Non-regenerative MIMO Relaying Strategies — from Single to Multiple Cooperative Relays

Chao Zhao and Benoît Champagne
Department of Electrical and Computer Engineering, McGill University
3480 University, Montreal, QC, Canada, H3A 2A7
Email: chao.zhao@mail.mcgill.ca, benoit.champagne@mcgill.ca

***Abstract*—In this paper, we study the performance of various amplify-and-forward (AF) relaying strategies in terms of ergodic capacity and bit-error-rate (BER) for single-user MIMO channels with single and multiple relays. We first consider some hybrid methods for the single-relay channel and compare their performance with previously proposed methods under various signal-to-noise ratio (SNR) regimes. We then extend these hybrid methods to multiple-relay channels and unveil that they can exceed SVD-based optimum methods in this case. Finally, we propose new multiple-relay strategies based on a cooperative minimum mean square error (CMMSE) criterion. Through simulations, we show that the new hybrid methods can outperform existing ones in the multiple-relay channel, at the price of a slight increase in implementation complexity.**

***Index Terms*—Non-regenerative relaying, MIMO systems, cooperative communications, ergodic capacity, MMSE criterion**

I. Introduction

Wireless relaying has been attracting interests in recent years as a promising strategy to overcome the impairments caused by multipath fading, shadowing and path losses in traditional communication systems. The introduction of multiple-input multiple-output (MIMO) techniques into the relaying framework, through the use of multiple antennas at the source, relays or destination, promises further leverage [1]. In [2], the authors established a capacity scaling law for the one-source–multiple-relays–one-destination (1S-MR-1D) channel, showing that multiple relays can induce distributed array/diversity gain in addition to the MIMO multiplexing gain. The use of multiple relays is also well-suited to cooperative communications, which is emerging as a promising alternative to centralized wireless communication systems in order to meet increasing demands for high data rates [3].

Relaying strategies can be *regenerative* such as decode-and-forward (DF), or *non-regenerative*, e.g. amplify-and-forward (AF). Non-regenerative strategies apply linear processing matrices to the received baseband signals. Regenerative strategies, which attempt to decode these signals, are more sophisticated but suffer from longer delays and higher overall costs. Accordingly, AF strategies have received more attention in the literature. In addition, relay stations can work in *full-duplex* or *half-duplex* mode. A half-duplex relay receives in a given time slot and transmits in the next one; full-duplex relays are generally difficult to implement. In this paper, we focus on half-duplex AF relaying strategies in single-user MIMO relaying channels, including one-source–one-relay–one-destination (1S-1R-1D) and 1S-MR-1D channels. We neglect the presence of a possible direct source-to-destination link, which is typically hindered by high levels of attenuation.

Two types of philosophy arise in the design of relaying strategies. The first category of approaches "borrow" ideas from MIMO transceiver design. These heuristic methods include simplistic amplify-and-forward (SAF) [2], matched filtering (MF) [1], zero-forcing (ZF) [1], linear minimum mean square error (MMSE) [1] [4] and methods based on matrix decompositions, including the QR [5] and singular value decomposition (SVD) [6]. The other type of approaches seek to optimize an objective function which can be of an information-theoretic nature, such as the maximum mutual information (MMI) [4] [7] [8], or based on a statistical criterion, such as the joint MMSE (JMMSE) [9], [10], the maximum SNR [4] or the minimum pairwise error probability (PEP) [6]. Interestingly, in the 1S-1R-1D case, a variety of these objective functions lead to the canonical SVD structure with different choices of singular values [6]–[9], [11]. The singular value matrix, which represents the power allocation strategy, can be determined by convex optimization [12]. Extension of the MMI and JMMSE criteria to the 1S-MR-1D case with relaxed constraints are considered in [4].

In this paper, we first consider hybrid strategies for the 1S-1R-1D MIMO relay configuration based on different combinations of the optimization criteria used for the backward and forward channels, including MF, ZF and MMSE. While these criteria have been studied individually before, our unified framework allows a classification and comparison of their tandem combinations. We then extend our methodology from 1S-1R-1D to 1S-MR-1D configurations. Existing works deal primarily with one scenario only, and it is not clear whether methods suited for one-relay channels can perform well under multiple-relay channels or vice versa. Our approach here complements the works in [2], [5] and [13] which cover some of these issues. In particular, we show that the SVD-based methods generally behave unsatisfactorily in multiple-relay scenarios and can be outperformed by the proposed hybrid strategies, which can achieve the distributed array gain. Finally, for the 1S-MR-1D channels, we also propose new cooperative relaying strategies based on tandem combinations of

This work was supported by InterDigital Canada Ltée., the Natural Sciences and Engineering Research Council of Canada, and the Government of Québec under the PROMPT program.

