A Fast RLS-CM Algorithm for Blind Adaptive Beamforming

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ABSTRACT

An approximation for the constant modulus (CM) cost function is proposed to allow the use of the fast recursive least squares (RLS) algorithm. Simulations are performed to compare the performance of the introduced RLS-CM and stochastic gradient descent (SGD) algorithms for blind adaptive beamforming. Results indicate that the introduced RLS-CM has faster convergence speed and good tracking ability.

KEY WORDS: Beamforming, Constant Modulus (CM), Recursive Least Squares (RLS), Adaptive Array.

1. INTRODUCTION

The problem of detecting and extracting communications signals from dense interference environments is important to the wireless communications systems. Adaptive beamforming techniques provide a potential solution to this problem, by forming high gain beams in the direction of arrival (DOA) of the signals of interest (SOI) and, ideally, direct nulls in the directions of arrivals of the interferences.

Conventional beamforming techniques make use of a known training sequence. However, this consumes a large amount of spectrum resource, especially when the users move fast or the channel variation is severe. To address this issue, various blind beamforming techniques [1-10], which can separate multiple cochannel signals that impinge on the antenna array from unknown source location, were proposed. There are two kinds of blind beamforming techniques based on the temporal features of the SOI.

One kind uses the cyclostationary properties of communication signals [4-7]. A cyclostationary signal has the statistical property of correlating with either a frequency-shifted or a complex conjugate version of itself. These beamforming techniques can suppress not only Gaussian but also non-Gaussian interferences by utilizing the signal cyclostationary properties. However, the cyclic correlation matrix in the cyclostationary beamformer leads to intensive computation. These algorithms also suffer from severe performance degradation even if there is a small mismatch in the cycle frequency of the desired signal.

The other kind of widely used approaches is based on the constant modulus (CM) beamforming [1-3, 8-10]. A CM beamforming technique exploits the low modulus fluctuation exhibited by most communication signals to extract them from the array output. This beamformer is based on minimizing the MSE between the array output after a modulus nonlinearity and a fixed real number. The cost function of the CM algorithm is:

$$J(p,q) = E\left[\left(\left|y(n)\right|^{p} - 1\right)^{q}\right] \tag{1}$$

where $E[\cdot]$ denotes statistical expectation and $y(n)$ denotes the array output.

Consequently, the CM beamformer does not require a special array geometry or knowledge of the array manifold or noise covariance matrix to adapt the array weights. In general, the CM cost function has a non-quadratic order and is solved via a stochastic gradient descent (SGD) approach. It is well known that stochastic gradient descent method is quite sensitive to the selected step size and has a slow convergence speed. To overcome this limitation, Agee [3] proposed a least squares CMA (LSCMA) for the $J(1,2)$ case, which is a block-update iterative algorithm.

In this paper, we introduce an approximation into the CM cost function $J(2,2)$ that enables the use of the rapidly converging RLS algorithm for the array weight adaptation. The introduced RLS-CM algorithm will be described in detail. Its convergence and tracking ability will be investigated by simulations for different cases.

2. SIGNAL & SYSTEM MODELING

Array signal processing involves the manipulation of signals induced onto the elements of an array. A constant modulus array, with adjustable element weights, is shown in Figure 1.

The signals from each element $x_l(n)$, $1 \leq l \leq L$ are scaled by a complex weight $w_l(n)$, $1 \leq l \leq L$, and summed to form the array output $y(n)$. From the diagram in Figure 1, an expression for the array output can be given by

$$y(n) = \sum_{l=1}^{L} w_l^* x_l(n) \tag{2}$$

where * denotes the complex conjugate. Using vector format to denote the beamformer weights and the signals induced on the antenna elements, i.e.

$$\mathbf{w} = [w_1, w_2, \cdots, w_L]^T \tag{3}$$

$$\mathbf{x}(n) = [x_1(n), x_2(n), \cdots, x_L(n)]^T \tag{4}$$

the output of the beamformer becomes

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \tag{5}$$

where the superscripts *T* and *H* stand for the transpose and complex conjugate transpose, respectively. The objective of an adaptive array is to extract the desired signal by finding the weight vector **w** according to a particular criterion.

