Adaptive Blind Widely Linear CCM Reduced-Rank Beamforming for Large-Scale Antenna Arrays

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***Abstract*—In this paper, we propose an adaptive blind reduced-rank beamforming algorithm based on Krylov-subspace (KS) techniques and widely linear (WL) processing for non-circular signals. In contrast to the conventional WL processing approach, the properties of the augmented covariance matrix are exploited to derive a new structured WL beamforming scheme based on the generalized sidelobe canceler (GSC) structure. We develop a recursive least square (RLS) algorithm according to the constrained constant modulus (CCM) criterion to update the reduced-rank beamformer so obtained. A detailed signal-to-interference-plus noise ratio (SINR) analysis and a computational complexity analysis are carried out. Simulation results show that the proposed algorithm outperforms its linear counterpart and the full-rank algorithms, achieving the best convergence performance among all the analyzed methods with a relatively low complexity.**[1]

Keywords*—Widely linear, Krylov-subspace, beamforming, constrained constant modulus

I. Introduction

Adaptive beamforming techniques can discriminate signals with different spatial characteristics and have been widely applied in various areas such as radar, sonar, and wireless communication systems [1]–[3]. In many practical situations, the received vector $\boldsymbol{r}$ is assumed to be second-order circular with its complementary covariance matrix $\boldsymbol{R}_c = \mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^T\} = \boldsymbol{0}$, and for this reason, only the covariance matrix $\boldsymbol{R} = \mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^H\}$ is utilized in conventional schemes. However, when the received vector is derived from non-circular modulated signals, such in the case of binary phase shift keying (BPSK) modulation, $\boldsymbol{R}_c$ is no longer a zero matrix. Under such circumstances, a more general estimation scheme, which takes into consideration both the received vector $\boldsymbol{r}$ and its conjugate $\boldsymbol{r}^*$, is needed to obtain superior performance. Referred to as widely linear (WL) beamformer, this more general scheme can lead to higher signal-to-interference-plus noise (SINR) or smaller mean square error (MSE) in the estimation of a desired signal [4].

Blind algorithms without any requirements for training symbols can significantly improve the information capacity of communication systems employing antenna arrays. The most popular design criteria for adaptive blind beamformers are the constrained minimum variance (CMV) [5], [6] and the constrained constant modulus (CCM) [7] due to their effectiveness and simplicity. The CCM criterion, which minimizes the mean deviation of the squared output from constant values, exploits additional information about the underlying signal constellation, and achieves superior performance as compared with the CMV criterion.

However, in a large-scale antenna array system with numerous filter coefficients to be estimated, the high computational complexity and the slow convergence speed of adaptive blind beamforming algorithms often prohibit the application of full-rank processing. Moreover, WL processing doubles the size of the received data vector which further motivates the use of reduced-rank techniques [8]–[11]. Various WL-based reduced-rank algorithms have been introduced in the previous studies, including the eigen-decomposition method [12], the multi-stage Wiener filter (MSWF) [13], and the auxiliary vector filtering (AVF) [14]. Both the MSWF and AVF methods involve the construction of the low rank Krylov-subspace (KS), which has shown excellent performance in several applications and can be combined with different design criteria.

In this paper, a new adaptive blind reduced-rank WL beamforming algorithm based on the KS technique is proposed which operates in the generalized sidelobe canceller (GSC) structure [15], [16]. We develop a recursive least square (RLS) algorithm according to the CCM criterion to update the reduced-rank filter, which is referred to as the direct WLCCM-KS-RLS. In order to reduce the computational complexity of the conventional realization scheme based on the stacking of the received data and its complex conjugate, the structure of the augmented covariance matrix is taken into consideration as prior information to devise a structured WLCCM-KS-RLS. A theoretical analysis of the achievable SINR of the WLCCM-KS scheme and its linear counterpart is given. In addition, we investigate the computational complexity of the proposed algorithm and compare it with other existing reduced-rank algorithms. Simulation results show that the proposed algorithm outperforms its linear counterpart and the full-rank algorithms, achieving the best convergence performance and steady-state SINR among all the analyzed methods with a relatively low complexity.

