Cooperative Spectrum Sensing Based on the Rao Test in Non-Gaussian Noise Environments

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***Abstract*—One of the key challenges in cognitive radio (CR) networks is to perform spectrum sensing in environments characterized by shadowing and fading effects as well as non-Gaussian noise distributions. Existing literature on spectrum sensing focuses mainly on the Gaussian noise model assumption, which does not properly characterize all the various noise types found in practical CR systems. This paper addresses the problem of spectrum sensing in the presence of non-Gaussian noise and interference for cognitive radio systems. A novel detector based on the Rao test is proposed for the detection of a primary user in the non-Gaussian noise environments described by the generalized Gaussian distribution (GGD). The test statistic of the proposed Rao detector is derived and its detection performance is analyzed and compared to that of the traditional energy detection. The Rao-based detection is then extended to a multi-user cooperative framework based on an improved decision fusion rule. It is shown through computer simulations that for a given probability of false alarm, the Rao detector can significantly enhance the spectrum sensing performance over conventional energy detection in non-Gaussian noise. Furthermore, the proposed cooperative detection scheme has a significantly higher global probability of detection than the non-cooperative scheme.**

I. INTRODUCTION

Cognitive radio (CR) has emerged as a key technology that can improve the spectrum utilization efficiency in next generation wireless networks through dynamic management and opportunistic use of radio resources. In this approach, a frequency band allocated to one or more high-priority, or so-called primary users (PU), can be accessed by other, secondary users (SU) provided that the PUs are temporally not using their spectrum or they can be adequately protected from the interference created by the SUs. Hence, the radio spectrum can be reused in an opportunistic manner or shared at all time, leading to increased capacity scaling in the network. Therefore it is very important to detect the absence ($\mathcal{H}_0$ = null hypothesis) or presence ($\mathcal{H}_1$ = alternative hypothesis) of a primary user (PU) in complicated noise environments for CR systems.

Several spectrum sensing methods have been proposed for single-user and cooperative detection under the white Gaussian noise (WGN) assumption, see e.g. [1], [2]. In practice, however, the problem is more challenging as we need to detect the various PU signals impaired by non-Gaussian noise and interference, as pointed out in [3]. Non-Gaussian noise impairments may include man-made impulsive noise, co-channel interference from other SUs, emission from microwave ovens, out of band spectral leakage, etc. [4], [5]. Furthermore, the performance of a spectrum detector optimized against Gaussian distribution noise may degrade drastically when non-Gaussian noise or interference signal are present because of the heavy tails characteristic of their probability density function (PDF) [6], [7]. In view of these problems, it is desirable to seek useful solutions to spectrum detection in practical non-Gaussian noises and to evaluate the detection performance.

Several standard models are currently available in the literature to fit non-Gaussian noise or interference distributions, such as the generalized Gaussian distribution (GGD) and the Gaussian mixture distribution (GMD). The GGD is a parametric family of distributions which can model both "heavier" and "lighter" than normal tails [8] through the selection of its shape parameter. In particular, it has been widely used to model man-made noise, impulsive phenomena [4], and certain types of ultra-wide band (UWB) interference [9].

Spectrum sensing for CR networks in the presence of non-Gaussian noise has been addressed by several researchers recently [10], [11]. However, the implementation of these detectors remains challenging as they require *a priori* knowledge of various side information by the CRs, such as the complex channel gain between the PU and the SU or the variance of the receiver noise at the SU, which may not be readily available in practice. To overcome this limitation, use of the generalized likelihood ratio test (GLRT) which combines unknown parameter estimation to the traditional likelihood ratio

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test, has been proposed in [12]. The GLRT is indeed an optimal detector, but it needs to perform the maximum likelihood estimation (MLE) of the received signal power under $\mathcal{H}_1$ and noise variance under $\mathcal{H}_0$ and, as such, it suffers from a large computational burden. The Rao test is an approximate form of the GLRT which only needs to estimate system model parameters under $\mathcal{H}_0$. Therefore, it has a simpler structure and can lower computational complexity than the GLRT [13], but its application to spectrum sensing has been limited to Gaussian noise [14]. Multi-user cooperation is a commonly used technique in spectrum sensing due to its overcoming the harmful effects of fading and shadowing by taking advantage of the spatial diversity. Although many recent works have explored the use of cooperation for improving the performance of spectrum sensing in the presence of Gaussian noise [15],[16], multi-user cooperation for spectrum sensing in the presence of non-Gaussian noise has not yet received much attention.