Under the assumption that the transmitted signal has a constant envelope, the array output should have constant envelope too. However, the multipath fading and interference can cause amplitude fluctuations in the received signals. The objective of CMA is therefore to restore the array output to a constant envelope signal on average. This is accomplished by adjusting the weight vector w to minimize the cost function *J* as defined by (1). For simplification, a simple SGD algorithm is generally employed to minimize the cost function *J(p,q)*. When $p = q = 2$, using complex matrix calculus and replacing the statistical expectation with an instantaneous value, a recursive update equation is obtained [1]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu(|y(n)|^2 - 1)y^*(n)\mathbf{x}(n) \tag{6}$$

where μ is the step size.

3. THE RLS-CM ALGORITHM

In the above SGD method (6), the step size μ should be carefully selected. A small step size will lead to slow convergence speed while a large step size will result in the adapted weight oscillation.

It is well known that RLS algorithm has a fast convergence. However, when p=q=2, the cost function (1) is non-quadratic in the array weights and cannot be solved by the RLS algorithm. In the following, we will introduce an approximation to derive a new cost function that enables the use of the RLS.

Replacing the time average operation in (1) with the exponentially weighted sum yields:

$$J = \sum_{n=1}^{t} \lambda^{t-n} \left(\left| \mathbf{w}^H(t)\mathbf{x}(n) \right|^2 - 1 \right)^2 \tag{7}$$

where λ is the forgetting factor and $0 \le \lambda \le 1$. Rewriting (7) as

$$J = \sum_{n=1}^{t} \lambda^{t-n} \left(\mathbf{w}^H(t)\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(t) - 1 \right)^2 \tag{8}$$

and replacing $\mathbf{x}^H(n)\mathbf{w}(t)$ by $\mathbf{x}^H(n)\mathbf{w}(n-1)$, we obtain the modified cost function:

$$J' = \sum_{n=1}^{t} \lambda^{t-n} \left(\mathbf{w}^H(t)\left(\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(n-1)\right) - 1 \right)^2 \tag{9}$$

The main advantage of (9) over (8) is that the former is now quadratic in the array weights; the weight vector $\mathbf{w}(n\text{-}1)$ in (9) is available from previous iterations and can be computed for $1 \le n \le t$ at the time instant t. For stationary or slowly varying signals, the difference between $\mathbf{x}^H(n)\mathbf{w}(t)$ and $\mathbf{x}^H(n)\mathbf{w}(n-1)$ is small, and the above approximation is justified. Defining $\mathbf{z}(n) = \mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(n-1)$, (9) can be re-written as:

$$J' = \sum_{n=1}^{t} \lambda^{t-n} \left(\mathbf{w}^H(t)\mathbf{z}(n) - 1 \right)^2 \tag{10}$$

The above modified cost function J' is an approximate of the original cost function J, and can be solved easily for $\mathbf{w}(t)$ by using RLS fast algorithm [11]. The resulting algorithm, called RLS-CM, is described in Table 1.

Table 1. RLS-CM algorithm

Initialization	$\mathbf{w}(0)=[1,\mathbf{0}_{1\times L}]^T$, $\mathbf{P}(0)=\delta\mathbf{I}$, δ = small positive constant
Approximation and RLS update (For each iteration n=1,2,…)	$\mathbf{z}(n)=\mathbf{x}(n)\mathbf{x}^H(n)\mathbf{w}(n\text{-}1)$ $\mathbf{h}(n)=\mathbf{z}^H(n)\times\mathbf{P}(n\text{-}1)$ $\mathbf{g}(n)=\mathbf{P}(n\text{-}1)\times\mathbf{z}(n)/(\lambda+\mathbf{h}(n)\times\mathbf{z}(n))$ $\mathbf{P}(n)=(\mathbf{P}(n\text{-}1)-\mathbf{g}(n)\times\mathbf{h}(n))/\lambda$ $e(n)=1-\mathbf{w}^H(n\text{-}1)\times\mathbf{z}(n)$ $\mathbf{w}^H(n)=\mathbf{w}^H(n\text{-}1)+\mathbf{g}(n)\times e^*(n)$

4. ILLUSTRATIVE SIMULATION RESULTS

A 10-element uniform linear array is employed. The distance between two adjacent sensors is half of the carrier wavelength of the desired signals. Initially all components of the weight vector are set to zero except one.