II. System Model and Problem Statement

Let us consider that K independent user signals, indexed by $k = 0, 1, ..., K-1$, impinge on a large-scale uniform linear array (ULA) system equipped with $M(K \leq M)$ sensor elements. The users are assumed to be in the far field with directions-of-arrival (DOAs) $\theta_0, ..., \theta_{K-1}$. The received $M \times 1$ complex vector at the ith snapshot can be modeled as

$$\boldsymbol{r}(i) = \sum_{k=0}^{K-1} b_k(i)\boldsymbol{a}(\theta_k) + \boldsymbol{n}(i), \qquad i = 1, ..., N \tag{1}$$

where $b_k(i)$ denotes the source signal for user k which takes value from the set $\{\pm 1\}$ with equal probability. The vector $\boldsymbol{a}(\theta_k)$ denotes the normalized $M \times 1$ signal steering vector, which is given by

$$\boldsymbol{a}(\theta_k) = \frac{1}{\sqrt{M}}[1, e^{-j2\pi\frac{d\cos\theta_k}{\lambda_c}}, ..., e^{-j2\pi\frac{(M-1)d\cos\theta_k}{\lambda_c}}]^T, \tag{2}$$

where λ_c is the wavelength and $d = \frac{\lambda_c}{2}$ is the inter-element distance of the ULA. In (1), $\boldsymbol{n}(i) \in \mathbb{C}^{M\times 1}$ is the white circular complex noise

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vector whose components are independent and identically distributed random Gaussian variables with zero-mean and variance σ_n^2. Without loss of generality, we assume that source $k = 0$ is the desired user while the remaining $K-1$ sources are interfering users. Our interest is focused on detection problems where the large-scale antenna array is used to extract information from the desired user with a known normalized steering vector $\boldsymbol{a}(\theta_0)$.

We define $\boldsymbol{w}(i) = \gamma \boldsymbol{a}(\theta_0) - \boldsymbol{B}\boldsymbol{w}_{gsc}(i)$ as the weight vector of the full-rank beamformer with the GSC structure [15], where γ is a real-valued scalar to guarantee the convexity of the optimization problem [9], and $\boldsymbol{B}$ is the signal blocking matrix, which spans a subspace that is orthogonal to the steering vector $\boldsymbol{a}(\theta_0)$ [17]. Consequently, the CCM beamformer is converted into an unconstrained optimization problem with the following cost function:

$$J_{CM}(\boldsymbol{w}_{gsc}(i)) = \mathbb{E}\{(|y(i)|^2 - 1)^2\}, \tag{3}$$

where $y(i) = (\gamma \boldsymbol{a}(\theta_0) - \boldsymbol{B}\boldsymbol{w}_{gsc}(i))^H \boldsymbol{r}(i)$ is the output of the GSC beamformer, and $\boldsymbol{w}_{gsc}(i)$ is a filter to be designed.

However, for a large-scale antenna array system with large dimension M, the convergence speed for the full-rank blind adaptive beamformer is typically rather slow, and we resort to reduced-rank techniques to solve this problem.

III. Krylov-Subspace based reduced-rank scheme with the GSC structure

The reduced-rank beamforming receiver reduces the number of adaptive filter coefficients by projecting the received signal onto a lower dimensional subspace. An illustration of the reduced-rank scheme operating in the GSC structure is shown in Fig. 1. As can be seen, for the auxiliary (i.e., bottom) branch, the received vector $\boldsymbol{r}(i)$ is successively processed by the signal blocking matrix $\boldsymbol{B}$, the transformation matrix $\boldsymbol{T}_r$, and the reduced-rank filter $\bar{\boldsymbol{w}}_{gsc}(i)$ to compute the unconstrained output. The auxiliary branch is devised to recover the interference-plus-noise component which has passed through the top branch and then cancel it.

For the construction of the transformation matrix $\boldsymbol{T}_r$, we utilize the KS technique. The standard rank-D ($1 \le D \ll M$) KS can be represented by

$$K_D = \text{Span}\{\boldsymbol{a}(\theta_0), \boldsymbol{R}\boldsymbol{a}(\theta_0), ..., \boldsymbol{R}^{D-1}\boldsymbol{a}(\theta_0)\}, \tag{4}$$