In this paper, we consider cooperative spectrum sensing for a CR sub-network comprised of one fusion center (FC) and multiple SUs, which together seek to detect the presence/absence of a PU over a given frequency band. Each SU employs a Rao detector to independently sense the PU signal in the presence of non-Gaussian noise characterized by a GGD. The local decisions of the SUs are then forwarded to the FC which finally provides a global decision based on this information. We analyze and derive the detection and false alarm probabilities of the proposed cooperative sensing scheme. Through numerical simulations, we show that the Rao detector can significantly enhance the local detection performance over conventional energy detection in non-Gaussian noise. Furthermore, for a given probability of false alarm, the proposed Rao-based cooperative spectrum sensing scheme has a significantly higher global probability of detection than the non-cooperative one.

The rest of the paper is organized as follows. The CR system and GGD noise models under consideration are presented in Section II. The local Rao-based detector used by the SUs is derived and analyzed in Section III, while the cooperative spectrum sensing scheme implemented at the FC is discussed in Section IV. Simulation results of the proposed schemes with comparison to traditional energy detection are provided in Section V. Conclusions are drawn in Section VI.

II. PROBLEM FORMULATION

In this section, we state the spectrum sensing problem in two steps, i.e., presentation of the CR system model followed by description of the non-Gaussian noise model.

A. System Model

We consider a CR sub-network comprised of M SUs and one FC. Each SU senses the presence of the PU signal over a limited time interval, through a wireless channel that is assumed to be frequency non-selective and time invariant. The local decisions from the SUs are forwarded to an FC where a final, or global decision is made. Within this general cooperative framework, spectrum sensing can be formulated as a binary hypothesis testing problem, with the null and alternative hypotheses respectively defined as $\mathcal{H}_0$: *PU absent* and $\mathcal{H}_1$: *PU present*. Under these two hypotheses, the baseband signal samples $z_m(n) \in \mathbb{C}$, $\mathbb{C}$ denotes the set of complex numbers, received by the m-th SU, where $m \in \{1, 2, \ldots, M\}$, at discrete-time $n \in \{1, 2, \ldots, N\}$, can be formulated as

$$\begin{cases} \mathcal{H}_0: & z_m(n) = w_m(n) \\ \mathcal{H}_1: & z_m(n) = u_m(n) + w_m(n) \end{cases} \tag{1}$$

where $w_m(n) \in \mathbb{C}$ is an additive background noise component present under both hypotheses and $u_m(n) \in \mathbb{C}$ is the PU signal component present only under $\mathcal{H}_1$. Considering the time-invariant, flat fading channel model, we can express the latter as $u_m(n) = h_m s(n)$ where $s(n) \in \mathbb{C}$ is the signal sample emitted by the PU at time n and $h_m \in \mathbb{C}$ is the channel gain between the PU's transmitter and the m-th SU's receiver.

Under both hypotheses, we model the noise sequence $w_m(n)$ as an independent and identically distributed (IID) random process, with zero-mean, variance σ_m^2 and circularly symmetric distribution, whose special form is further discussed below; the noise sequences observed by different SUs are mutually independent. The PU signal $s(n)$ is modeled as an IID process with zero-mean but otherwise arbitrary distribution; it is assumed to be independent of the noise processes $\{w_m(n)\}$. The channel coefficients h_m are assumed to be IID over the spatial index m, with zero-mean but arbitrary distribution; they are independent of the PU signal and SU noises.