The performance of the beamformer is measured by the output Signal-to-Interference-Noise-Ratio(SINR), defined as

$$SINR_i = \frac{\mathbf{w}^H \mathbf{a}_i p_{s_i s_i} \mathbf{a}_i^H \mathbf{w}}{\mathbf{w}^H \mathbf{R}_I \mathbf{w}} \tag{11}$$

where $p_{s_i s_i}$ is the true power of the *i*-th source, and $\mathbf{R}_I$ is the true autocorrelation matrix of the interference (noise and other signals) in the environment.

Example 1: To compare the tracking ability of the proposed RLS-CM algorithm and the SGD algorithm, we abruptly change the number of sources at iteration 5000. In all simulations, we assume that the sources have a normalized variance of 1 and the noise power is $\sigma_n^2 = 0.1$. The phase $\psi_i(n)$, $i \le M$, of each source $s_i(n) = e^{j\psi_i(n)}$ is independently and uniformly distributed over $[-\pi, \pi]$, where M is the number of sources. In the first 5000 iterations, there are 2 sources with directions of arrival (DOA) θ_1 = 10° and θ_2 = -30°; in the subsequent 5000 iterations, there are two additional sources impinging onto the array with DOA θ_3 = -45° and θ_4 = -25°.

In Fig. 2, the forgetting factor of the RLS-CM algorithm is chosen as λ = 0.99, and the step size of SGD algorithm is set to μ = 0.005 and μ = 0.009. In the initial convergence phase, the RLS-CM algorithm has a much faster convergence speed than that of the SGD algorithm. The SGD algorithm exhibits a two-stage convergence behavior. In the first stage, it quickly approaches a certain error level. However, in the second stage, it takes a much longer time to converge to the minimum MSE. By increasing the step size of the SGD, the first stage gets shorter but the misadjustment becomes larger. After a sudden change, i.e. two sources added at iteration 5000, both the RLS-CM and the SGD algorithms can quickly track the change. However, the SGD has a lower steady-state SINR than the proposed RLS-CM algorithm. This lower SINR becomes more pronounced as the SGD step size increases due to the corresponding increase in misadjustment. Figures 3(a) and 3(b) show the beam patterns of the proposed RLS-CM and SGD algorithms (μ = 0.005) at iterations 5000 and 10,000, respectively. The results indicate that the proposed RLS-CM algorithm provides deeper nulls in the directions of the other interfering sources.

Example 2: In this example, initially, there are two sources with θ_1 = 10° and θ_2 = -30°. However, at iteration 5000, the situation is changed to four different sources with θ_1 = -5°, θ_2 = 20°, θ_3 = 45° and θ_4 = -25°. The forgetting factor in the RLS-CM is still λ = 0.99 and the SGD step size is μ = 0.005. The other parameters are the same as in Example 1. The results are shown in Figures 4 and 5. Because the source configuration is totally changed, both algorithms will take longer time to converge (see Fig. 4). The results again indicate that the proposed RLS-CM algorithm has a much faster tracking ability (Fig. 4), and provides deeper nulls (Fig. 5), than the SGD approach.

5. CONCLUSION

This paper proposed an approximation to the CM cost function and the corresponding RLS-CM algorithm for blind-adaptive beamforming. Simulations were performed to investigate its convergence behavior and tracking ability. The results indicate that the proposed RLS-CM algorithm has good initial convergence behavior and tracking ability under sudden change conditions, better than the SGD.