where $\boldsymbol{R} = \mathbf{E}\{\boldsymbol{r}(i)\boldsymbol{r}^H(i)\}$ denotes the array covariance matrix [18]. The dth projection vector $\boldsymbol{R}^{d-1}\boldsymbol{a}(\theta_0)$ ($2 \le d \le D$), maximizes the magnitude of the correlation between its output $(\boldsymbol{R}^{d-1}\boldsymbol{a}(\theta_0))^H \boldsymbol{r}(i)$ and the output of the previous projection vector $(\boldsymbol{R}^{d-2}\boldsymbol{a}(\theta_0))^H \boldsymbol{r}(i)$. Considering the GSC structure, the product between the first basis vector and the blocked array signal is zero, that is $\boldsymbol{a}^H(\theta_0)\boldsymbol{B}^H\boldsymbol{r}(i) = 0$. Noting this, we define a modified rank-D transformation matrix that is well-suited to the GSC structure via the following expression

$$\begin{aligned} \boldsymbol{T}_r &= [\boldsymbol{R}\boldsymbol{a}(\theta_0), \boldsymbol{R}^2\boldsymbol{a}(\theta_0), ..., \boldsymbol{R}^D\boldsymbol{a}(\theta_0)] \\ &\doteq [\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, ..., \boldsymbol{\rho}_D], \end{aligned} \tag{5}$$

which can be formed iteratively with $\boldsymbol{\rho}_1 = \boldsymbol{R}\boldsymbol{a}(\theta_0)$, and recursively applying $\boldsymbol{\rho}_k = \boldsymbol{R}\boldsymbol{\rho}_{k-1}$. The first projection vector maximizes the magnitude of the correlation between its output $(\boldsymbol{B}\boldsymbol{\rho}_1)^H\boldsymbol{r}(i)$ and the output of the top branch $\gamma\boldsymbol{a}^H(\theta_0)\boldsymbol{r}(i)$. Similar optimization problem has appeared in [11], [10]. Thus, the reduced-rank estimation can capture most, in the maximum correlation sense, of the interference-plus-noise signal that has passed over the top branch. Besides, the reduced-rank scheme with the GSC structure requires the concatenation of the blocking matrix $\boldsymbol{B}$ and the transformation matrix $\boldsymbol{T}_r \in \mathbb{C}^{M\times D}$, and accordingly, the blocking matrix $\boldsymbol{B}$ deserves proper design. One suitable method is the application of the correlation subtractive structure (CSS) [19], where

$$\boldsymbol{B} = \boldsymbol{I} - \boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0) \in \mathbb{C}^{M\times M}. \tag{6}$$

In this work, we use the CSS structure of the blocking matrix and directly cascade it with the reduced-rank transformation matrix $\boldsymbol{T}_r$. We note that, the computational complexity of the product $\boldsymbol{B}\boldsymbol{r}(i)$ is restricted to $O(M)$ instead of $O(M^2)$ for a general matrix $\mathbf{B}$.

After these operations, the received vector $\boldsymbol{r}(i)$ is mapped into a lower dimensional version termed the reduced-rank vector, which is described by

$$\bar{\boldsymbol{r}}(i) = (\boldsymbol{B}\boldsymbol{T}_r)^H \boldsymbol{r}(i). \tag{7}$$

Finally, the beamformer output is obtained as the difference $y(i) = \gamma\boldsymbol{a}^H(\theta_0)\boldsymbol{r}(i) - \bar{\boldsymbol{w}}_{gsc}^H(i)\bar{\boldsymbol{r}}(i)$, where $\bar{\boldsymbol{w}}_{gsc}(i)$ is the reduced-rank filter to be designed.

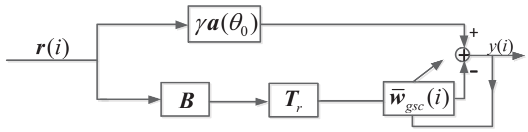

Fig. 1: Reduced-rank beamforming scheme with the GSC structure

IV. Proposed Blind Adaptive Widely linear reduced-rank Algorithm

For many applications with non-circular sources, the second-order statistics are fully described by both the covariance matrix $\boldsymbol{R} = \mathbf{E}\{\boldsymbol{r}\boldsymbol{r}^H\}$ and the complementary covariance matrix $\boldsymbol{R}_c = \mathbf{E}\{\boldsymbol{r}\boldsymbol{r}^T\} \neq \mathbf{0}$. In order to exploit the additional information contained in $\boldsymbol{R}_c$, we combine the received signal $\boldsymbol{r}$ with its complex conjugate $\boldsymbol{r}^*$ into an augmented vector $\tilde{\boldsymbol{r}}$ using a bijective transformation $\mathcal{T}$ shown below

$$\boldsymbol{r} \xrightarrow{\mathcal{T}} \tilde{\boldsymbol{r}}: \qquad \tilde{\boldsymbol{r}} = \frac{1}{\sqrt{2}}[\boldsymbol{r}^T, \boldsymbol{r}^H]^T \in \mathbb{C}^{2M\times 1}. \tag{8}$$

In the WL case, the size of the augmented vector obtained by (8) is twice that of the observed signal. It is therefore crucial to take full advantage of the reduced-rank signal processing techniques to achieve a faster convergence, robustness to interference and a lower complexity.

