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Robust Transmit Beamforming for Collaborative MIMO-OFDM Systems

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***Abstract*— *We present a robust beamforming scheme for collaborative multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) wireless systems. Optimum collaborative transmit beamforming requires knowledge of channel state information (CSI) at the transmitters (collaborative nodes). In practice, however, exact knowledge of CSI is not available at the transmitters. To mitigate the effects of the channel mismatch, we consider a max-min beamforming design approach for collaborative transmission by maximizing the minimum (worst-case) received signal-to-noise ratio (SNR) within a predefined uncertainty region at each OFDM subcarrier. In addition, several subcarrier power allocation strategies are investigated to further improve the performance of collaborative systems.*

I. Introduction

Distributed nodes in wireless networks are required to transmit to or receive from a remote location. Most often, the communication range is limited by the transmission power level of the individual network nodes. In this energy-constrained network, cooperative communication techniques can greatly increase the energy efficiency and range of communication. One technique of cooperative communication that has recently received great attention is *collaborative* or *distributed beamforming* [1]–[4]. Indeed, collaborative beamforming can produce up to an M-fold increase in signal-to-noise ratio (SNR) for a network of M distributed nodes.

Recent studies on collaborative beamforming are based on simple channel and system models. In addition, accurate knowledge of channel state information (CSI) is assumed to be available at the transmitters (collaborative nodes), typically through channel feedback. In a practical situation, however, the transmitters cannot have exact knowledge of instantaneous CSI due to the time-varying characteristics of channels, channel estimation error, and delay in channel feedback. In such a situation, a robust design is required to mitigate the effect of CSI uncertainties. There are two different robust design approaches. In the Bayesian approach, the statistics of the error are utilized to design robust beamformers (*e.g.*, [5], [6]). In the max-min approach, robust beamformers are designed based on the worst-case performance optimization (*e.g.*, [7], [8]).

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In this paper, we focus on the design and evaluation of a beamforming scheme that is robust against CSI mismatches and applicable to practical collaborative wireless systems. We consider a distributed transmission model in which a network of nodes can cooperatively form a *virtual antenna array* to transmit a common message to a remote receiver. Our aim is to extend the max-min robust beamforming design approach of [8] into a multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) framework for collaborative transmission systems. The robust beamformer is designed to provide the best performance in worst-case scenario by maximizing the minimum received SNR within a predefined uncertainty set associated with the current CSI estimate. The robust design takes full advantage of the available estimated eigenmodes of the channel while the nonrobust design uses only the maximum one. In addition, several subcarrier power allocation strategies are investigated to further improve the performance of collaborative systems. Numerical simulation results show that the robust beamformer offers improved performance over the nonrobust beamformers and power allocation strategies among subcarriers further improve the system performance.

The rest of this paper is organized as follows: Section II describes a system model for MIMO-OFDM systems using collaborative transmission. In Section III, a robust beamforming scheme based on worst-case performance optimization is introduced. Section IV describes simple power allocation strategies. Numerical simulation results are presented in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

We consider an N-subcarrier OFDM based distributed wireless system consisting of a network of M_t nodes that transmits a common message to a distant receiver with M_r antennas. Assuming that each node is equipped with a single antenna, a virtual MIMO-OFDM system can be used to model the distributed communication system as shown in Fig. 1. In this paper, we assume that all nodes are perfectly synchronized with each other and data are shared *a priori* among nodes. At the kth subcarrier of the mth node, the complex message signal $s(k)$ is first modulated by the transmit vector $\mathbf{w}^{(m)}(k) = [w_1^{(m)}(k), w_2^{(m)}(k), \ldots, w_P^{(m)}(k)]^T$ of length P ($P \geq M_t$), where $(\cdot)^T$ denotes the transpose operator. After the inverse