REFERENCES

[1] R.P. Gooch and J.D. Lundell, The CM array: an adaptive beamformer for constant modulus signals, *Proc. ICASSP'86*, Tokyo, Japan, Apr., 1986, 2523-2526

[2] J.J. Shynk, R.P. Gooch, The constant modulus array for cochannel signal copy and direction finding, *IEEE Trans. on Signal Processing*, *44*(3), Mar., 1996, 652-660

[3] B.G. Agee, Blind separation and capture of communication signals using a multitarget constant modulus beamformer, *IEEE Military Communications Conference*, 1989, 340-346

[4] B.G. Agee, S.V. Schell, and W.A. Gardner, Spectral self-coherence restoral: A new approach to blind adaptive signal extraction using antenna arrays, *Proc. IEEE*, *78*(4), 1990, 753-767

[5] Q. Wu, K.M. Wong, Blind adaptive beamforming for cyclostationary signals, *IEEE Trans. on Signal Processing*, *44*(11), 1996, 2757-2767

[6] L. Castedo, A.R. Figueiras-Vidal, An adaptive beamforming technique based on cyclostationary signal properties, *IEEE Trans. on Signal Processing, 43*(7), 1995, 1637-1650

[7] J.H. Lee, Y.T. Lee, W.H. Shih, Efficient robust adaptive beamforming for cyclostationary signals, *IEEE Trans. on Signal Processing, 48*(7), 2000, 1893-1901

[8] A. Mathur, A.V. Keerthi, and J.J. Shynk, A variable step-size CM array algorithm for fast fading channels, *IEEE Trans. on Signal Processing, 45*(4), 1997, 1083-1087

[9] H. Furukawa, Y. Kamio, and H. Sasaoka, Cochannel interference reduction and path-diversity reception technique using CMA adaptive array antenna in digital land mobile communications, *IEEE Trans. on Vehicular Technology, 50*(2), 2001, 605-616

[10] J.J. Shynk, A.V. Keethi, and A. Mathur, Steady-state analysis of the multistage constant modulus array, *IEEE Trans. on signal processing, 44*(4), 1996, 948-962

[11] S. Haykin, *Adaptive Filter Theory*, (3rd Edition, Printice-Hall Inc., 1996)