The block diagram of the WLCCM reduced-rank beamforming algorithm is similar to Fig. 1. The difference lies in that all the elements including $\boldsymbol{r}(i), \boldsymbol{a}(\theta_0), \boldsymbol{B}, \boldsymbol{T}_r$ are extended to their WL variants $\tilde{\boldsymbol{r}}(i), \tilde{\boldsymbol{a}}(\theta_0), \tilde{\boldsymbol{B}}, \tilde{\boldsymbol{T}}_r$. The augmented vectors $\tilde{\boldsymbol{r}}(i)$ and $\tilde{\boldsymbol{a}}(\theta_0)$ are obtained by (8), whereas $\tilde{\boldsymbol{B}}$ and $\tilde{\boldsymbol{T}}_r$ are developed by substituting their conventional components $\boldsymbol{r}(i)$, $\boldsymbol{a}(\theta_0)$ in expressions (6) and (5) for the augmented ones $\tilde{\boldsymbol{r}}(i)$ and $\tilde{\boldsymbol{a}}(\theta_0)$, respectively. However, this direct WL scheme does not fully exploit the structure of $\tilde{\boldsymbol{B}}$ and $\tilde{\boldsymbol{T}}_r$, and consequently, the procedure of cascading them to obtain the reduced-rank vector requires a large number of operations. In this section, we employ the property of the augmented covariance matrix and propose a structured WLCCM reduced-rank beamforming scheme. Furthermore, an analysis of the proposed algorithm is given.

A. The proposed WLCCM-KS-RLS algorithm

Development of the proposed WLCCM-KS-RLS beamforming algorithm involves two steps, that are: the construction of reduced-rank vector $\bar{\boldsymbol{r}}(i)$ and the design of an RLS algorithm to iteratively update the reduced-rank filter $\bar{\boldsymbol{w}}_{gsc}(i)$. In the first step, we take advantage of the structure of the WL covariance matrix to reduce the computational complexity. Firstly, the augmented covariance matrix can be written as

$$\tilde{\boldsymbol{R}} = \frac{1}{2}\begin{bmatrix} \boldsymbol{R} & \boldsymbol{R}_c \\ \boldsymbol{R}_c^* & \boldsymbol{R}^* \end{bmatrix}. \tag{9}$$

In practice, $\boldsymbol{R}$ and $\boldsymbol{R}_c$ are often estimated by the time average of N received snapshots, that is

$$\hat{\boldsymbol{R}}(i) = \frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{r}(n)\boldsymbol{r}^H(n) \quad \hat{\boldsymbol{R}}_c(i) = \frac{1}{N}\sum_{n=0}^{N-1}\boldsymbol{r}(n)\boldsymbol{r}^T(n). \tag{10}$$

Let us rewrite the augmented steering vector as

$$\tilde{\boldsymbol{a}}(\theta_0) = \mathcal{T}\{\boldsymbol{a}(\theta_0)\} = \frac{1}{\sqrt{2}}[\boldsymbol{a}^T(\theta_0), \boldsymbol{a}^H(\theta_0)]^T, \tag{11}$$

then, the WL transformation matrix can be written as

$$\tilde{\boldsymbol{T}}_r(i) = \mathcal{T}\{\boldsymbol{P}(i)\} \doteq \mathcal{T}\{[\bar{\boldsymbol{\rho}}_1(i), \bar{\boldsymbol{\rho}}_2(i), ..., \bar{\boldsymbol{\rho}}_D(i)]\}, \tag{12}$$

