l)}^2$
  $\beta_k = \frac{\sigma_{v(k)}^{-2}}{\sigma_{v(k)}^{-2}+\sigma_{v(\mathcal{N}_k)}^{-2}}$
  $\boldsymbol{\phi}_k(i) = \beta_k \boldsymbol{\psi}_k(i-1) + (1-\beta_k)\bar{\boldsymbol{\phi}}_k(i)$
  $e_k(i) = d_k(i) - \boldsymbol{\phi}_k^T(i)\mathbf{u}_k(i)$
  $\boldsymbol{\psi}_k(i) = \boldsymbol{\phi}_k(i) + \mu \mathbf{u}_k^*(i) e_k(i)$
end for

Algorithm 3 Diffusion LMS for Rayleigh channel Type II

for $k = 1 : N$ **do**
  $\{$Diffusion process$\}$
  for $l \in \mathcal{N}_k$ **do**
    $\mathbf{r}_{k,l}(i) = h_{k,l}(i)\boldsymbol{\psi}_l(i-1) + \mathbf{n}_{k,l}(i)$
    $\hat{\boldsymbol{\psi}}_l(i-1) = h_{k,l}^*(i)\mathbf{r}_{k,l}(i)/|h_{k,l}(i)|^2$
  end for
  $\{$Local parameters update$\}$
  $\bar{\boldsymbol{\phi}}_k(i) = \frac{\sum_{l\in\mathcal{N}_k} \hat{\boldsymbol{\psi}}_l(i-1)\sigma_{h(k,l)}^2\sigma_{n(k,l)}^{-2}}{\sum_{l\in\mathcal{N}_k} \sigma_{h(k,l)}^2\sigma_{n(k,l)}^{-2}}$
  $\beta_k(i) = \frac{1}{(1+|\exp(-\alpha_k(i-1))|^2)}$
  $\boldsymbol{\phi}_k(i) = \beta_k(i)\boldsymbol{\psi}_k(i-1) + (1-\beta_k(i))\bar{\boldsymbol{\phi}}_k(i)$
  $e_k(i) = d_k(i) - \boldsymbol{\phi}_k^T(i)\mathbf{u}_k(i)$
  $\boldsymbol{\psi}_k(i) = \boldsymbol{\phi}_k(i) + \mu \mathbf{u}_k^*(i) e_k(i)$
  $\gamma_k(i) = [\boldsymbol{\psi}_k(i) - \boldsymbol{\phi}_k(i)]\, e_k(i)\beta_k(i)(1-\beta_k(i))$
  $\alpha_k(i) = \alpha_k(i-1) + \frac{\mu_\alpha \gamma_k^T(i)\mathbf{u_k(i)}}{\|\mathbf{u}_k(i)\|^4}$
end for

In certain applications, the network may not have access to measurement noise variances of the sensors. In this scenario, we can still implement the third step of the proposed approach, i.e. data fusion by using an *adaptive* convex combination of $\bar{\phi}_k(i)$ and $\psi_k(i-1)$. Indeed, these vectors can be viewed as the coefficients of two adaptive filters, aiming to estimate the same system parameters. Therefore, the required time-varying convex coefficient, $\beta_k(i)$, can be obtained based on the suggested technique in [14]. Making these modifications to Algorithm 2 (leaving the other steps of the procedure unchanged), results into Algorithm 3. This new algorithm, referred to as diffusion LMS Type II, does not require prior knowledge of the measurement noise variances; however in compared with Type I, it needs more computational power and demonstrates slightly lower performance efficiency.

C. Diffusion LMS algorithm with channel indicator

The performance of the proposed diffusion LMS algorithms can be further improved if we use a channel quality indicator at each node to limit the impact of "bad" channels on the overall performance of diffusion algorithms. Specifically, node k can monitor the magnitude of its inter-sensor channels at each time iteration, and discard data transmitted from neighboring nodes $l \in \mathcal{N}_k$ in the adaptive process whenever $|h_{k,l}(i)| < h_{\min}$, where $h_{\min}$ is a preset threshold. This way, we can avoid the noise enhancement phenomenon due to bad channels or deep fading conditions in the network. Numerical results (see below) show that by implementing a such channel indicator, the level of residual estimation error in the steady state can be reduced. Intuitively, excluding the processing of data from bad channels, reduces the overall computation load of the algorithm.