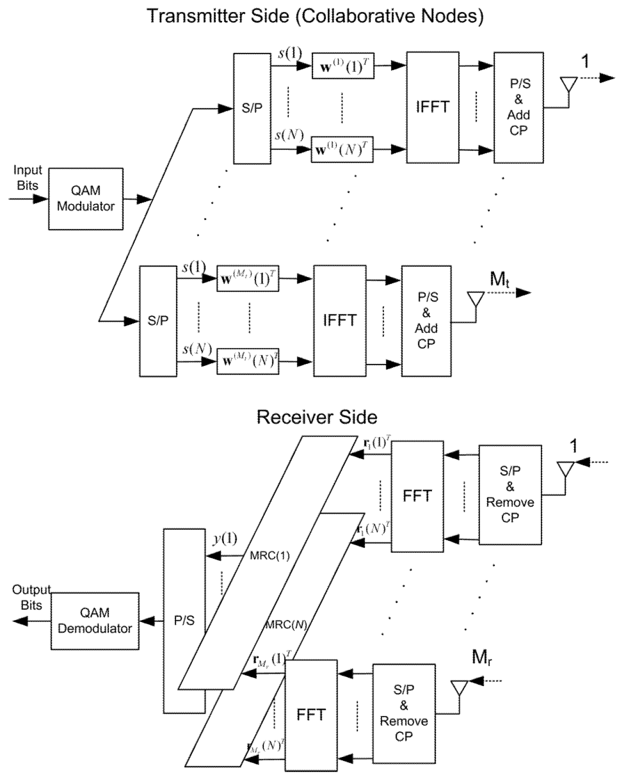

Fig. 1. Block diagram of a virtual MIMO-OFDM system with collaborative beamforming.

fast Fourier transform (IFFT) and cyclic prefix (CP) insertion, the M_t nodes cooperatively transmit OFDM symbols into the radio channels. Note that one OFDM symbol is sent over P time slots due to the spreading. Assuming that the length of the CP is longer than that of the delay spread of the channel, the fast Fourier transform (FFT) output of the jth receive antenna at the kth subcarrier can be expressed as

$$\mathbf{r}_j(k) = \mathbf{W}(k)\mathbf{h}_j(k)s(k) + \mathbf{n}_j(k), \quad 1 \le k \le N, \quad 1 \le j \le M_r \tag{1}$$

where

$$\mathbf{W}(k) = [\mathbf{w}^{(1)}(k), \mathbf{w}^{(2)}(k), \ldots, \mathbf{w}^{(M_t)}(k)] \tag{2}$$

$$= \begin{bmatrix} w_1^{(1)}(k) & w_1^{(2)}(k) & \cdots & w_1^{(M_t)}(k) \\ w_2^{(1)}(k) & w_2^{(2)}(k) & \cdots & w_2^{(M_t)}(k) \\ \vdots & \vdots & \ddots & \vdots \\ w_P^{(1)}(k) & w_P^{(2)}(k) & \cdots & w_P^{(M_t)}(k) \end{bmatrix} \tag{3}$$

is the beamforming (or precoding) matrix and the entry $w_p^{(m)}(k)$ represents the pth transmit beamforming weight at the mth node, $\mathbf{h}_j(k) = [h_j^{(1)}(k), h_j^{(2)}(k), \ldots, h_j^{(M_t)}(k)]^T$ is the frequency domain channel response vector and the entry $h_j^{(m)}(k)$ represents a complex gain at the jth receive antenna from the mth transmit node, $s(k)$ is the complex symbol, and $\mathbf{n}_j(k) = [n_j^{(1)}(k), n_j^{(2)}(k), \ldots, n_j^{(P)}(k)]^T$ is the zero mean circularly symmetric complex Gaussian noise vector with covariance matrix $N_0\mathbf{I}_P$ at the jth receive antenna. Note that the rows of $\mathbf{W}(k)$ correspond to spatial beamforming vectors across M_t collaborative nodes during P time slots. Assuming that the receiver has perfect knowledge of $\mathbf{W}(k)$ and $\mathbf{h}_j(k)$, the output of the maximum ratio combiner (MRC) at the kth subcarrier for the jth receive antenna is given by

$$y_j(k) = \mathbf{h}_j^H(k)\mathbf{W}^H(k)\mathbf{W}(k)\mathbf{h}_j(k)s(k) + \mathbf{h}_j^H(k)\mathbf{W}^H(k)\mathbf{n}_j(k) \tag{4}$$

where $(\cdot)^H$ denotes the Hermitian transpose operator. The SNR for subcarrier k at the jth receive antenna can be written as

$$\mathrm{SNR}_j(k) = \frac{E\left\{|\mathbf{h}_j^H(k)\mathbf{W}^H(k)\mathbf{W}(k)\mathbf{h}_j(k)s(k)|^2\right\}}{E\left\{|\mathbf{h}_j^H(k)\mathbf{W}^H(k)\mathbf{n}_j(k)|^2\right\}} = \frac{E_s}{N_0}\gamma_j(k) \tag{5}$$