where we define $\bar{\boldsymbol{\rho}}_1(i) = \frac{1}{2}(\hat{\boldsymbol{R}}(i)\boldsymbol{a}(\theta_0) + \hat{\boldsymbol{R}}_c(i)\boldsymbol{a}^*(\theta_0))$ and $\bar{\boldsymbol{\rho}}_k(i) = \frac{1}{2}(\hat{\boldsymbol{R}}(i)\bar{\boldsymbol{\rho}}_{k-1}(i) + \hat{\boldsymbol{R}}_c(i)\bar{\boldsymbol{\rho}}_{k-1}^*(i))$, for $k = 2, ..., D$. In that sense, $\boldsymbol{P}(i) \in \mathbb{C}^{M\times D}$ contains the same information as $\tilde{\boldsymbol{T}}_r(i) \in \mathbb{C}^{2M\times D}$. In addition, the blocking matrix can be partitioned into four sub-matrices

$$\begin{aligned}\tilde{\boldsymbol{B}} &\doteq \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 \\ \boldsymbol{B}_2^* & \boldsymbol{B}_1^* \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{I} - \frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)}{2} & -\frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^T(\theta_0)}{2} \\ (-\frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^T(\theta_0)}{2})^* & (\boldsymbol{I} - \frac{\boldsymbol{a}(\theta_0)\boldsymbol{a}^H(\theta_0)}{2})^* \end{bmatrix}.\end{aligned} \tag{13}$$

After further matrix manipulations, the reduced-rank vector can be rewritten as

$$\begin{aligned}\bar{\boldsymbol{r}}(i) &= (\tilde{\boldsymbol{B}}\tilde{\boldsymbol{T}}_r(i))^H\tilde{\boldsymbol{r}}(i) \\ &= \mathcal{R}\{(\boldsymbol{B}_1\boldsymbol{P}(i) + \boldsymbol{B}_2\boldsymbol{P}^*(i))^H\boldsymbol{r}(i)\},\end{aligned} \tag{14}$$

where $\mathcal{R}\{\}$ denotes the real part of a value. The block diagram is depicted as Fig. 2.

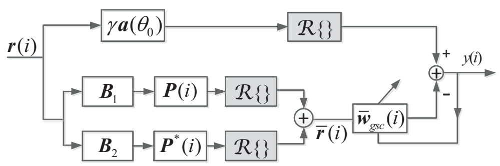

Fig. 2: Proposed WL reduced-rank scheme with the GSC structure.

Then, we derive the structured RLS algorithm for the reduced-rank filter. The reduced-rank weighting vector $\bar{\boldsymbol{w}}_{gsc}(i)$ is optimized by minimizing the unconstrained least squares (LS) cost function

$$J_{CM}(\bar{\boldsymbol{w}}_{gsc}(i)) = \sum_{n=1}^{i}\alpha^{i-n}(|y(n)|^2 - 1)^2, \tag{15}$$

where $|y(n)|^2 = y^*(n)(\gamma\mathcal{R}\{\boldsymbol{a}^H(\theta_0)\boldsymbol{r}(n)\} - \bar{\boldsymbol{w}}_{gsc}^H(i)\bar{\boldsymbol{r}}(n))$, α is a forgetting factor chosen as a positive scalar, close to, but less than 1. Letting $\tilde{\boldsymbol{x}}(n) = y^*(n)\bar{\boldsymbol{r}}(n)$ and $\tilde{d}(n) = \gamma y^*(n)\mathcal{R}\{\boldsymbol{a}^H(\theta_0)\boldsymbol{r}(n)\} - 1$, (15) can be rewritten as

$$J_{CM}(\bar{\boldsymbol{w}}_{gsc}(i)) = \sum_{n=1}^{i}\alpha^{i-n}[\tilde{d}(n) - \bar{\boldsymbol{w}}_{gsc}^H(i)\tilde{\boldsymbol{x}}(n)]^2. \tag{16}$$