In general, the SUs have no *a priori* knowledge about the emitted PU signal $s(n)$ nor the channel gains h_m, although they can extract relevant information about the noise $w_m(n)$ through measurement under $\mathcal{H}_0$ and local processing.

B. Non-Gaussian Noise Model

In this paper, we assume that the probability density function (PDF) of the additive background noise $w_m(n)$ is known up to a variance parameter $\sigma_{w_m}^2$, which will be estimated by the SUs as part of the proposed approach. Specifically, we consider the GGD model in the context of CR, which allows to control the degree of non-Gaussianity in the noise distribution efficiently through a shape factor. The PDF of the complex circularly symmetric GGD with zero-mean, variance $\sigma_{w_m}^2 > 0$ and shape factor $\beta > 0$, is obtained from [17] as

$$p(w_m(n);\beta,\sigma_{w_m}^2) = \frac{\beta^2}{[2B(\beta,\sigma_{w_m}^2/2)\Gamma(1/\beta)]^2} \exp\Big(-\frac{|w_m^{\Re}(n)|^\beta + |w_m^{\Im}(n)|^\beta}{B^\beta(\beta,\frac{\sigma_{w_m}^2}{2})}\Big) \tag{2}$$

where $w_m(n) \in \mathbb{C}$, $w_m^{\Re}(n) = \mathrm{Re}\{w_m(n)\}$ and $w_m^{\Im}(n) = \mathrm{Im}\{w_m(n)\}$ denote the real and imaginary parts of $w_m(n)$,

$$B(\beta,\sigma_{w_m}^2) = \sigma_{w_m}\Big(\frac{\Gamma(1/\beta)}{2\Gamma(3/\beta)}\Big)^{1/2} \tag{3}$$

is a scaling factor and $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$. It is easily seen that the GGD reduces to the Gaussian distribution for $\beta = 2$ and to the Laplacian distribution for $\beta = 1$. By varying β, different tail behaviours can be obtained: for $\beta > 2$, the tail decays faster than for the normal, while for $0 < \beta < 2$, it decays more slowly. The GGD with $0 < \beta < 2$ is therefore well suited to fit the "heavier" than normal tail behavior found in practical CR systems.

Then, spectrum sensing for CR applications in non-Gaussian noise must take into account these large magnitude noise samples with heavier-than-normal tail distribution, in order to improve the detection performance, e.g., increasing the probability of detection under a given probability of false alarm. To this end, a good detector for non-Gaussian noise typically utilizes nonlinearities or clippers to reduce the noise spikes, as will be seen in below for the proposed Rao detector.

III. Rao Detector for Local Spectrum Sensing

In this section, we propose a nonlinear detector based on the Rao test which will allow the SUs to make a preliminary, local decision on the channel occupancy by the PU. The derivation is carried on for a selected SU, say the one with index m.

Referring to the system model equation (1), we begin by introducing some necessary definitions and notations for convenience in analysis. We define $u_m^{\Re}(n) = \mathrm{Re}\{u_m(n)\}$, $u_m^{\Im}(n) = \mathrm{Im}\{u_m(n)\}$, $z_m^{\Re}(n) = Re\{z_m(n)\}$ and $z_m^{\Im}(n) = Im\{z_m(n)\}$. The complete vector of signal samples observed by the SU is denoted as $\boldsymbol{z}_m = \big[z_m(1),\ldots,z_m(N)\big]^T$. Adopting the notations from [18], we define the parameter vector

$$\boldsymbol{\theta}_r = \big[u_m^{\Re}(1),\ldots,u_m^{\Re}(N),u_m^{\Im}(1),\ldots,u_m^{\Im}(N)\big]^T \tag{4}$$

which contains the real and imaginary parts of the PU signal samples. We also let $\theta_s = \sigma_{w_m}^2$ denote the nuisance parameter for the detection problem at hand. Finally, we define $\boldsymbol{\theta} = [\boldsymbol{\theta}_r^T\ \theta_s]^T$, which is a $(2N+1)$-dimensional real vector.