IV. Numerical Experiments

In this section, we evaluate the performance of the developed algorithms through numerical simulations. In our experiments, the measurement data $d_k(i)$ is generated according to the model given in (1) where the unknown system parameter vector $\mathbf{w} \in \mathbb{C}^M$ has $M = 4$ components. The entry of the regressor vector $\mathbf{u}_k(i) = [u_k(i), u_k(i-1), \ldots, u_k(i-M+1)]^T$, is generated by means of the following first order autoregressive (AR) process equation:

$$u_k(i) = \eta_k u_k(i-1) + \zeta_k z_k(i) \tag{15}$$

where $z_k(i)$ is a spatially independent white Gaussian process with zero-mean and unit variance. For given values of $\eta_k \in [0,1)$ and ζ_k, the regressor variance can be computed as $\sigma^2_{u,k} = \zeta_k^2/(1-\eta_k^2)$. For our experiments, a connected ad-hoc wireless sensor network with $N = 20$ nodes is generated as a realization of a random geometric graph on a unit square, with maximum normalized communication range of $r = 0.4$ as shown in Fig. 4. In this approach, nodes are deployed uniformly and randomly over $[0,1]^2$ in two dimensions; an edge joining two nodes is drawn whenever their Euclidean distance does not exceed r. To model the inter-sensor wireless links in the presence of fading, independent random variables $h_{k,l}(i)$

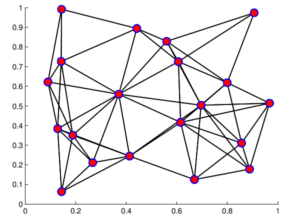

Fig. 4. Realization of the simulated network with $r = 0.4$

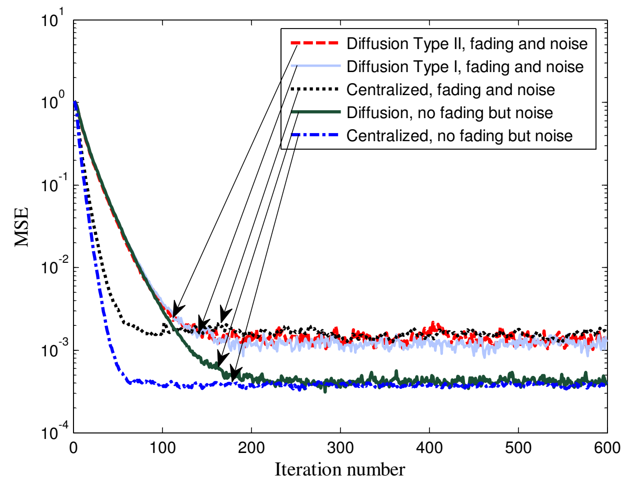

Fig. 5. MSE performance of diffusion and centralized LMS algorithms

with Rayleigh distribution and unit variance are generated, one for every non-zero entry of the network adjacency matrix. For simplicity, the values of the channel gains are kept fixed during the during the adaptation process. The variance of the zero mean link noise $\mathbf{n}_{k,l}(i)$ in (2), i.e. $\sigma^2_{n(k,l)}$, is calculated based on the channel SNRs, drawn from a uniform distribution within the range of $[15, 25]dB$.