978-1-4244-7555-1/10/$26.00

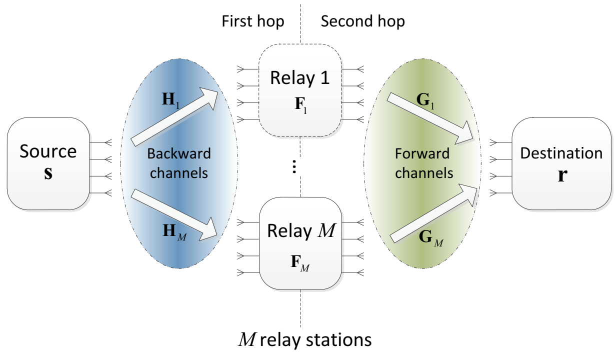

Fig. 1. A point-to-point MIMO relaying channel.

the cooperative MMSE (CMMSE) structure in [4], [14]. These methods prove to be more efficient than previous methods at the price of slight increase in complexity due to the exchange of channel state information (CSI). The proposed approaches are evaluated via simulations over 1S-1R-1D and 1S-MR-1D channels and their relaying performance is compared in terms of ergodic capacity and theoretical bit-error rate (BER).

The organization of this paper is as follows. Section II describes the system and signal model along with some basic assumptions. Section III presents the new hybrid relaying strategies and Section IV analyze their performance in multi-relay configurations. The cooperative relaying strategies are developed in Section V. Results of simulation studies are given in Section VI and conclusions are drawn in Section VII. Two important notations are conjugate transpose $()^H$ and pseudo-inverse $()^{\dagger}$.

II. System Model

Fig. 1 illustrates a point-to-point MIMO relaying channel between a source and a destination, going through M parallel relay stations via a pair of hops. The source and destination are equipped with N_S and N_D antennas, while for simplicity, each relay is equipped with the same number of antennas, denoted by N_R. [1] We make the common-sense assumption that $N_R \geq N_S = N_D$ [1]. We consider a half-duplex mode of operation where the relay antennas are used for both transmitting and receiving purposes during different time slots. The direct link between the source and destination is unavailable or too weak to be taken into consideration.

In order to introduce a suitable system model for the above MIMO relaying channel, we make the following general assumptions: (1) flat-fading radio propagation, which means that the operating system bandwidth is less than the coherence bandwidth of the wireless channels, (2) the channel characteristics remain constant during the transmission period of interest, and (3) all the channel and signal representations are baseband equivalence of their bandpass radio frequency (RF) counterparts. That is, standard operations of demodulation, down-conversion, filtering and A/D conversion are assumed on the receivers of the relays, with dual operations applied on the transmitters. As a result, all processing takes place on complex-valued, baseband signal samples.

Under the above assumptions, the received baseband signal vector $\mathbf{x}_j \in \mathbb{C}^{N_R \times 1}$ at the jth relay takes the form

$$\mathbf{x}_j = \mathbf{H}_j \mathbf{s} + \mathbf{w}_j, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_S \times 1}$ is the vector of transmitted source symbols, $\mathbf{H}_j \in \mathbb{C}^{N_R \times N_S}$ is the matrix corresponding to the baseband-equivalent *backward* channel between the source user and the jth relay and $\mathbf{w}_j \in \mathbb{C}^{N_R \times 1}$ is an additive noise component. The signal and noise vectors, $\mathbf{s}$ and $\mathbf{w}_j$ for $j = 1, \ldots, M$, are modelled as independent, circularly symmetric complex Gaussian random vectors with zero-mean and correlation matrices $\mathbf{R_s} = \mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \sigma_s^2 \mathbf{I}$ and $\mathbf{R}_{\mathbf{w}_j} = \mathrm{E}\{\mathbf{w}_j \mathbf{w}_j^H\} = \sigma_w^2 \mathbf{I}$, respectively, where σ_s^2 is the average transmitted power per antenna at the source, σ_w^2 is the average noise power received at the individual relay antennas and $\mathbf{I}$ denotes an identity matrix of appropriate dimension. That is, we assume that the noise is spatially white and independent across different relays. We can also multiply $\mathbf{s}$ by a precoding matrix if necessary, with corresponding changes in the definition of $\mathbf{R_s}$.

With linear processing at the relays defined in terms of the transformation matrices $\mathbf{F}_j \in \mathbb{C}^{N_R \times N_R}$ for $j = 1, \ldots, M$, the jth relay retransmits its received noisy signal $\mathbf{x}_j$ as

$$\mathbf{y}_j = \mathbf{F}_j \mathbf{x}_j. \tag{2}$$

The received signal vector at the destination, denoted by $\mathbf{r} \in \mathbb{C}^{N_D \times 1}$, can be expressed as

$$\mathbf{r} = \sum_{j=1}^{M} \mathbf{G}_j \mathbf{F}_j \mathbf{H}_j \mathbf{s} + \sum_{j=1}^{M} \mathbf{G}_j \mathbf{F}_j \mathbf{w}_j + \mathbf{n}, \tag{3}$$

where $\mathbf{G}_j \in \mathbb{C}^{N_D \times N_R}$ is the baseband *forward* channel matrix from relay j to the destination user and $\mathbf{n} \in \mathbb{C}^{N_D \times 1}$ is and additive noise component. The latter, which is assumed to be independent from $\mathbf{s}$ and $\{\mathbf{w}_j\}$, is modeled as a circularly symmetric complex Gaussian random vectors with zero-mean and correlation matrix $\mathbf{R_n} = \mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2 \mathbf{I}$, where σ_n^2 is the average noise power received at the individual destination antennas. Equation (3) for the general 1S-MR-1D MIMO relay channel can also be expressed in a "block-diagonal" form as