TABLE I: The proposed WLCCM-KS-RLS algorithm

Initialization with a specified rank D:
$\tilde{\boldsymbol{Q}}^{-1}(0) = \delta\boldsymbol{I}_D, \bar{\boldsymbol{w}}_{gsc}(0) = [1, 0, ..., 0]^T, \alpha = 0.998$
For the ith snapshot $i = 1, 2, ..., N$
Compute $\hat{\boldsymbol{R}}(i)$ and $\hat{\boldsymbol{R}}_c(i)$ according to (10)
Calculate $\boldsymbol{P}(i)$ according to (12)
$\bar{\boldsymbol{r}}(i) = \mathcal{R}\{(\boldsymbol{B}_1\boldsymbol{P}(i) + \boldsymbol{B}_2\boldsymbol{P}^*(i))^H\boldsymbol{r}(i)\}$
$y(i) = \gamma\mathcal{R}\{\boldsymbol{a}^H(\theta_0)\boldsymbol{r}(i)\} - \bar{\boldsymbol{w}}_{gsc}^H(i-1)\bar{\boldsymbol{r}}(i)$
$\tilde{\boldsymbol{x}}(i) = y^*(i)\bar{\boldsymbol{r}}(i)$, $\tilde{d}(i) = \gamma y^*(i)\mathcal{R}\{\boldsymbol{a}^H(\theta_0)\boldsymbol{r}(i)\} - 1$
Update the reduced-rank coefficient $\bar{\boldsymbol{w}}_{gsc}$ according to (18)-(21).

By taking the gradient of (16) with respect to $\bar{\boldsymbol{w}}_{gsc}^*(i)$ and equating it to zero, after further manipulations we obtain

$$\bar{\boldsymbol{w}}_{gsc}(i) = \tilde{\boldsymbol{Q}}^{-1}(i)\tilde{\boldsymbol{p}}(i), \tag{17}$$

where $\tilde{\boldsymbol{Q}}(i) = \sum_{n=1}^{i}\alpha^{i-n}\tilde{\boldsymbol{x}}(n)\tilde{\boldsymbol{x}}^H(n)$ and $\tilde{\boldsymbol{p}}(i) = \sum_{n=1}^{i}\alpha^{i-n}\tilde{\boldsymbol{x}}(n)\tilde{d}^*(n)$.

To avoid the matrix inversion and reduce the complexity, we apply the matrix inversion lemma to (17), and obtain the following recursive expression

$$\bar{\boldsymbol{w}}_{gsc}(i) = \bar{\boldsymbol{w}}_{gsc}(i-1) + \tilde{\boldsymbol{k}}(i)\tilde{\xi}^*(i), \tag{18}$$

where

$$\tilde{\boldsymbol{k}}(i) = \frac{\tilde{\boldsymbol{Q}}^{-1}(i-1)\tilde{\boldsymbol{x}}(i)}{\alpha + \tilde{\boldsymbol{x}}^H(i)\tilde{\boldsymbol{Q}}^{-1}(i-1)\tilde{\boldsymbol{x}}(i)}, \tag{19}$$

$$\tilde{\xi}(i) = \tilde{d}(i) - \bar{\boldsymbol{w}}_{gsc}^H(i-1)\tilde{\boldsymbol{x}}(i), \tag{20}$$

$$\tilde{\boldsymbol{Q}}^{-1}(i) = \alpha^{-1}(\tilde{\boldsymbol{Q}}^{-1}(i-1) - \tilde{\boldsymbol{k}}(i)\tilde{\boldsymbol{x}}^H(i)\tilde{\boldsymbol{Q}}^{-1}(i-1)). \tag{21}$$

Based on (18)-(21), we obtain the reduced-rank filter updating procedure for the proposed adaptive WLCCM-KS-RLS algorithm with the GSC structure. The algorithm is summarized in Table I.

With this new structured scheme, we need not use the bijective transform and all the calculations are processed with vectors of lengths less than or equal to M, thereby significantly reducing the computational complexity as compared to the conventional direct WL scheme.

B. Analysis of the proposed algorithm

1) SINR analysis: From the block diagram shown in Fig. 2, we explicitly note that the WL reduced-rank vector $\bar{\boldsymbol{r}}(i)$ is real-valued. Thus, the filter coefficient $\bar{\boldsymbol{w}}_{gsc}(i)$ and the output of the filter $y(i)$ are also real-valued. Then the optimum augmented WL weighting vector $\tilde{\boldsymbol{w}}_o = \gamma\tilde{\boldsymbol{a}}(\theta_0) - \tilde{\boldsymbol{B}}\tilde{\boldsymbol{T}}_r(i)\bar{\boldsymbol{w}}_{gsc,o}$ is conjugate symmetric, and can be expressed as $\tilde{\boldsymbol{w}}_o = \mathcal{T}\{\boldsymbol{w}_{o,\mathrm{WL}}\}$, which means that $\boldsymbol{w}_{o,\mathrm{WL}} \in \mathbb{C}^{M\times 1}$ contains all the information of $\tilde{\boldsymbol{w}}_o \in \mathbb{C}^{2M\times 1}$. The corresponding optimal weight vector $\boldsymbol{w}_{o,\mathrm{WL}}$ minimizes the cost function $E\{(|\mathcal{R}\{y(i)\}|^2 - 1)^2\}$, where $y(i) = \boldsymbol{w}^H\boldsymbol{r}(i)$. The optimum output SINR can be equivalently expressed as