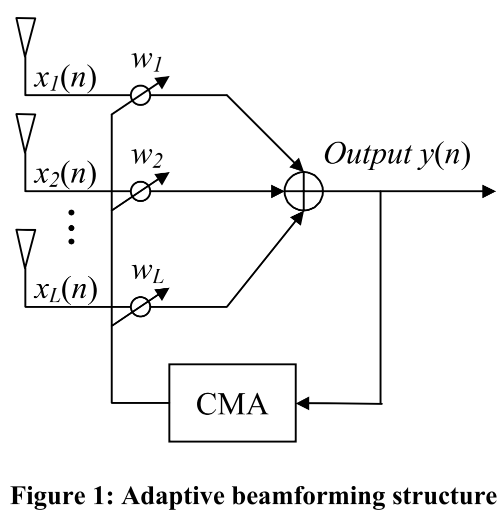


Figure 1: Adaptive beamforming structure

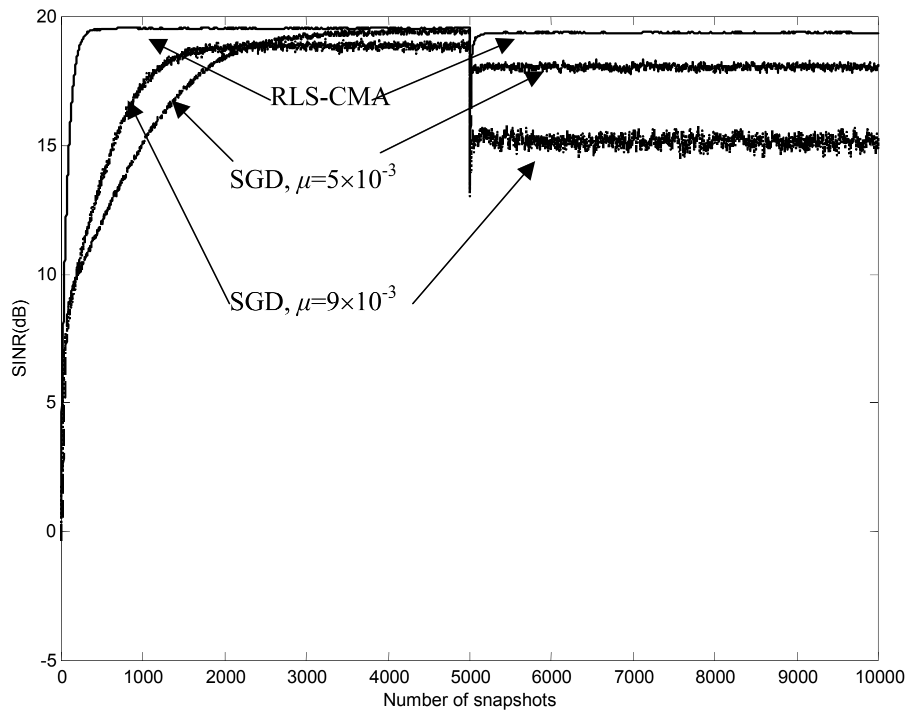


Figure 2: SINR versus iteration number for RLS-CM and SGD in Example 1

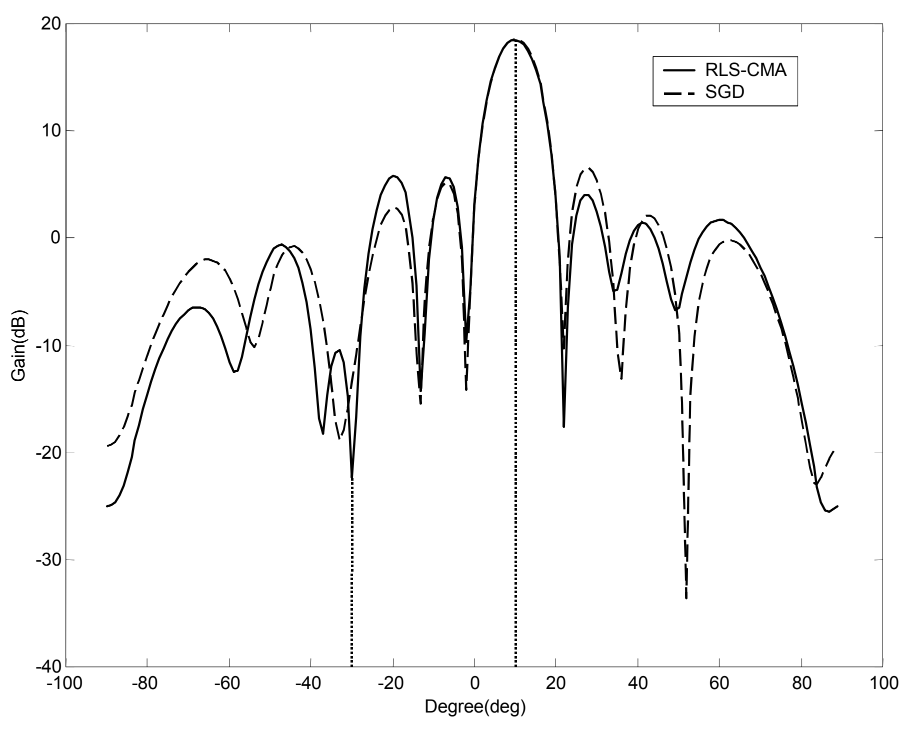


Figure 3 (a): Beam patterns obtained with the RLS-CM and SGD in Example 1 at iteration 5000.