where

$$\gamma_j(k) = \mathbf{h}_j^H(k)\mathbf{W}^H(k)\mathbf{W}(k)\mathbf{h}_j(k) \tag{6}$$

is the effective channel gain, $E_s = E\{|s(k)|^2\}$, and $E\{\cdot\}$ denotes the expectation operator. With M_r receive antennas, the combined signal is given by

$$y(k) = \sum_{j=1}^{M_r} y_j(k). \tag{7}$$

Finally, the overall effective channel gain (OECG) for the kth subcarrier can be expressed as

$$\Gamma(k) = \sum_{j=1}^{M_r} \gamma_j(k) = \mathrm{tr}\left\{\mathbf{H}^H(k)\mathbf{W}^H(k)\mathbf{W}(k)\mathbf{H}(k)\right\} \tag{8}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix and $\mathbf{H}(k) = [\mathbf{h}_1(k), \mathbf{h}_2(k), \ldots, \mathbf{h}_{M_r}(k)]$ is the $M_t \times M_r$ channel matrix.

III. Robust Transmit Beamforming for Collaborative Transmission

In practice, perfect CSI, *i.e.*, knowledge of $\{\mathbf{H}(k)\}_{k=1}^N$, is not available at the transmitters due to the time varying condition of channels, channel estimation errors, and channel feedback delay. Hence, the design of the beamforming matrices $\{\mathbf{W}(k)\}_{k=1}^N$ must be based on the estimated CSI at the transmitters. The *true* channel matrix for the kth subcarrier can be expressed as

$$\mathbf{H}(k) = \hat{\mathbf{H}}(k) + \mathbf{E}(k), \quad 1 \le k \le N \tag{9}$$

where $\hat{\mathbf{H}}(k)$ is the *presumed* channel matrix at the transmitters and $\mathbf{E}(k)$ is the unknown channel estimation error matrix whose norm is assumed to be bounded by some known constant $\varepsilon(k) > 0$, *i.e.*, $\|\mathbf{E}(k)\|_F \le \varepsilon(k)$ where $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. Hence, we can rewrite the OECG in (8) as

$$\Gamma(k) = \mathrm{tr}\left\{(\hat{\mathbf{H}}(k) + \mathbf{E}(k))^H \mathbf{W}^H(k)\mathbf{W}(k)(\hat{\mathbf{H}}(k) + \mathbf{E}(k))\right\}. \tag{10}$$

Considering (10) as a system performance criterion and the max-min robust design approach of [8], the beamforming matrix for the kth subcarrier can be obtained by maximizing the minimum (worst performance over all channel errors) of the OECG of that subcarrier, *i.e.*,

$$\begin{aligned} &\underset{\mathbf{W}(k)}{\text{maximize}} \quad \min_{\mathbf{E}(k)\in\mathcal{R}_k} \Gamma(k) \\ &\text{subject to} \quad \mathrm{tr}\{\mathbf{W}^H(k)\mathbf{W}(k)\} \le p(k) \end{aligned} \tag{11}$$

where $p(k)$ represents the maximum available transmit power at the kth subcarrier and $\mathcal{R}_k = \{\mathbf{E}(k) : \|\mathbf{E}(k)\|_F \le \varepsilon(k)\}$ is the uncertainty region. In order to simplify the cost function, the positive semi-definite matrices $\mathbf{W}^H(k)\mathbf{W}(k)$ and $\hat{\mathbf{H}}(k)\hat{\mathbf{H}}^H(k)$ are expressed in terms of their eigenvalue decomposition (EVD) as

$$\mathbf{W}^H(k)\mathbf{W}(k) = \mathbf{U}_w(k)\mathbf{D}_w(k)\mathbf{U}_w^H(k) \tag{12}$$

$$\hat{\mathbf{H}}(k)\hat{\mathbf{H}}^H(k) = \mathbf{U}_{\hat{h}}(k)\mathbf{D}_{\hat{h}}(k)\mathbf{U}_{\hat{h}}^H(k) \tag{13}$$