$$\mathbf{r} = \mathbf{GFHs} + \mathbf{GFw} + \mathbf{n}, \tag{4}$$

where we define $\mathbf{G} = [\mathbf{G}_1, \ldots, \mathbf{G}_M]$, $\mathbf{H} = \left[\mathbf{H}_1^H, \ldots, \mathbf{H}_M^H\right]^H$, $\mathbf{F} = \mathrm{diag}(\mathbf{F}_1, \ldots, \mathbf{F}_M)$, and $\mathbf{w} = \left[\mathbf{w}_1^H, \ldots, \mathbf{w}_M^H\right]^H$, When $M = 1$, this signal model reduces to the 1S-1R-1D case. Note that in designing the linear relaying matrices $\mathbf{F}_j$, one is actually selecting an equivalent channel $\mathbf{GFH}$, aiming to maximize signal-relaying efficiency while taking into account the effects of the relayed noise term $\mathbf{GFw}$. Transceiver design for the source and destination user is an independent issue and does not affect the relaying performance.

In discussing the properties of the proposed hybrid MIMO relaying strategies, it is convenient to introduce two SNR parameters as follows. We first define $\rho_1 = {\sigma_s}^2/\sigma_w^2$ which,

[1] Generalization to different numbers of antennas at the relays, i.e. N_R, j, is straightforward.

in the special case of a normalized backward channel with $\mathrm{tr}(\mathbf{H}\mathbf{H}^H) = N_R$, would represent the average received signal-to-noise power ratio at the relay antennas. The average power transmitted by the jth relay can be expressed as

$$P_j = E[\|\mathbf{y}_j\|^2] = \mathrm{tr}\left(\mathbf{F}_j \mathbf{R}_{\mathbf{x}_j} \mathbf{F}_j^H\right), \quad (5)$$

where $\mathbf{R}_{\mathbf{x}_j} = \mathrm{E}\{\mathbf{x}_j \mathbf{x}_j^H\} = \sigma_s^2 \mathbf{H}_j \mathbf{H}_j^H + \sigma_w^2 \mathbf{I}$, while the total transmitted power by the M relays is $P = \sum_{j=1}^{M} P_j$. The second SNR parameter, defined in terms of P as $\rho_2 = P/(M N_R \sigma_n^2)$, gives the ratio of average transmitted power per relay antenna to the power of the noise induced at the individual destination antennas. Note that P represents the total power consumed by the relay stations to transmit both the desired signal component $\mathbf{s}$ and the additive relay noise $\mathbf{w}_j$ to the destination user.

As ρ_1 becomes much larger than ρ_2, the noise introduced by the relay station ($\mathbf{GFw}$) is dominated by the noise at the destination receiver $\mathbf{n}$. The signal model reduces to $\mathbf{r} = \mathbf{GFHs} + \mathbf{n}$ and the power constraint is $\sigma_s^2 \mathrm{tr}\left(\mathbf{F}\mathbf{H}\mathbf{H}^H\mathbf{F}^H\right) = P$. The product $\mathbf{FH}$ can be seen as a linear precoder matrix for the channel $\mathbf{G}$ and the asymptotic behavior depends on the relaying strategy. As ρ_2 goes much larger than ρ_1, $\mathbf{n}$ can be ignored and the signal model is $\mathbf{r} = \mathbf{GF}(\mathbf{Hs} + \mathbf{w})$. Assuming $\mathbf{GF}$ is well-conditioned and can be equalized by an inverse system, the asymptotic performance does not depend on $\mathbf{F}$ in spite of the fact that the values of ρ_2 needed to converge to this limit may vary for different strategies.

III. Linear Relaying Strategies for 1S-1R-1D

A. Previous Linear Relaying Strategies

Simplistic amplify-and-forward (SAF) does not use any CSI knowledge and its relaying matrix is a scale identity matrix, i.e. $\mathbf{F} = \eta \mathbf{I}$, where here and in subsequent equations, η is a scale parameter introduced to satisfy constraints on the transmit power consumed by the relay stations. The relaying matrices for MF, ZF and MMSE [1] are, respectively, $\mathbf{F} = \eta \mathbf{G}^H \mathbf{H}^H$, $\mathbf{F} = \eta \mathbf{G}^\dagger \mathbf{H}^\dagger$ and

$$\mathbf{F} = \eta \mathbf{G}^H \left(\mathbf{I} + \rho_2 \mathbf{G}\mathbf{G}^H\right)^{-1} \left(\mathbf{I} + \rho_1 \mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H. \quad (6)$$