The Rao test is asymptotically equivalent to the GLRT, yet it does not require the MLE of the unknown parameters under $\mathcal{H}_1$ and is computationally simpler [18]. In order to formulate the Rao test, we first recast the detection model (1) in the following equivalent form:

$$\begin{cases} \mathcal{H}_0: & \boldsymbol{\theta}_r = \mathbf{0}, \quad \theta_s > 0 \\ \mathcal{H}_1: & \boldsymbol{\theta}_r \neq \mathbf{0}, \quad \theta_s > 0 \end{cases} \tag{5}$$

Within this framework, the Rao test statistic $T(\boldsymbol{z}_m)$ at the m-th SU for composite binary parameter test can be expressed as

$$T(\boldsymbol{z}_m) = \nabla \ln p(\boldsymbol{z}_m;\hat{\boldsymbol{\theta}}_0)^T \big[I^{-1}(\hat{\boldsymbol{\theta}}_0)\big]_{rr} \nabla \ln p(\boldsymbol{z}_m;\hat{\boldsymbol{\theta}}_0) \tag{6}$$

where $p(\boldsymbol{z}_m;\boldsymbol{\theta})$ is the PDF of the received complex-valued observation vector $\boldsymbol{z}_m$ under $\mathcal{H}_1$, ∇ denotes the gradient operator with respect to the entries of vector $\boldsymbol{\theta}_r$, defined as

$$\nabla = \Big[\frac{\partial}{\partial u_m^{\Re}(1)},\ldots,\frac{\partial}{\partial u_m^{\Re}(N)},\frac{\partial}{\partial u_m^{\Im}(1)},\ldots,\frac{\partial}{\partial u_m^{\Im}(N)}\Big]^T, \tag{7}$$

$\hat{\boldsymbol{\theta}}_0 = [\hat{\boldsymbol{\theta}}_{r0}^T\ \hat{\theta}_{s0}]^T$ is the MLE of $\boldsymbol{\theta}$ under $\mathcal{H}_0$, $\boldsymbol{I}(\boldsymbol{\theta})$ is the $(2N+1)\times(2N+1)$ Fisher information matrix (FIM) [19] associated to the PDF $p(\boldsymbol{z}_m;\boldsymbol{\theta})$, and $[\boldsymbol{I}^{-1}(\hat{\boldsymbol{\theta}}_0)]_{rr}$ is the $2N\times 2N$ matrix obtained as the upper-left block partition of the inverse FIM $\boldsymbol{I}^{-1}(\boldsymbol{\theta})$ under $\mathcal{H}_0$, i.e. when evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}_0$.

According to the system model defined in Section II, the PDF of the received signal vector $\boldsymbol{z}_m$, with IID samples, can be expressed as

$$p(\boldsymbol{z}_m;\boldsymbol{\theta}) = \prod_{n=1}^{N}\frac{\beta^2}{[2B(\beta,\frac{\sigma_{w_m}^2}{2})\Gamma(1/\beta)]^2} \exp\{-\frac{|z_m^{\Re}(n)-u_m^{\Re}(n)|^\beta + |z_m^{\Im}(n)-u_I^{\Im}(n)|^\beta}{B^\beta(\beta,\frac{\sigma_{w_m}^2}{2})}\}. \tag{8}$$

Taking the natural logarithm of (8), we obtain

$$\ln p(\boldsymbol{z}_m;\boldsymbol{\theta}) = 2N\ln\frac{\beta}{[2B(\beta,\frac{\sigma_{w_m}^2}{2})\Gamma(1/\beta)]} - \frac{\sum_{n=1}^{N}(|z_m^{\Re}(n)-u_m^{\Re}(n)|^\beta + |z_m^{\Im}(n)-u_m^{\Im}(n)|^\beta)}{B^\beta(\beta,\frac{\sigma_{w_m}^2}{2})} \tag{9}$$