$$\mathbf{D}_w(k) = \mathrm{diag}(d_{w_1}(k), \cdots, d_{w_{M_t}}(k)) \tag{14}$$

$$\mathbf{D}_{\hat{h}}(k) = \mathrm{diag}(d_{\hat{h}_1}(k), \cdots, d_{\hat{h}_{M_t}}(k)) \tag{15}$$

where the columns of unitary matrices $\mathbf{U}_w(k)$ and $\mathbf{U}_{\hat{h}}(k)$ are the orthonormal eigenvectors of $\mathbf{W}^H(k)\mathbf{W}(k)$ and $\hat{\mathbf{H}}(k)\hat{\mathbf{H}}^H(k)$, respectively, and $\mathbf{D}_w(k)$ and $\mathbf{D}_{\hat{h}}(k)$ contain the corresponding eigenvalues sorted in non-increasing order. Considering eigen beamforming in which symbols are transmitted along the eigenvectors of the presumed channel correlation matrix $\hat{\mathbf{H}}(k)\hat{\mathbf{H}}^H(k)$, the beamforming matix for the kth subcarrier can be expressed as

$$\mathbf{W}(k) = \mathbf{\Phi}(k)\mathbf{D}_w^{1/2}(k)\mathbf{U}_{\hat{h}}^H(k) \tag{16}$$

where $\mathbf{\Phi}(k)$ is a $P \times M_t$ arbitrary maxtix whose columns are orthonormal, *i.e.*, $\mathbf{\Phi}^H(k)\mathbf{\Phi}(k) = \mathbf{I}_{M_t}$ [5].

Using the proof in [8], the method of Lagrange multipliers, and the above EVDs, the minimum of the OECG under the structural constraint in (16) is obtained as

$$\Gamma_{\min}(k) = \sum_{i=1}^{M_t} d_{\hat{h}_i}(k)\left[d_{w_i}(k)\left(\frac{\lambda(k)}{d_{w_i}(k)+\lambda(k)}\right)^2\right] \tag{17}$$

where the Lagrange multiplier $\lambda(k)$ satisfies

$$\varepsilon^2(k) = \sum_{i=1}^{M_t} d_{\hat{h}_i}(k)\left(\frac{d_{w_i}(k)}{d_{w_i}(k)+\lambda(k)}\right)^2. \tag{18}$$

Then, we can rewrite the optimization problem (11) as

$$\begin{aligned} &\underset{d_{w_i}(k)}{\text{maximize}} \quad \Gamma_{\min}(k) \\ &\text{subject to} \quad \sum_{i=1}^{M_t} d_{w_i}(k) \le p(k),\, d_{w_i}(k) \ge 0,\ \forall i. \end{aligned} \tag{19}$$

Using the Karush-Kuhn-Tucker (KKT) optimality conditions, the closed-form solution can be obtained as [8]

$$d_{w_i}(k) = \begin{cases} \lambda(k)\left(\sqrt{\frac{d_{\hat{h}_i}(k)}{v(k)}} - 1\right), & v(k) < d_{\hat{h}_i}(k) \\ 0, & v(k) \ge d_{\hat{h}_i}(k) \end{cases} \tag{20}$$

where the constants $\lambda(k)$ and $v(k)$ are chosen such that the total transmit power for a given subcarrier k is satisfied, *i.e.*, $\sum_{i=1}^{M_t} d_{w_i}(k) = p(k)$ and that (18) is also satisfied. Interestingly, the optimal power loading solution can be interpreted as a form of water-filling [8].

Letting $n(k) \in \{1, 2, \cdots, M_t\}$ be the number of the active eigenmodes for the kth subcarrier and substituting (20) into (18), we can obtain the minimum root $v(k)$ of the following second order equation in $\sqrt{v(k)}$:

$$\varepsilon^2(k) = \sum_{i=1}^{n(k)} \left(\sqrt{d_{\hat{h}_i}(k)} - \sqrt{v(k)}\right)^2 \tag{21}$$

The value of $n(k)$ for which $d_{\hat{h}_{n(k)+1}}(k) \le v(k) < d_{\hat{h}_{n(k)}}(k)$ is determined by a simple finite iteration. Then, $\lambda(k)$ is found such that $\sum_{i=1}^{n(k)} d_{w_i}(k) = p(k)$.