Another category of methods are based on the SVD. In these methods, the relay station rotates the eigenmodes of the backward and forward channels via the transformation matrix

$$\mathbf{F} = \mathbf{V}_2 \mathbf{\Lambda} \mathbf{U}_1^{\mathrm{H}}. \quad (7)$$

where $\mathbf{H} = \mathbf{U}_1 \mathbf{\Sigma}_1 \mathbf{V}_1^{\mathrm{H}}$ and $\mathbf{G} = \mathbf{U}_2 \mathbf{\Sigma}_2 \mathbf{V}_2^{\mathrm{H}}$ are the SVD of $\mathbf{H}$ and $\mathbf{G}$, respectively, while the diagonal matrix $\mathbf{\Lambda}$ depends on the power allocation strategy. With this choice of $\mathbf{F}$, the signal model reduces to

$$\mathbf{r} = \mathbf{U}_2 \mathbf{\Sigma}_2 \mathbf{\Lambda} \mathbf{\Sigma}_1 \mathbf{V}_1^{\mathrm{H}} \mathbf{s} + \mathbf{U}_2 \mathbf{\Sigma}_2 \mathbf{\Lambda} \mathbf{U}_1^{\mathrm{H}} \mathbf{w} + \mathbf{n}. \quad (8)$$

One can use precoder $\mathbf{V}_1$ and receiver $\mathbf{U}_2^{\mathrm{H}}$ to diagonalize the equivalent channel.

The SVD-based canonical form was initially used to simplify the matrix design problem [15]. Interestingly, this structure turns out to be the optimal form for a wide class of criterions for the 1S-1R-1D channel, including MMI and JMMSE. Ref. [11] used majorization theory to prove that most objective functions lead to the SVD form in a unified framework .

TABLE I
List of hybrid relaying strategies.

Rx ($\mathbf{B}$) \ Tx ($\mathbf{A}$)	$\mathbf{G}^H$	$\mathbf{G}^\dagger$	$\mathbf{G}^H\left(\mathbf{I}+\rho_2\mathbf{G}\mathbf{G}^H\right)^{-1}$
$\mathbf{H}^H$	MF-MF	MF-ZF	MF-MMSE
$\mathbf{H}^\dagger$	ZF-MF	ZF-ZF	ZF-MMSE
$\left(\mathbf{I}+\rho_1\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$	MMSE-MF	MMSE-ZF	MMSE-MMSE

B. Hybrid Methods - A Unified Framework

The hybrid relaying matrices proposed in this work are composed of two components, $\mathbf{B} \in \mathcal{C}^{N_S \times N_R}$ and $\mathbf{A} \in \mathcal{C}^{N_R \times N_D}$, and can be expressed as

$$\mathbf{F} = \eta \mathbf{A}\mathbf{B}. \quad (9)$$

$\mathbf{B}$ equalizes the source-to-relay MIMO channel and generates N_S sufficient statistics, each of which is a signal stream impaired by interferences from other streams and noise. $\mathbf{A}$ is the MIMO precoder (or transmit beamforming matrix) for the relay-to-destination channel.

We "borrow" the substructures of the MF, ZF and MMSE approaches and combine them in a tandem manner as shown in Table I. In the sequel, we use the terminology Rx-Tx to designate these hybrid strategies. In particular, MF-MF, ZF-ZF and MMSE-MMSE refer to the previously published forms of the MF, ZF and linear MMSE strategies. The MMSE-ZF relaying matrix minimizes the MSE at the destination user without power constraint if no source precoder and destination MIMO equalizer are employed [4], [9].

In a linear MIMO receiver or transmitter, linear MMSE offers a tradeoff between noise and interference cancelation, and outperforms both MF and ZF over the complete SNR range. However, a hasty generalization of this conclusion to the scenario of linear relaying is inappropriate because two noise vectors arise in the signal model in this case, one of which (i.e. $\mathbf{w}$) is affected by the relaying matrix. The precoder $\mathbf{A}$ in (9) processes the sufficient statistics vector which is impaired by noise and interferences instead of noiseless signal streams.

IV. Noncooperative Strategies for 1S-MR-1D

A. SVD-based Methods

For 1S-MR-1D, when using the relaying matrices $\mathbf{F}_j = \mathbf{V}_{2j} \mathbf{\Lambda}_j \mathbf{U}_{1j}^H$, the received signal model is

$$\mathbf{r} = \sum_{j=1}^{M} \mathbf{U}_{2j} \mathbf{\Sigma}_{2j} \mathbf{\Lambda}_j \mathbf{\Sigma}_{1j} \mathbf{V}_{1j}^H \mathbf{s} + \sum_{j=1}^{M} \mathbf{U}_{2j} \mathbf{\Sigma}_{2j} \mathbf{\Lambda}_j \mathbf{U}_{1j}^H \mathbf{w}_j + \mathbf{n}, \quad (10)$$

where $\mathbf{H}_j = \mathbf{U}_{1j} \mathbf{\Sigma}_{1j} \mathbf{V}_{1j}^H$ and $\mathbf{G}_j = \mathbf{U}_{2j} \mathbf{\Sigma}_{2j} \mathbf{V}_{2j}^H$ are SVD forms. Signals transmitted by different relays add in a noncoherent way at the destination user. Indeed, the matrices $\mathbf{V}_{1j}$ are different for $j = 1, \ldots, M$, and similarly for the matrices $\mathbf{U}_{2j}$, and hence the eigenmodes of different branches are not aligned. There are no unique unitary precoder and equalizer matrices that can diagonalize all branches simultaneously so that the signals from all relays enhance each other constructively. This explains the poor performance of SVD-based methods observed experimentally in 1S-MR-1D configurations.