$$\begin{aligned}\mathrm{SINR}_{\mathrm{WL}} &= \frac{\mathbb{E}\{|\mathcal{R}\{\boldsymbol{w}_{o,\mathrm{WL}}^H\boldsymbol{s}\}|^2\}}{\mathbb{E}\{|\mathcal{R}\{\boldsymbol{w}_{o,\mathrm{WL}}^H\boldsymbol{v}\}|^2\}} \\ &= \frac{\gamma^2}{\mathbb{E}\{|\mathcal{R}\{\boldsymbol{w}_{o,\mathrm{WL}}^H\boldsymbol{v}\}|^2\}},\end{aligned} \tag{22}$$

TABLE II: Real operations of reduced-rank algorithms per snapshot

Algorithms	Real multiplications	Real additions
Structured WLCCM-KS-RLS	$8DM^2 + 12M^2 + 6DM$ $+18M + 3D^2 + 5D + 5$	$8DM^2 + 8M^2 + 2DM$ $+16M + 2D^2 + D - 4$
Direct WLCCM-KS-RLS	$16DM^2 + 24M^2 + 8DM$ $+32M + 3D^2 + 5D + 1$	$16DM^2 + 16M^2$ $+4DM + 24M + 2D^2 - 3$
WL-AVF	$32DM^2 + 24M^2$ $+40DM + 8M + 4D$	$32DM^2 + 16M^2$ $+32DM - 4D$
LCCM-KS-RLS	$4DM^2 + 6M^2 + 4DM$ $+12M + 10D^2 + 20D + 2$	$4DM^2 + 4M^2 + 2DM$ $+12M + 8D^2 + 12D - 3$
L-AVF	$8DM^2 + 6M^2$ $+20DM + 4D$	$8DM^2 + 4M^2$ $+16DM - 4D$

where $\boldsymbol{s}$ and $\boldsymbol{v}$ denote the desired signal and the interference-plus-noise component, respectively. On the one hand, the optimum solution of the linear algorithm is defined as $\boldsymbol{w}_{o,L}$, and the corresponding optimum SINR is given by

$$\mathrm{SINR_L} = \frac{\gamma^2}{\mathbb{E}\{|\boldsymbol{w}_{o,L}^H \boldsymbol{v}|^2\}}. \tag{23}$$

According to (22), if we substitute $\boldsymbol{w}_{o,\mathrm{WL}}$ for $\boldsymbol{w}_{o,L}$, the resulting $\mathrm{SINR}' = \frac{\gamma^2}{\mathbb{E}\{|\mathcal{R}\{\boldsymbol{w}_{o,L}^H \boldsymbol{v}\}|^2\}} \leq \mathrm{SINR_{WL}}$. On the other hand, the operation $\mathcal{R}\{\}$ nearly reduces the interference-plus-noise power by half, that is $\mathrm{SINR}' \approx 2\mathrm{SINR}_L$. Consequently, the optimum SINR of the WL processing exhibits an almost $3dB$ gain over that of the linear one [14].

2) Complexity analysis: We investigate the computational complexity of the proposed WLCCM-KS-RLS algorithm with the GSC structure, where the complexity is evaluated in terms of the number of real additions and real multiplications for each snapshot of size M. We compare the complexity of the proposed algorithms with that of the conventional WL processing scheme, the WL-AVF reduced-rank algorithm [14] and their linear counterparts, referred to by the acronyms LCCM-KS-RLS and L-AVF, respectively. The complexity figures are listed in Table II. In particular, given the rank $D = 2$ and $M = 32$, the total number of operations (real multiplications plus real additions) per snapshot for the proposed structured WLCCM-KS-RLS algorithm is 54881, whereas the direct scheme and WL-AVF require 109084 and 176896 operations, respectively. In general, we can verify that the proposed structured WLCCM-KS-RLS algorithm reduces the computational complexity compared with the WL-AVF algorithm and further saves nearly half of the operations with respect to the direct scheme.