Since $v(k)$ is inversely proportional to the size of the uncertainty region $\varepsilon(k)$ as in (21), the robust beamformer tends to use more eigenmodes to transmit data as $\varepsilon(k)$ increases. The robust beamformer takes full advantage of the available estimated eigenmodes of the channel and distributes the total transmit power across the eigenmodes in a water-filling fashion. In contrast, the nonrobust beamformer ($\varepsilon(k) = 0$) uses only the maximum eigenmode. A similar closed-form solution using convex optimization theory is found in [9].

IV. Power Allocation

We consider power allocation strategies to distribute the total available transmit power P_0 to the N subcarriers. We apply the strategies considered in [10], [11] for the nonrobust design approach to the case of robust beamforming.

Taking into account the partial knowledge of the CSI at the transmitters, the estimated OECG at the kth subcarrier can be expressed as

$$\tilde{\Gamma}(k) = z(k)p(k) \tag{22}$$

where $z(k) = \mathrm{tr}\{\hat{\mathbf{H}}^H(k)\tilde{\mathbf{W}}^H(k)\tilde{\mathbf{W}}(k)\hat{\mathbf{H}}(k)\}$, $\tilde{\mathbf{W}}(k) = (1/\sqrt{p(k)})\mathbf{W}(k)$ such that $\|\tilde{\mathbf{W}}(k)\|_F = 1$, and $p(k)$ satisfies the global power constraint $\sum_{k=1}^{N} p(k) = P_0$. One simple optimization criterion is to maximize the arithmetic mean of the estimated OECG. In this criterion, information is transmitted through only one subchannel. The system becomes a single carrier system that wastes the remaining bandwidth. In addition, information on other subcarriers will be lost. Therefore, this criterion is not feasible.

A. Maximization of Geometric Mean (GEOM)

Considering the maximization of the geometric mean of the estimated OECG, we can formulate the optimization problem as

$$\begin{array}{ll} \underset{p(k)}{\text{maximize}} & \sum_{k=1}^{N} \ln\tilde{\Gamma}(k) \\ \text{subject to} & \sum_{k=1}^{N} p(k) = P_0, \quad p(k) \geq 0, \quad 1 \leq k \leq N \end{array}$$

where $\ln(\cdot)$ denotes the natural logarithm. The solution is a uniform power allocation over all subcarriers:

$$p(k) = \frac{P_0}{N}. \tag{23}$$

B. Maximization of Harmonic Mean (HARM)

Consider the maximization of the harmonic mean of the estimated OECG. The optimization problem can be expressed in the equivalent form as

$$\begin{array}{ll} \underset{p(k)}{\text{minimize}} & \sum_{k=1}^{N} \frac{1}{\tilde{\Gamma}(k)} \\ \text{subject to} & \sum_{k=1}^{N} p(k) = P_0, \quad p(k) \geq 0, \quad 1 \leq k \leq N. \end{array}$$

The solution of this problem can be readily shown as

$$p(k) = \frac{P_0}{\sum_{i=1}^{N} z^{-\frac{1}{2}}(i)} \frac{1}{\sqrt{z(k)}}. \tag{24}$$

C. Maximization of Minimum (MAXMIN)

The performance of the OFDM system can be substantially degraded by the subchannels with low SNRs. Similar to the robust beamforming design approach, we can maximize the minimum of the estimated OECG, *i.e.*,

$$\begin{array}{ll} \underset{p(k)}{\text{maximize}} & \min_{k} \tilde{\Gamma}(k) \\ \text{subject to} & \sum_{k=1}^{N} p(k) = P_0, \quad p(k) \geq 0, \quad 1 \leq k \leq N. \end{array}$$

The solution is then given by

$$p(k) = \frac{P_0}{\sum_{i=1}^{N} z^{-1}(i)} \frac{1}{z(k)}. \tag{25}$$

V. Numerical Simulations

We consider 4 collaborative nodes ($M_t = 4$) in the x-y plane and a receiver with 2 antennas ($M_r = 2$) located in the far-field along the direction of $\phi = 0°$, where the azimuth angle ϕ is measured from the x-axis. Fig. 2 shows the location of the collaborative nodes. The simulation parameters are: $N = 64$, CP length $= 16$, L (Channel length) $= 3$, B (system bandwidth) $=$ 20Mhz, f_c (carrier frequency) $=$ 5GHz,