B. Hybrid methods

In contrast, hybrid methods can benefit from distributed array gain. For example, the received signal when using ZF-ZF relaying matrices in a 1S-MR-1D system is

$$\mathbf{r} = \left(\sum_{k=1}^{M} \eta_k\right)\mathbf{s} + \sum_{k=1}^{M} \eta_k \mathbf{H}_k^{\dagger}\mathbf{w}_k + \mathbf{n}. \tag{11}$$

We see that signals from different relay stations add coherently, while the noise vectors $\mathbf{w}_k$ generally do not. Consequently, nn M-fold SNR gain can be obtained in this case.

For the MF-MF hybrid method, the equivalent channel matrix seen by the source component is $\mathbf{H}_{eq} = \sum_{j=1}^{M} \mathbf{G}_j \mathbf{G}_j^H \mathbf{H}_j^H \mathbf{H}_j$. Its entries can be expanded as

$$\mathbf{H}_{eq}(p,q) = \sum_{j=1}^{M}\sum_{l=1}^{N_S} \mathbf{g}_{j,p}^H \mathbf{g}_{j,l} \mathbf{h}_{j,l}^H \mathbf{h}_{j,q}, \tag{12}$$

where $\mathbf{H}_j = [\mathbf{h}_{j,1}, \ldots, \mathbf{h}_{j,N_S}]$, and $\mathbf{G}_j^H = [\mathbf{g}_{j,1}, \ldots, \mathbf{g}_{j,N_D}]$. Assuming a rich-scattering Rayleigh-fading scenario, where the entries of $\mathbf{G}_k$ and $\mathbf{H}_k$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, i.e. $\mathcal{CN}(0,1)$, the mean of the diagonal entries $\mathbf{H}_{eq}(p,p)$ is MN_R^2 while the off-diagonal entries $\mathbf{H}_{eq}(p,q)$ $(p \neq q)$ have zero mean. That is, the matrices $\mathbf{G}_k \mathbf{G}_k^H \mathbf{H}_k^H \mathbf{H}_k$ have positive diagonal entries whose sum ensures that the signal components add coherently, while the off-diagonal entries add noncoherently. This explains the distributed array gain of the MF-MF strategy and similar considerations extend to the other hybrid methods.

V. Cooperative relaying strategies

Although hybrid strategies can achieve distributed array gain, there is no cooperation between relay stations. Introducing cooperation can enable further performance improvement. One way is to follow the "optimization" approach. However, the block-diagonal matrix $\mathbf{F}$ in (4) complicates this problem because the optimal SVD form for 1S-1R-1D does not have the block-diagonal structure. The works in [4] solve the MMSE, MMI and JMMSE problems but their power are constrained at the destination receiver instead of the relay transmitters.

A reasonable design approach is to consider $\mathbf{F}_k$ collaboratively such that the overall MSE at the destination is minimized (Cooperative MMSE, CMMSE). The minimum norm solution of this underdetermined problem is given by [4] as

$$\mathbf{F}_k = \eta \mathbf{G}_k^{\dagger}\left(\mathbf{I} + \rho_1 \sum_{j=1}^{M} \mathbf{H}_j^H \mathbf{H}_j\right)^{-1} \mathbf{H}_k^H, k = 1, \ldots, M. \tag{13}$$

where η is used here to specify the power level at the destination receiver. However, if we attempt to adjust η in this structure to satisfy transmit power constraints at the relays, this solution is merely suboptimal.

Here, further expanding the concept of an hybrid structure introduced earlier, we consider various tandem combinations of this structure for the backward and forward channels in the multiple-relay context. In this respect, we note that (13) is composed of two parts — CMMSE and ZF. In MIMO receiver design, ZF is inferior to MMSE, especially under low SNR conditions. It is therefore natural to replace the ZF factor with MMSE or MF in this equation. Indeed, as we will show below, these new strategies, referred to as CMMSE-MF and CMMSE-MMSE, will outperform the CMMSE-ZF. The expression for CMMSE-MMSE is

$$\mathbf{F}_k = \eta \mathbf{G}_k^H \left(\mathbf{I} + \rho_2 \mathbf{G}_k \mathbf{G}_k^H\right)^{-1} \left(\mathbf{I} + \rho_1 \sum_{j=1}^{M} \mathbf{H}_j^H \mathbf{H}_j\right)^{-1} \mathbf{H}_k^H. \tag{14}$$

The CMMSE structure can also be generalized to the transmitting side in the form of CMMSE-CMMSE:

$$\mathbf{F}_k = \eta \mathbf{G}_k^H \left(\mathbf{I}+\rho_2 \sum_{j=1}^{M} \mathbf{G}_j \mathbf{G}_j^H\right)^{-1} \left(\mathbf{I}+\rho_1 \sum_{j=1}^{M} \mathbf{H}_j^H \mathbf{H}_j\right)^{-1} \mathbf{H}_k^H, \tag{15}$$

which reduces to MMSE-MMSE when $M = 1$. For large M, we can show that the sums of matrix products in 15 approach identity matrices and the method reduces to MF-MF.