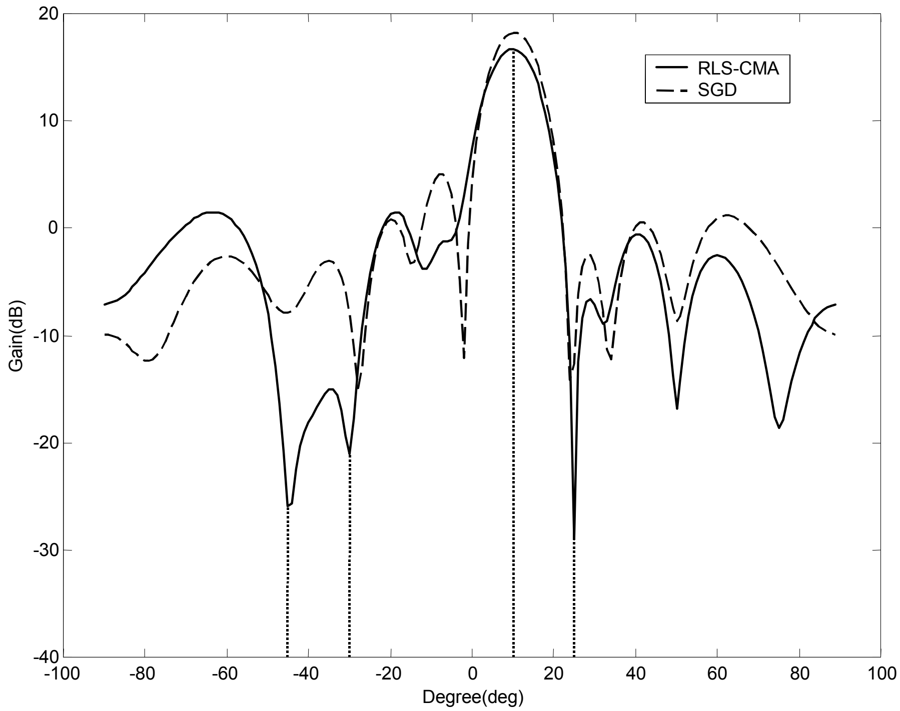


Figure 3 (b): Beam patterns obtained with the RLS-CM and SGD in Example 1 at iteration 10,000.