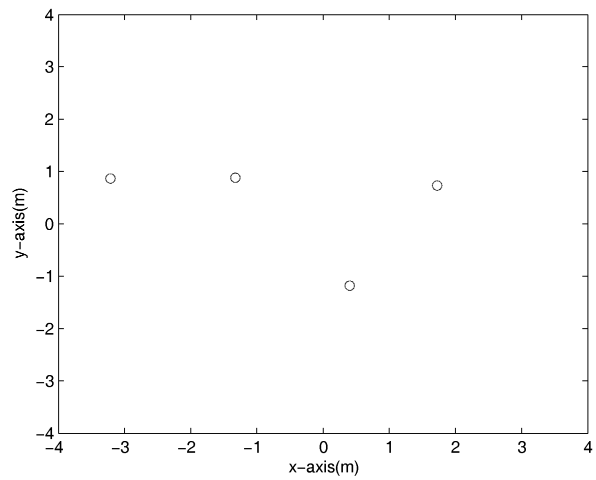

Fig. 2. Location of collaborative nodes

f_d (Doppler frequency) $=$ 10Hz, and 16-QAM signal constellation with a normalized energy, *i.e.*, $E_s = E\{|s(k)|^2\} = 1$. The transmission channels between the collaborative nodes and the receiver are generated using a statistical multi-path vector channel simulator [12]. For simulations, we assume that the channel is fixed for a frame but can vary between successive frames. Furthermore, we assume that the estimated CSIs $\{\hat{\mathbf{H}}(k)\}_{k=1}^{N}$ are only available at the transmitters and the uncertainty matrices $\{\mathbf{E}(k)\}_{k=1}^{N}$ are generated using complex Gaussian independent and identically distributed matrices whose entries have zero mean and variance $\sigma_e^2 = 0.4$. Without loss of generality, the estimated CSI matrices are normalized to 1 such that $0 \leq \{\varepsilon(k)\}_{k=1}^{N} \leq 1$. As in [8], we consider the sizes of the uncertainty region $\{\varepsilon(k)\}_{k=1}^{N}$ as robust design parameters. At low SNRs, the performance of the system is mainly dominated by the noise. It is sufficient to use small values of $\varepsilon(k)$ such that the nonrobust beamforming is applied. In contrast, at high SNRs, the mismatch between the presumed and true CSIs is the dominant factor that degrades the system performance. Hence, larger values of $\varepsilon(k)$ should be used.

Fig. 3 shows the uncoded symbol error rate (SER) averaged over 5000 Monte-Carlo runs. In this simulation, we linearly increase $\{\varepsilon(k)\}_{k=1}^{N}$ from 0 to 0.95 over the E_s/N_0 range of 10 to 17dB. In case of the equal power allocation strategy among subcarriers (GEOM), it can be seen that the robust beamformer offers performance gains over the conventional one-directional (1-D) beamformer and the equal-power beamformer (transmit power is evenly distributed among all eigenmodes), which are the nonrobust approaches. This is due to the fact that the robust beamformer effectively distributes the transmit power among available estimated eigenmodes of the channel whereas the 1-D beamformer uses only the maximum estimated eigenmode and the equal-power beamformer does not utilize the CSI. It is evident that combining HARM and MAXMIN power allo-

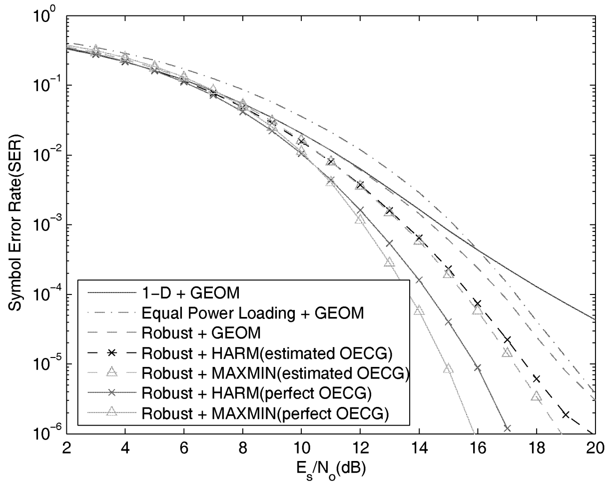

Fig. 3. SER versus E_s/N_0 ($M_t = 4$, $M_r = 2$, 16-QAM).