The cost of cooperation can be broken down as follows: Firstly, the relay stations feedback all $\mathbf{H}_k$ (possibly $\mathbf{G}_k$) to a fusion center. Secondly, the fusion center computes the required matrix sums and broadcasts them to the relay stations. Lastly, each relay implements its own relaying matrix based on available CSI and the broadcasted information.

VI. Comparative Performance Evaluation

We compare the performance of different MIMO relaying strategies in terms of ergodic capacity and theoretical BER. We use the expression for capacity from [8]

$$C = \frac{1}{2}\log\det\left(\mathbf{I} + \rho_1 \mathbf{H}\mathbf{H}^H - \rho_1 \mathbf{H}\mathbf{H}^H \mathbf{S}^{-1}\right). \tag{16}$$

where $\mathbf{S} = \mathbf{I} + \frac{\sigma_w^2}{\sigma_n^2}\mathbf{F}^H \mathbf{G}^H \mathbf{G}\mathbf{F}$. The ergodic capacity is obtained as the expected value of (16) over all possible channel realizations. When computing theoretical BER, we assume that each source antenna transmits independent uncoded QPSK-modulated streams and the destination user applies MMSE equalizer to demultiplex all streams and uses single-stream maximum likelihood decoding. According to a derivation similar to that in [11], [16], we can compute the MSE matrix, single-stream SINRs and henceforth theoretical BER.

According to the above settings, we use a Monte-Carlo approach to obtain the required expected values over channel realization but do not need to simulate the complete transceiver chain. To this end, we generate 2000 realizations of i.i.d. normalized Rayleigh-fading channels. Both ergodic capacity

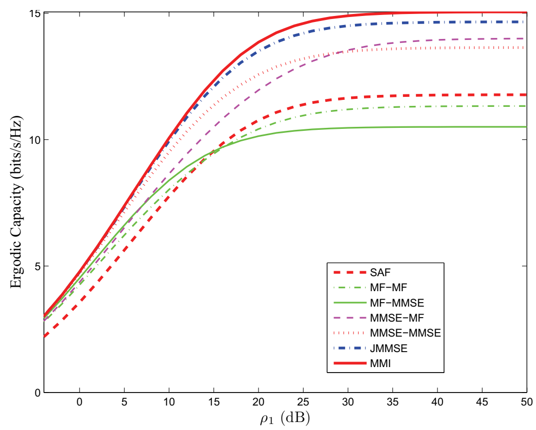

Fig. 2. Ergodic capacity versus ρ_1 for 1S-1R-1D ($N_R = 6$ and $\rho_2 = 15$dB).

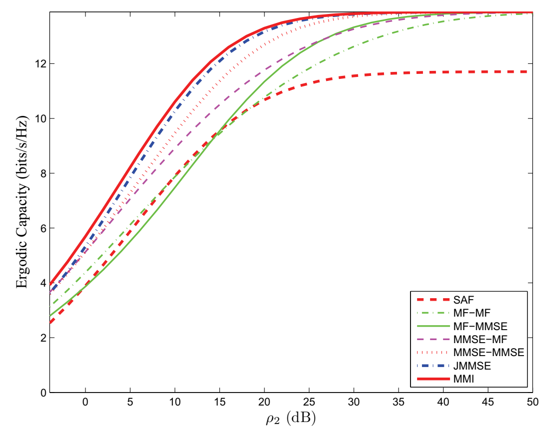

Fig. 3. Ergodic capacity versus ρ_2 for 1S-1R-1D ($N_R = 6$ and $\rho_1 = 15$dB).

and BER are functions of the SNR parameters ρ_1 and ρ_2; in our analysis, we fix one of them and change the other. We choose the other parameters as follows: $N_S = N_D = 4$, N_R varies between 4 and 8. For the 1S-MR-1D configuration, we set $M = 3$. The noises induced at the relay and destination have the same power: $\sigma_w^2 = \sigma_n^2$.

A. 1S-1R-1D

For hybrid methods, when fixing one of the matrices $\mathbf{A}$ or $\mathbf{B}$ and altering the other, results show that MMSE is always superior to ZF. Accordingly, we only consider MMSE-MMSE, MMSE-MF, MF-MF, and MF-MMSE in the following discussions. Fig. 2 and 3 show ergodic capacity results. We can observe that optimum MMI relaying attains the highest capacity while JMMSE, which benefits from the same SVD structure, has a slightly inferior performance since its power allocation strategy is not designed to maximize capacity. MMSE-MMSE outperforms all other three hybrid methods except when $\rho_1 \gg \rho_2$ or $\rho_2 \gg \rho_1$. In practice, large gap between ρ_1 and ρ_2 is undesirable and not energy efficient due to a "bottleneck effect". We can observe that capacity is insensitive with either ρ_1 or ρ_2 as $|\rho_1 - \rho_2|$ grows very large. Besides, we can observe that for a fixed ρ_2 and $\rho_1 \to \infty$, the asymptotic capacity depends on the relaying strategy. BER results are shown in Fig. 4 and 5. [2] In this case, JMMSE outperforms MMI due to their difference in power allocation. MMSE-MMSE performs much better than other hybrid methods in this transceiver architecture. Asymptotic performance is in conformity with discussion in Section II.