From (5), it follows that the MLE of $\boldsymbol{\theta}_r$ under $\mathcal{H}_0$ is simply $\hat{\boldsymbol{\theta}}_{r0} = \mathbf{0}$. The MLE of $\theta_s = \sigma_{w_m}^2$ under $\mathcal{H}_0$ is found by computing the derivative of (9) with respect to $\sigma_{w_m}^2$, assuming $\boldsymbol{\theta}_r = \mathbf{0}$, and setting the result to zero. This yields

$$\hat{\theta}_{s0} = \hat{\sigma}_{w_m}^2 = \Big(\frac{\beta(\frac{2\Gamma(3/\beta)}{\Gamma(1/\beta)})^{\beta/2}}{2N}\sum_{n=1}^{N}(|z_m^{\Re}(n)|^\beta + |z_m^{\Im}(n)|^\beta)\Big)^{\frac{2}{\beta}} \tag{10}$$

The gradient of (9) with respect to $\boldsymbol{\theta}_r$, as defined in (7), can be expressed as

$$\nabla \ln p(\boldsymbol{z}_m;\boldsymbol{\theta}) = [\boldsymbol{\nu}^{\Re}(\boldsymbol{z}_m;\boldsymbol{\theta}), \boldsymbol{\nu}^{\Im}(\boldsymbol{z}_m;\boldsymbol{\theta})]^T \tag{11}$$

where $\boldsymbol{\nu}^{\Re}(\boldsymbol{z}_m;\boldsymbol{\theta}) = [\nu_1^{\Re}, \dots, \nu_N^{\Re}]$ and $\boldsymbol{\nu}^{\Im}(\boldsymbol{z}_m;\boldsymbol{\theta}) = [\nu_1^{\Im}, \dots, \nu_N^{\Im}]$. In turn, the entries of these vectors are defined as

$$\nu_n^{\Re} = \frac{\beta|z_m^{\Re}(n) - u_m^{\Re}(n)|^{\beta-1}\text{sgn}(u_m^{\Re}(n) - z_m^{\Re}(n))}{B^{\beta}(\beta, \frac{\sigma_{w_m}^2}{2})} \tag{12}$$

$$\nu_n^{\Im} = \frac{\beta|z_m^{\Im}(n) - u_m^{\Im}(n)|^{\beta-1}\text{sgn}(u_m^{\Im}(n) - z_m^{\Im}(n))}{B^{\beta}(\beta, \frac{\sigma_{w_m}^2}{2})} \tag{13}$$

where $\text{sgn}(x)$ is 1 if $x > 0$ and -1 if $x \le 0$.

We now proceed to calculate the submatrix $[\boldsymbol{I}^{-1}(\boldsymbol{\theta})]_{rr}$, which appears in (6). Here, it is convenient to partition the FIM $\boldsymbol{I}(\boldsymbol{\theta})$ as [19]

$$\boldsymbol{I}(\boldsymbol{\theta}) = \begin{bmatrix} \boldsymbol{I}_{rr}(\boldsymbol{\theta}) & \boldsymbol{I}_{rs}(\boldsymbol{\theta}) \\ \boldsymbol{I}_{sr}(\boldsymbol{\theta}) & \boldsymbol{I}_{ss}(\boldsymbol{\theta}) \end{bmatrix}, \tag{14}$$

where the upper left block $\boldsymbol{I}_{rr}(\boldsymbol{\theta})$ has dimension $2N \times 2N$. Using the definition of the FIM, we find

$$\begin{aligned} \boldsymbol{I}_{rr}(\hat{\boldsymbol{\theta}}_0) &= -E\big[\nabla[\nabla \ln p(\boldsymbol{z}_m;\hat{\boldsymbol{\theta}}_0)]^T\big] \\ &= \frac{2\beta(\beta-1)\Gamma(1-1/\beta)\Gamma(3/\beta)}{\hat{\sigma}_{w_m}^2\Gamma^2(1/\beta)}\boldsymbol{\mathcal{I}}_{2N} \end{aligned} \tag{15}$$