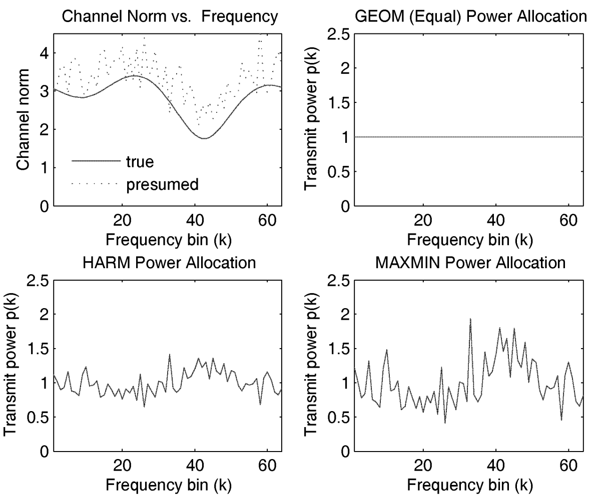

Fig. 4. Power allocation strategies (P_0 = 64W).

cation strategies with the robust beamformer can substantially increase the system performance by injecting more power to subchannels experiencing deep fades as shown in Figs. 3 and 4. Fig. 4 shows the allocated power to the N subcarriers when the total transmit power is $P_0 = 64$W. The top left figure shows the Frobenius norm of the true channel $\|\mathbf{H}(k)\|_F$ and the presumed channel $\|\hat{\mathbf{H}}(k)\|_F$ for a given channel realization. The top right figure shows GEOM(Equal) power allocation. The bottom left and right figures show the HARM and MAXMIN power allocation strategies, respectively. However, it should be noted that power allocation strategies based on perfect CSI (perfect OECG) can further improve the system performance by approximately 2dB at high SNRs as shown in Fig. 3.

Fig. 5 depicts the probability density function (pdf) of the SNR at the MRC output of the receiver when $E_s/N_0 = 17$dB, $k = 40$. Note that the vertical lines represent the corresponding SNR means and the pdf is obtained from 50000 Monte-Carlo runs. This figure verifies that the 1-D beamformer is optimal in terms of the expected SNR. However, the SER performance of the 1-D beamformer is poor as seen in the previous simulation. This is because the SER performance is dominated by worst-case errors (performance). Thus, maximizing the expected SNR may not guarantee the lowest SER performance. The SNR variance must be kept as small as possible to improve the SER [5]. The robust beamformer indeed provides a tradeoff between the SNR mean and SNR variance by maximizing worst-case performance.

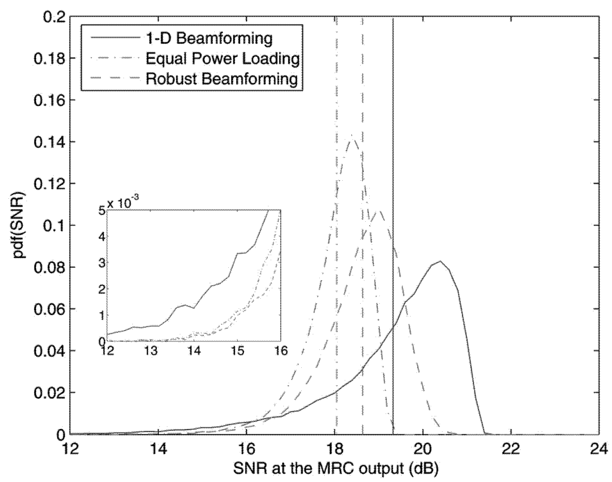

Fig. 5. SNR pdf at the MRC output (E_s/N_0 = 17dB, k = 40).

VI. Conclusions

In this paper, we have designed and evaluated a beamforming scheme that is robust against CSI mismatches and applicable to collaborative MIMO-OFDM wireless systems using the max-min robust beamforming design approach. At each OFDM subcarrier, the robust beamformer transmits complex symbols along the eigenmodes of the estimated channel correlation matrix with the water-filling-type power distribution. It has been shown that the robust beamformer offers performance gains over the conventional one-directional and equal-power beamformers and provides a tradeoff between the SNR mean and SNR variance. Moreover, numerical simulation results have shown that in the presence of CSI errors simple power allocation strategies among subcarriers further improve the system performance.

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