B. 1S-MR-1D

When it comes to multiple relays, it can be seen from Fig. 6 that hybrid methods outperform SVD-based methods. If $\rho_2 \gg \rho_1$, the best strategy MF-MMSE applies MF receiver to the first low-SNR hop and MMSE transmitter to the second high-SNR hop; for $\rho_2 \ll \rho_1$, MMSE-MF has the best capacity

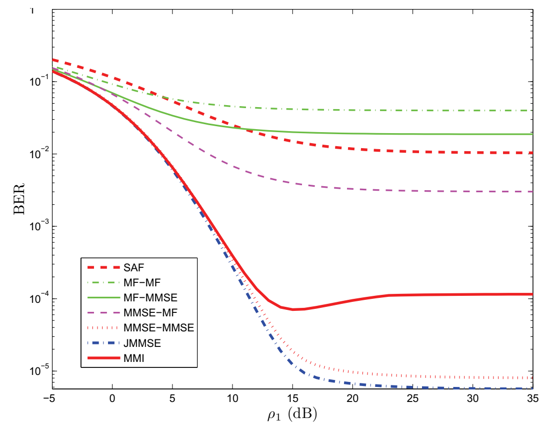

Fig. 4. Theoretical BER versus ρ_1 for 1S-1R-1D ($\rho_2 = 12$dB, $N_R = 6$).

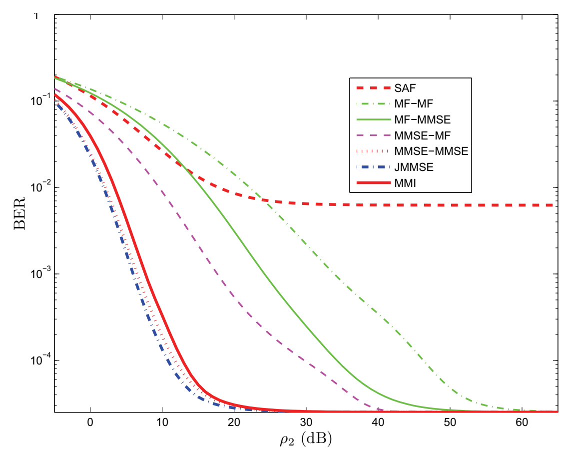

Fig. 5. Theoretical BER versus ρ_2 for 1S-1R-1D ($\rho_1 = 12$dB, $N_R = 6$).

performance. MF-MF has the highest capacity results among these methods if ρ_1 and ρ_2 are close. In real applications, method selection highly depends on the relationship between ρ_1 and ρ_2, computational costs, sensitivity with respect to CSI uncertainty and other implementation issues.

Next, we study our proposed cooperative hybrid MIMO relaying strategies, i.e. CMMSE-MMSE, CMMSE-CMMSE and CMMSE-MF and compare them with noncooperative methods. Fig.7 shows that CMMSE-CMMSE and CMMSE-MF have

[2] We note that BER curve for MMI is not a monotonically decreasing function of ρ_1. In effect, $\mathbf{F}$ and hence the signal model are both changing as ρ_1 increases, which cannot guarantee the monotonicity of the curve. We have been able to explain this phenomenon theoretically but this falls outside the scope of this paper.

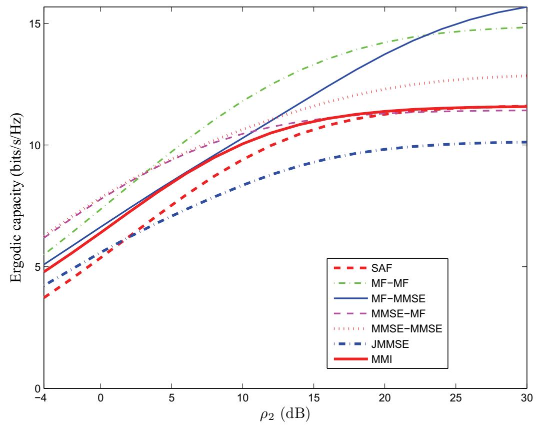

Fig. 6. Comparision between SVD-based methods and hybrid methods for 1S-3R-1D ($\rho_1 = 15$dB, $N_R = 4$).

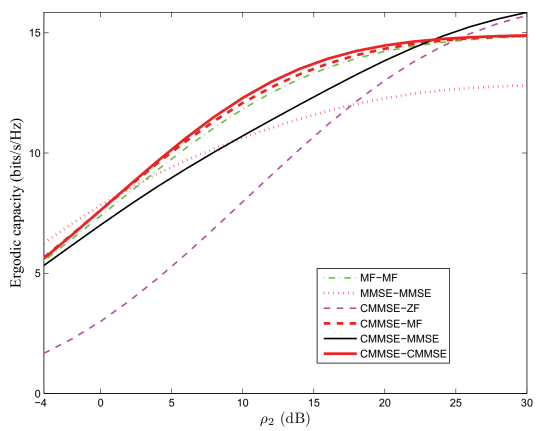

Fig. 7. Ergodic capacity for 1S-3R-1D ($\rho_1 = 15$ dB, $N_R = 4$).