V. Simulations

In this section, the SINR performance of the proposed WLCCM-KS-RLS algorithm and that of other analyzed schemes is evaluated. The output SINR of the WL processing is given by

$$\mathrm{SINR}(i) = \frac{\tilde{\boldsymbol{w}}^H(i)\tilde{\boldsymbol{R}}_s\tilde{\boldsymbol{w}}(i)}{\tilde{\boldsymbol{w}}^H(i)\tilde{\boldsymbol{R}}_{in}\tilde{\boldsymbol{w}}(i)}, \tag{24}$$

where $\tilde{\boldsymbol{R}}_s$ and $\tilde{\boldsymbol{R}}_{in}$ denote the augmented covariance matrice of the desired signal and the interference-plus-noise in the observation space, respectively. In our simulations, we consider a large-scale ULA system equipped with $M = 32$ sensor elements. The DOA of the desired user is $\theta_0 = 50^\circ$ and the whole interfering signals impinge on the array with DOAs evenly distributed on both sides of the desired user. All the users are assumed to have equal power. The performance of the reduced-rank algorithm depends on the specific rank D. Interestingly, it has been observed that in various scenarios that with the KS based reduced-rank technique, the optimal rank D does not scale significantly with the number of users K and the length of the observation vector M. For a blind algorithm, generally $D \leq 5$ can be chosen [10]. In our simulations, an appropriate value of the rank $D = 2$ was determined experimentally so as to ensure that all the algorithms achieve a good performance for a fair comparison. All the results given below are averaged over 100 simulation runs.

Fig. 3(a) illustrates the output SINR performance against the number of snapshots N. There are 10 interferers with DOAs of $[40^\circ, 30^\circ, 20^\circ, 10^\circ, 0^\circ, 60^\circ, 70^\circ, 80^\circ, 90^\circ, 100^\circ]$ (the DOA separation is 10°) and the input SNR is set as 10 dB. The performance of the optimum minimum variance distortionless response (MVDR) filter is also given for comparison. Obviously, the proposed WLCCM-KS-RLS algorithm exhibits a faster convergence and a higher steady-state SINR compared with the WL-AVF algorithm and the full-rank schemes. Basically, it can be observed that the WL algorithms outperform their linear counterparts. This can be explained by the fact that the augmented vectors provide more information. Moreover, the SINR performance of the CCM-based full-rank algorithm is superior to that of the CMV-based one.

In Fig. 3(b), we show the steady-state SINR performance as a function of the input SNR, where the simulation scenario is the same as that of Fig. 3(a). Generally, the SINR increases monotonically with the input SNR, and our proposed algorithm has a better performance with a smaller gap from the optimum MVDR results. The conclusion is consistent with the results in Fig. 3(a). Besides, the WL algorithms obtain an additional gain compared with the conventional linear algorithms.

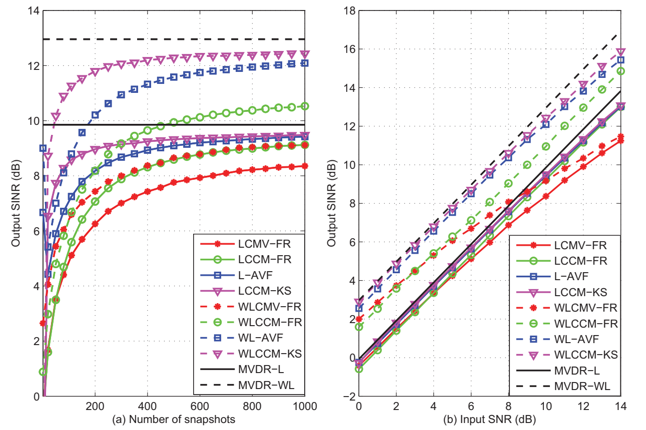

Fig. 3: Output SINR performance versus: (a) number of snapshots; (b) input SNR.

VI. Conclusion

In this paper, we proposed a novel blind reduced-rank WL beamforming receiver based on the KS technique. Inspired by the CCM criterion, an RLS algorithm was developed for adaptive implementation of the new beamformer within an extended GSC structure. The proposed scheme exploits the structure of the augmented covariance matrix, therefore can reduce the computational complexity compared with conventional direct WL processing. The performance analysis in terms of output SINR and also the computational complexity analysis of the new algorithms were carried out. Simulation results have shown that the proposed algorithm outperforms the existing WL reduced-rank and full-rank beamforming algorithms in terms of convergence speed and steady-state SINR.

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