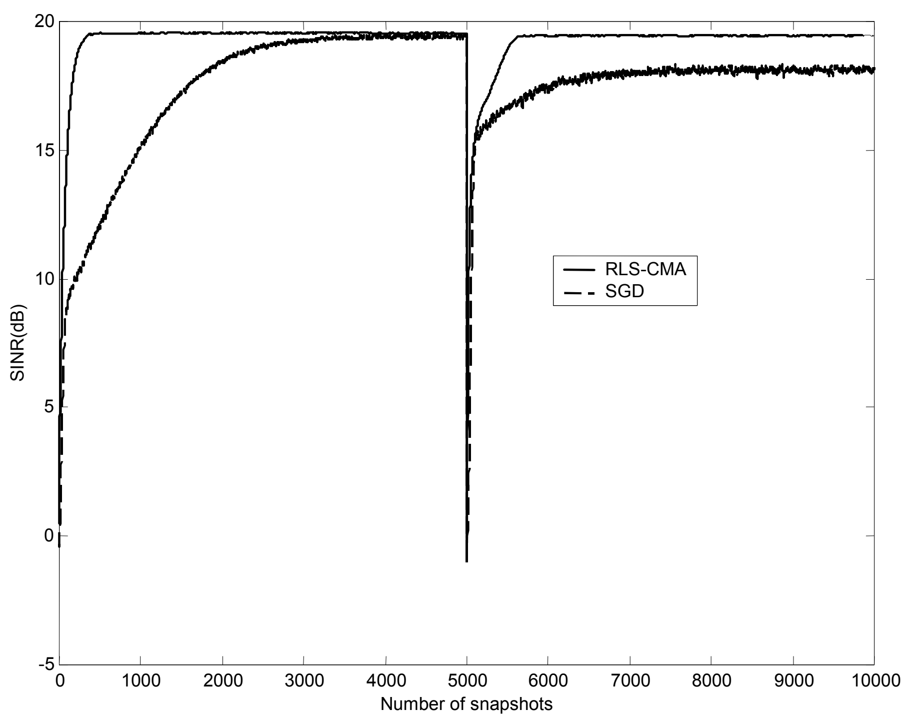
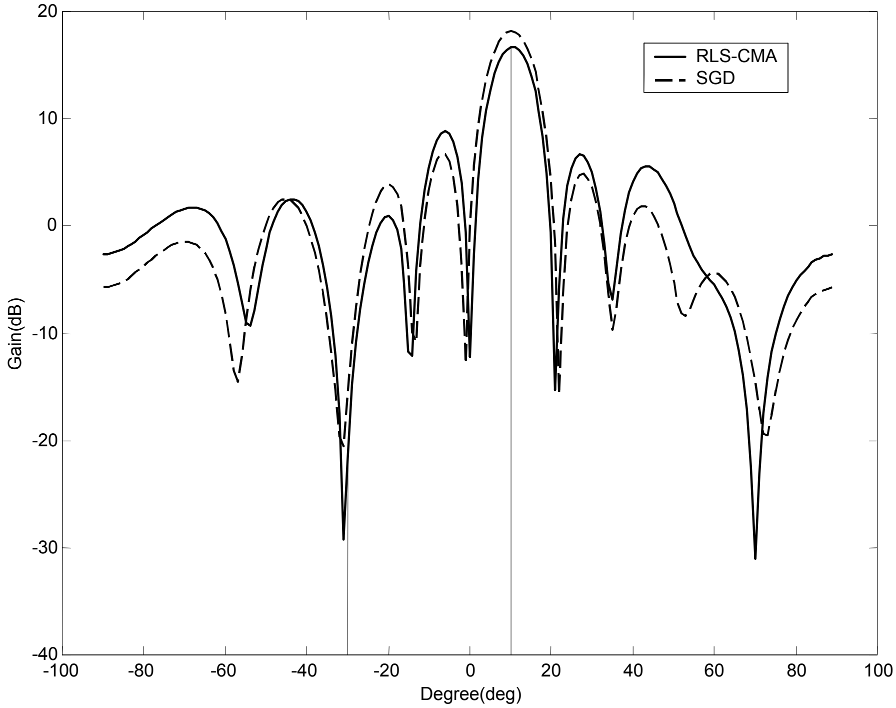
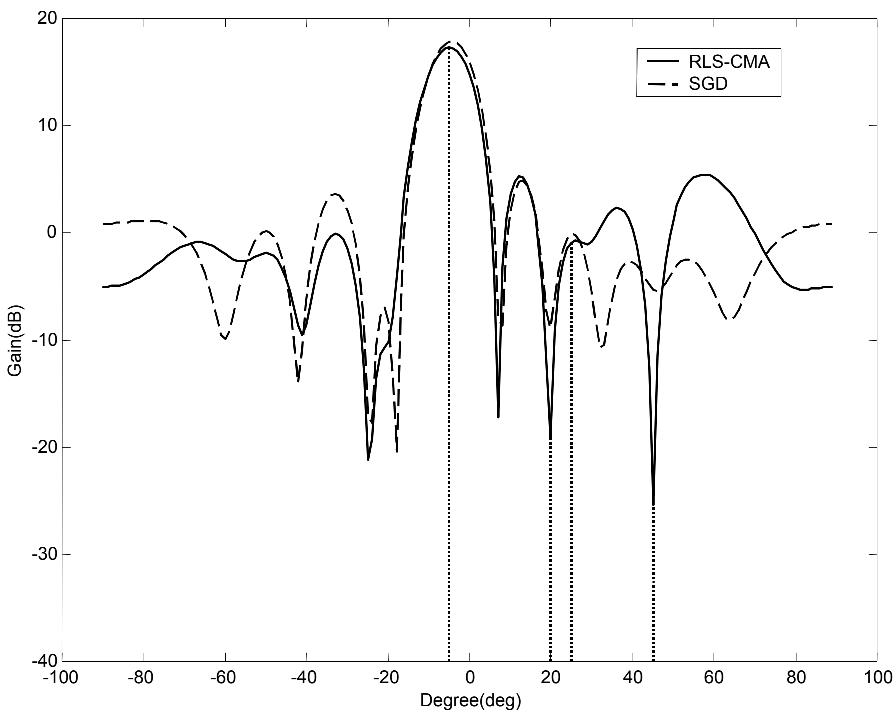


Figure 4: SINR versus iteration number for RLS-CM and SGD in Example 2



(a) Beam pattern at iteration 5000



(b) Beam pattern at iteration 10,000

Figure 5: Beam patterns obtained with the RLS-CM and SGD in example2