When the MLE of $\sigma_{w_m}^2$ is sufficiently accurate, we can show that

$$\boldsymbol{I}_{rs}(\hat{\boldsymbol{\theta}}_0) \approx \mathbf{0}_{2N,1} \tag{16}$$

where $\mathbf{0}_{2N,1}$ is a $2N \times 1$ zero vector. Next, applying a well-known inversion formula for block partitioned matrices [20], the $2N \times 2N$ upper left block of the inverse FIM can be expressed as

$$[\boldsymbol{I}^{-1}(\boldsymbol{\theta})]_{rr} = \big[\boldsymbol{I}_{rr}(\boldsymbol{\theta}) - \boldsymbol{I}_{rs}(\boldsymbol{\theta})\boldsymbol{I}_{ss}^{-1}(\boldsymbol{\theta})\boldsymbol{I}_{sr}(\boldsymbol{\theta})\big]^{-1}. \tag{17}$$

Consequently, using (15) and (16) in (17), we have

$$[\boldsymbol{I}^{-1}(\hat{\boldsymbol{\theta}}_0)]_{rr} = \boldsymbol{I}_{rr}^{-1}(\hat{\boldsymbol{\theta}}_0) \tag{18}$$

Finally, by substituting (10), (11) and (18) into (6), we obtain the Rao detection statistic, i.e.:

$$T(\boldsymbol{z}_m) = \phi(\beta)\sum_{n=1}^{N}\big[|z_m^{\Re}(n)|^{2(\beta-1)} + |z_m^{\Im}(n)|^{2(\beta-1)}\big] \tag{19}$$

where $\phi(\beta)$ is a scaling factor defined as

$$\phi(\beta) = \frac{\beta\Gamma(\frac{3}{\beta})^{\beta-1}}{(\beta-1)(\frac{\hat{\sigma}_{w_m}^2}{2})^{\beta-1}\Gamma(\frac{1}{\beta})^{\beta-2}\Gamma(1-\frac{1}{\beta})} \tag{20}$$

From (19), we can see the statistic of Rao detector is the function of β and sample values. So for the GGD noise with a given β, our proposed detector does not require any *a priori* knowledge of PU signal, channel gain and the variance of noise. Accordingly, the Rao detector gives a binary decision y_m for the m-th SU as

$$y_m = \begin{cases} 1, & T(\boldsymbol{z}_m) \ge \gamma_m \\ 0, & T(\boldsymbol{z}_m) < \gamma_m \end{cases} \tag{21}$$

where γ_m is a threshold, usually pre-determined according to the desired probability of false alarm requirement for the m-th SU.

IV. Cooperative spectrum sensing

Each cognitive user needs to conduct the MLE of $\sigma_{w_m}^2$ and the Rao detection locally, yielding local one-bit hard decision result y_m and the corresponding SNR_m. Let the decisions and SNRs of all the M SUs be denoted as $\mathbf{y} = [y_1, y_2, ..., y_M]$ and $\mathbf{SNR} = [\text{SNR}_1, \text{SNR}_2, ..., \text{SNR}_M]$, which will be input to the FC. The FC chooses the SUs whose SNRs are above the average SNR value of all the SUs to make a global decision. Suppose a total of M' SUs are selected. Then, the global decision to be made by the FC is given as

$$T_{FC}(y) = \sum_{m=1}^{M'} y_m \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{FC} \tag{22}$$

