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ROBUST CHANNEL FEEDBACK FOR COLLABORATIVE UPLINK BEAMFORMING

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ABSTRACT

We present a new algorithm for collaborative uplink transmit beamforming with robustness against mismatches in the channel state information. The beamforming coefficients are computed at the base station using the uplink measurements and fed back to the cooperating terminals. This can be considered as a robust feedback of the channel state. Our beamformer is derived by minimizing the total transmitted power while preserving the received signal at the base station for all the channel realizations within a prescribed uncertainty set. The problem is formulated as a second-order cone program that can be efficiently solved using interior point methods. Simulations results are presented showing the superior performance of our technique compared to classical transmit beamforming.

1. INTRODUCTION

Multihop relaying is one of the major modifications to the architecture of wireless cellular networks to achieve the high data rates envisioned for fourth generation wireless systems. In these systems, multiple relay terminals can collaboratively transmit the signal of a nearby user to a distant base station. Hence, multiple antenna signaling techniques can be used to exploit the spatial characteristics of the channel. Transmit beamforming is one of the approaches to exploit these characteristics as it is capable of providing spatially matched transmission that increases the received signl-to-noise ratio (SNR) at the target destination and reduces interference to non-targeted base stations [1].

Optimum collaborative transmit beamforming requires exact knowledge of the channel state at the transmitting relay terminals. However, this information might be difficult to acquire at the terminals due to the time varying nature of the channel [2], and/or relative phase and frequency offsets between the various terminals [3]. Many adaptive beamforming algorithms have been recently proposed to provide robustness against various mismatches in the array manifold (e.g., [4], [5] and the references therein). These algorithms are based on preserving all the received signals within a predefined uncertainty set centered around the channel state estimate. However, only line-of-sight (LOS) propagation was considered in all these receive beamforming algorithms. Moreover, it was assumed that the array elements are located within a single processing unit, and hence, these algorithms are not suitable for collaborative transmission scenarios where the array elements are distributed among different relay terminals with each terminal having an estimate (together with its associated uncertainty) of its channel vector only. This limits the ability of these approaches to exploit the good estimates that some terminals may have of their channels [6].

In this paper, we consider the problem of robust collaborative beamforming for uplink transmission. First, we present a unified signal model for both LOS and flat fading channels. Our signal model divides the available channel information into two parts: a perfectly known part that corresponds to the second-order statistics of the channel or the local array manifolds of the cooperating terminals, and a possibly erroneous estimate of the *channel realization driving vector* that captures the channel randomness and is assumed to belong to a prescribed uncertainty set. We formulate our beamforming problem as minimizing the total transmitted power by the cooperating terminals subject to a constraint that preserves the received signal at the target base station for all the channel vectors in the uncertainty set. The problem is converted to a convex optimization problem that can be solved efficiently with polynomial complexity using interior point methods [7]. We also introduce additional convex constraints that limit the interference received by nearby stations even in the presence of channel mismatches. Hence, using the uplink measurements, the base station can compute and feedback the uplink beamforming coefficients to the cooperating terminals. This can be viewed as a robust feedback of the channel state. Simulation results are presented showing the superior performance of our beamformer compared to classical beamforming techniques in both LOS and flat fading channels.

2. SIGNAL MODEL

We consider the uplink of a narrowband wireless communication system with M relay terminals collaboratively transmitting a common signal to the base station. The mth terminal

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is equipped with a k_m-element antenna array. The received baseband signal at the base station at the ith time instant is

$$y(i) = \sum_{m=1}^{M} \boldsymbol{w}_m^H \boldsymbol{h}_m s(i) + w(i) = \boldsymbol{w}^H \boldsymbol{h} s(i) + w(i) \qquad (1)$$

where $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively, $s(i)$ is the common information signal transmitted by the M relay terminals, $\boldsymbol{h}_m$ is the $k_m \times 1$ vector containing the channel coefficients from the mth terminal to the base station, $\boldsymbol{w}_m$ is the $k_m \times 1$ beamforming vector of the mth terminal, and $w(i)$ is white Gaussian noise with zero mean and variance σ_w^2. The $K \times 1$ stacked channel vector $\boldsymbol{h}$ is given by $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_M^T]^T$ where $K = \sum_{m=1}^{M} k_m$ and the $K \times 1$ vector $\boldsymbol{w} = [\boldsymbol{w}_1^T, \ldots, \boldsymbol{w}_M^T]^T$ is the beamforming vector.

2.1. Line-of-Sight Propagation Environment

In the case of LOS propagation, the channel vector of the mth terminal can be written as $\boldsymbol{h}_m = e^{-j2\pi f_0 T_m} \boldsymbol{a}_m(\theta_m)$ [3], where $\boldsymbol{a}_m(\theta) = [1, e^{-j2\pi f_0 \tau_{m,2}(\theta_m)}, \ldots, e^{-j2\pi f_0 \tau_{m,k_m}(\theta_m)}]^T$, f_0 is the carrier frequency, $\tau_{m,i}(\theta_m)$ is the propagation delay of the signal transmitted from the ith antenna of the mth terminal towards the base station, located in the direction θ_m, relative to that of the signal transmitted from the first antenna of the mth terminal, and T_m is the propagation delay of the signal transmitted from the first antenna of the mth terminal relative to that transmitted from a common reference point. We can write the stacked channel vector as

$$\boldsymbol{h} = \boldsymbol{V}\boldsymbol{n} \qquad (2)$$

where $\boldsymbol{n} = \left[e^{-j2\pi f_0 T_1}, \ldots, e^{-j2\pi f_0 T_M}\right]^T$ is the channel realization driving vector, the $K \times M$ matrix $\boldsymbol{V}$ is given by