In our experiments, the proposed algorithms are compared

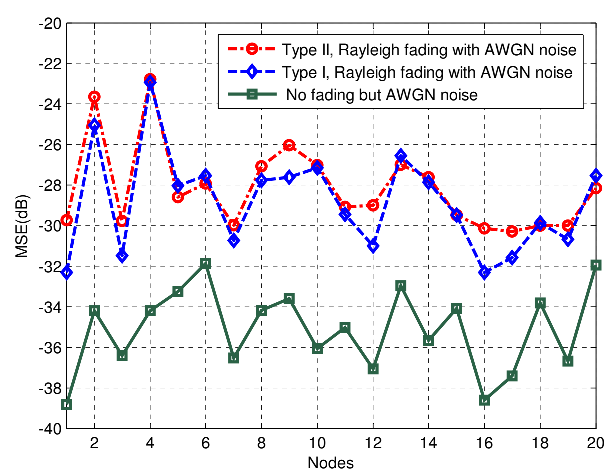

Fig. 6. MSE of proposed diffusion algorithms at each node in different scenarios

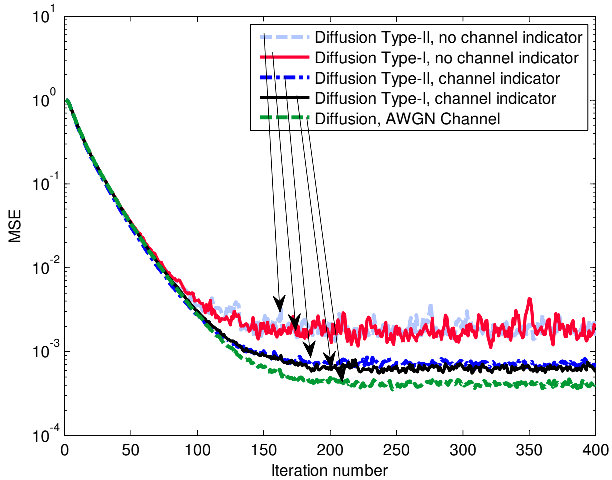

Fig. 7. MSE performance of diffusion LMS algorithms with channel indicator

to the centralized LMS algorithms, which is used as a benchmark in the evaluation. The latter refers to a distributed LMS algorithm that runs incrementally [4] at the central processor that has access to data from all over the network. The wireless channels from each node to the central processor are generated using the same mechanisms as for the distributed algorithms. The step size μ of distributed and central adaptive algorithms are constant and chosen as 0.05 and 0.008 respectively.

The MSE performance of the various algorithms are shown in Fig. 5, where the results are averaged over 200 independent runs. Following an initial period of rapid learning, the centralized LMS with AWGN channels achieves its steady-state level of residual error faster than the other algorithms. The centralized LMS with Rayleigh channels also converges rapidly, however the level of its residual estimation error significantly increases due to fading effect in wireless channels. It can be also observed that the proposed diffusion algorithms converge to nearly the same steady state level as the centralized LMS algorithm, although the speed of convergence is reduced. Indeed, the faster convergence of the centralized algorithm is because of the extensive processing of the data collected from all over network at each time i, while the volume of data used by the diffusion LMS at node k is restricted by the number of its neighbors, L_k. The results also indicate that diffusion LMS Type I reaches a slightly lower steady state error value than Type II. Similar conclusions can be made from Fig. 6, which shows the steady-state MSE performance of the diffusion LMS algorithms for the entire network. Again, the diffusion LMS algorithm in AWGN channels shows superior performance than diffusion LMS in Rayleigh channels.

Finally, Fig. 7 shows the results of implementing channel quality indicators on the performance of the developed algorithms. The results suggest, in this scenario, the algorithms can attain a significantly smaller level of residual error.

V. Conclusion

In this paper, we formulated new distributed LMS adaptive algorithms that can operate under the detrimental effects of channel fading and noise in wireless sensor networks. The proposed algorithms are diffusion-based and obtained as a combination of diffusion LMS and linear estimation approaches such as best linear unbiased estimation (BLUE). In particular, the developed algorithms estimate and track the unknown system parameters at each node, by processing the sensor current measurement, the received data from the neighboring nodes, and exploiting the channel state information and statistical data of their neighbors. These algorithms work based on diffusion strategy where nodes exchange their information with only a single-hop transmission step in their communication range. Simulation results demonstrate the effectiveness of the proposed algorithms, and show their satisfactory performance when compared with the costly centralized adaptive approaches.

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