The threshold γ_{FC} in (22) may be set to two extreme values:1 (OR rule) and M' (AND rule). With M' SUs participating in the cooperation based on the OR rule, the cooperative probability of detection and that of false alarm are, respectively, given by

$$P_{\text{d,OR}} = 1 - \prod_{m=1}^{M'}(1 - P_{\text{d},m}), \quad P_{\text{fa,OR}} = 1 - \prod_{m=1}^{M'}(1 - P_{\text{fa},m}), \tag{23}$$

where$P_{\text{d},m}$ and $P_{\text{fa},m}$ denote the probability of detection and the probability of false alarm of the m-th SU with Rao detector. Similarly, when the AND rule is employed, we obtain the cooperative probability of detection and that of false alarm as given below,

$$P_{\text{fa,AND}} = \prod_{m=1}^{M'} P_{\text{fa},m}, \quad P_{\text{d,AND}} = \prod_{m=1}^{M'} P_{\text{d},m}. \tag{24}$$

V. Simulation results

In this section, simulation results are provided to illustrate the performances of the proposed detector and the cooperative scheme in different situations.

A. Generation of the GGD noise

Let F_X be the cumulative distribution function (CDF) of a random variable X and F_X^{-1} be its inverse. If F_X^{-1} has a closed-form expression, such as in the case of Laplacian distribution, we can obtain a large number of realizations of X as given by $x_i = F_X^{-1}(g_i)$, where $g_i(i = 1, 2, ..., n)$ are random numbers uniformly distributed over $[0, 1]$. However, generating the samples of a general GGD is complicated since the inverse CDF may not exist. Here, we use the three-step method [21] for the generation of the samples of GGD with $0 < \beta < 2$.

B. Energy Detection

Assume that the primary user signal $s(n)$ is a zero-mean white Gaussian random variable, and the noise is a zero-mean WGN or GGD noise. The receiver operation characteristics (ROC) are computed based on 5000 Monte Carlo runs and the sample size is set to $N = 1000$. Fig. 1 shows the energy detection performances for Gaussian noise and GGD noise at SNR $= -10$dB. When β decreases, the degree of non-Gaussianity of the GGD noise increases. Clearly, the performance under GGD noise is worse than that under WGN, and the detection performance gets worse with increasing the degree of non-Gaussianity of the GGD noise.

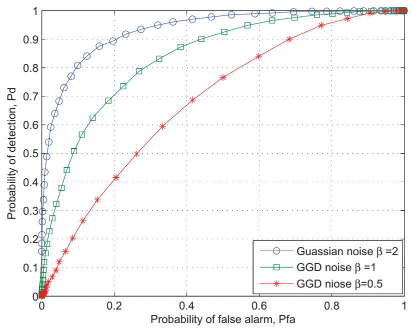

Fig. 1. ROC of energy detector for WGN and GGD noise

C. Rao Detection

Assume that the primary user is a PSK signal, $s(n) = \cos\phi(n) + j\sin\phi(n)$, $\phi(n) \in [0, 2\pi)$, and the noise is GGD with $\beta = 1.1$. Simulations are carried out with $N = 1000$ and $M = 1$. The performance of the proposed detector under SNR $= -15$dB is shown in Fig.2 with comparison to that of the energy detection with the same degree of non-Gaussianity. It is seen that when $P_{\text{fa}} = 0.1$, the probability of detection of our detector is 70%, but that of the energy detector is 28% only, which fails to meet the requirement of spectrum sensing. Again, Fig.3 shows that our proposed detector has a much better detection performance than the energy detector under almost all levels of SNR. In other words, with the same signal and probability of detection, the proposed detector has gained almost a 5dB SNR when $P_{\text{fa}} = 0.1$ and $\beta = 1.1$.