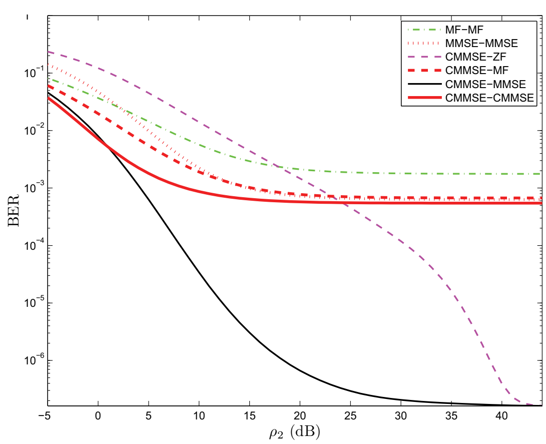

Fig. 8. Theoretical BER for 1S-3R-1D ($\rho_1 = 8$ dB, $N_R = 4$).

higher capacities than the CMMSE-ZF method proposed in [4] and [14], which is far from optimal in this sense. MF-MF is still attractive in that it is comparable in performance to CMMSE-CMMSE but do not need any CSI exchange. From a signal processing point of view, Fig. 8 shows CMMSE-MMSE has a much lower BER than CMMSE-CMMSE and other hybrid methods.

VII. Conclusion

For single-relay systems, SVD-based MMI and JMMSE methods perform best in the sense of capacity and BER, respectively. Although inferior to optimal methods, MMSE-MMSE is usually the best hybrid strategy if $\rho_1 \approx \rho_2$. SVD-based optimum methods perform unsatisfactorily when they are applied to the 1S-MR-1D channel without any adjustments. On the other hand, hybrid methods can provide distributed array gain without exchanging any CSI between relays. Introducing CSI exchange can further improve performance. The CMMSE-based relaying strategies proposed in this paper, such as CMMSE-CMMSE and CMMSE-MMSE exhibit better performance in terms of ergodic capacity or theoretical BER.

References

[1] O. Oyman and A. J. Paulraj, "Design and analysis of linear distributed MIMO relaying algorithms," *IEE Proc.-Communications*, vol. 153, pp. 565–572, Apr. 2006.

[2] H. Bolcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling law's in MIMO relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 1433–1444, Jun. 2006.

[3] A. Scaglione, D. L. Goeckel, and J. N. Laneman, "Cooperative communications in mobile *ad hoc* networks," *IEEE Signal Process. Mag.*, vol. 23, pp. 18–29, Sep. 2006.

[4] A. S. Behbahani, R. Merched, and A. M. Eltawil, "Optimizations of a MIMO relay network," *IEEE Trans. Signal Process.*, vol. 56, pp. 5062–73, Oct. 2008.

[5] H. Shi, T. Abe, T. Asai, and H. Yoshino, "A relaying scheme using QR decomposition with phase control for MIMO wireless networks," in *Proc. ICC*, vol. 4, 2005, pp. 2705–2711.

[6] P. Coronel and W. Schott, "Spatial multiplexing in the single-relay MIMO channel," in *Proc. IEEE Sarnoff Symp.*, Mar. 2006.

[7] O. Munoz, J. Vidal, and A. Agustin, "Non-regenerative MIMO relaying with channel state information," in *Proc. ICASSP*, 2005, pp. 361–364.

[8] X. J. Tang and Y. B. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1398–1407, Apr. 2007.

[9] W. Guan and H. Luo, "Joint MMSE transceiver design in non-regenerative MIMO relay systems," *IEEE Commun. Lett.*, vol. 12, pp. 517–519, Jul. 2008.

[10] C. Xing, S. Ma, and Y.-C. Wu, "Robust joint design of linear relay precoder and destination equalizer for dual-hop amplify-and-forward MIMO relay systems," *IEEE Trans. Signal Process.*, vol. 58, pp. 2273–2283, April 2010.

[11] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear nonregenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Process.*, vol. 57, pp. 4837–4851, Dec. 2009.

[12] C. Li, X. Wang, L. Yang, and W.-P. Zhu, "A joint source and relay power allocation scheme for a class of MIMO relay systems," *IEEE Trans. Signal Process.*, vol. 57, pp. 4852–4860, Dec. 2009.

[13] P. U. Sripathi and J. S. Lehnert, "A throughput scaling law for a class of wireless relay networks," in *Proc. of 38th Asilomar Conf. Signals, Systems and Computers*, vol. 2, 2004, pp. 1333–1337.

[14] W. Guan, H. Luo, and W. Chen, "Linear relaying scheme for MIMO relay system with QoS requirements," *IEEE Signal Process. Lett.*, vol. 15, pp. 697–700, 2008.

[15] I. Hammerstrom and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2798–2802, Aug. 2007.

[16] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, pp. 2381–2401, Sep. 2003.