D. Cooperative Detection

Assuming the number of SUs is $M = 4$, we consider the SUs have the same degrees of non-Gaussianity with $\beta = 1.1$. The corresponding SNRs are assumed as

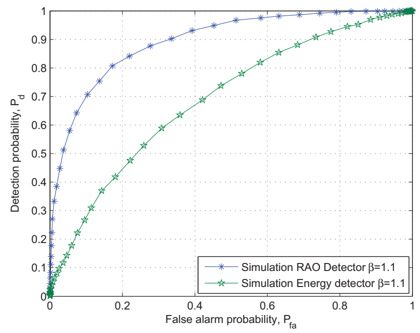

Fig. 2. ROC of Rao and Energy detectors for GGD (SNR $= -15$dB)

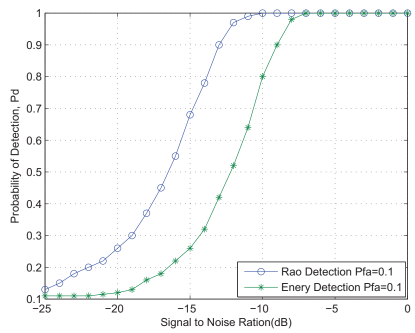

Fig. 3. Probability of detection versus SNR ($\beta = 1.1$)

-30dB, -15dB, -13dB, and -12dB. According to the proposed cooperative scheme in Section IV, the 1st SU will not be selected, so $M' = 3$. Simulations are carried out with $N = 1000$ and 5000 Monte Carlo runs. Fig.4 shows the ROC curves of the cooperative detection as compared with single SU local detection and the traditional AND and OR rules. The probability of detection of the proposed OR rule is the same as that resulting from the traditional OR rule in this case, but the probability of detection of the proposed AND rule is higher than that of the traditional AND rule. In particular, for $P_{\text{fa}} = 0.1$, P_{d} with our proposed AND rule is increased from 10% (the min P_{d} of the four SUs) to 95%, but with the traditional AND rule, P_{d} is only increased to 53%.

Assume that the number of observations is $N = 1000$, the number of SUs is $M = 4$. We consider four different degrees of non-Gaussianity, corresponding to $\beta_1 = 2$, $\beta_2 = 1.8$,$\beta_3 = 1.5$ and $\beta_4 = 1.1$, for the SUs, respectively. The SNRs are assumed as $\text{SNR}_1 = -30$dB, $\text{SNR}_2 = \text{SNR}_3 = \text{SNR}_4 = -15$dB. It can be seen from Fig. 1 and Fig.4 that as the degree of the non-Gaussianity increases, the performance of the energy detector decreases while the performance of the proposed Rao detector is greatly improved. Fig.4 shows that the

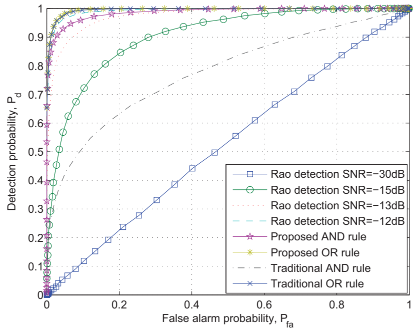

Fig. 4. ROC of cooperative detection with same β

proposed OR rule results in the best performance, by improving the probability of detection from 10% with $\beta = 2$ to 70% as the global performance when $P_{\text{fa}} = 0.1$.

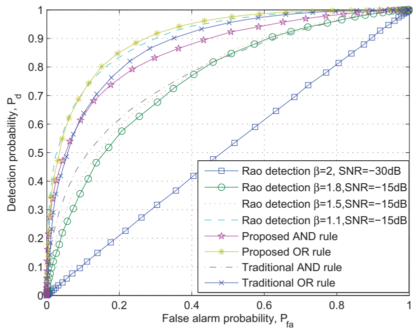

Fig. 5. ROC of cooperative detection with different β

VI. Conclusion

We have studied the Rao detector based spectrum sensing for cognitive radio in non-Gaussian noise environment that is modeled by GGD. We have focused on a scenario where the PU signal, the fading channel gain and the noise variance are unknown to the CR users. A new cooperative detection scheme for spectrum sensing in the non-Gaussian noise has been derived via the Rao detector and the decision fusion. Simulation results have shown that the proposed Rao detector yields large performance gains over the traditional energy detector, and the proposed cooperative scheme exhibits a much better detection performance than the traditional cooperative scheme.

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