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{a}_1(\theta_1) & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{a}_2(\theta_2) & \boldsymbol{0} & \vdots \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{a}_M(\theta_M) \end{bmatrix}, \qquad (3)$$

and $\boldsymbol{0}$ is column vector of zeros with appropriate dimension. We can see from (2) that the channel vector can be decomposed into the product of a matrix $\boldsymbol{V}$ that contains the local array manifold vectors of each terminal and a vector $\boldsymbol{n}$ containing the relative phase offsets between different terminals. The uncertainty in the location and/or the synchronization error of the mth terminal can be modeled as an error in the propagation delay T_m, and hence, as an error in the vector $\boldsymbol{n}$ [3]. Thus, we can model the channel vector $\boldsymbol{h}$ as

$$\boldsymbol{h} = \boldsymbol{V}(\hat{\boldsymbol{n}} + \boldsymbol{\Delta}) \qquad (4)$$

where $\hat{\boldsymbol{n}} = [e^{-j2\pi f_0 \hat{T}_1}, \ldots, e^{-j2\pi f_0 \hat{T}_M}]^T$ is the estimate of the vector $\boldsymbol{n}$ and $\{\hat{T}_m\}$ are the presumed delay offsets.

2.2. Flat Fading Propagation Environment

In the case of multipath flat fading channels, the channel vector of the mth terminal can be written as $\boldsymbol{h}_m = \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{n}_m$ [2], where $\boldsymbol{R}_m$ is the covariance matrix of the channel vector of the mth terminal, and $\boldsymbol{n}_m$ is a $k_m \times 1$ vector of independent zero mean, unit variance, circular Gaussian random variables. Note that we have assumed that the channel vector of each relay terminal is independent of that of the other terminals, i.e., the terminals are well-separated in space. Therefore, we can write the stacked channel vector as $\boldsymbol{h} = \boldsymbol{V}\boldsymbol{n}$ where the $K \times 1$ channel realization driving vector $\boldsymbol{n}$ is given by $\boldsymbol{n} = \left[\boldsymbol{n}_1^T, \ldots, \boldsymbol{n}_M^T\right]^T$ and the $K \times K$ matrix $\boldsymbol{V}$ is given by

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{R}_1^{\frac{1}{2}} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_2^{\frac{1}{2}} & \boldsymbol{0} & \vdots \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{R}_M^{\frac{1}{2}} \end{bmatrix}. \qquad (5)$$

In practice, we can assume that the channel is quasi-stationary, i.e., the second-order statistics of the channel are approximately constant within a certain stationarity period [2]. Hence, we can model the stacked channel vector $\boldsymbol{h}$ by the same model as that in Eq. (4) where $\hat{\boldsymbol{n}} = [\hat{\boldsymbol{n}}_1^T, \ldots, \hat{\boldsymbol{n}}_M^T]^T$ is the estimate of $\boldsymbol{n}$, e.g., obtained by (delayed) feedback from the base station, and $\boldsymbol{\Delta}$ is the corresponding error vector.

3. ROBUST TRANSMIT BEAMFORMING

If the cooperating terminals have perfect knowledge of the channel vector $\boldsymbol{h}$, the optimum (power-constrained) beamformer that maximizes the received signal-to-noise ratio (SNR) at the base station can be found by solving the equivalent minimum variance distortionless response problem [4]

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \qquad \text{s.t.} \quad \boldsymbol{w}^H \boldsymbol{V}\boldsymbol{n} = 1, \qquad (6)$$

whose solution is given by $\boldsymbol{w} = \frac{1}{\beta}\boldsymbol{V}\boldsymbol{n}$ where $\beta = \|\boldsymbol{V}\boldsymbol{n}\|^2$. However, at the transmission instant each terminal has a possibly erroneous estimate $\hat{\boldsymbol{n}}$ of the channel realization vector. This estimate is used instead of the actual vector $\boldsymbol{n}$ which might lead to considerable degradation in the received SNR at the target base station [6].

We define the uncertainty set $\mathcal{A}$ associated with the estimate of the channel realization driving vector as

$$\mathcal{A} = \left\{ \tilde{\boldsymbol{n}} = \left[\hat{\boldsymbol{n}}_1^T + \boldsymbol{\Delta}_1^T, \ldots, \hat{\boldsymbol{n}}_M^T + \boldsymbol{\Delta}_M^T\right]^T \middle| \, \|\boldsymbol{\Delta}_m\| \le \varepsilon_m \right\} \quad (7)$$

where $\varepsilon_m \ge 0$ reflects the uncertainty in the channel estimate of the mth terminal. Note that in the case of LOS propagation, the vectors $\{\boldsymbol{\Delta}_m\}$ decompose into scalar quantities that reflect the amount of error in the phase offset of each terminal. Hence, ε_m can be estimated given the amount of uncertainty in the location of the mth terminal and the phase error due to its local oscillator imperfections. In the case of fading channels, the parameter ε_m is a function of the feedback delay and the coherence time of the channel of the mth terminal.

In order to provide robustness against errors in the channel realization driving vector, we will modify the constraint in (6) such that a high gain is provided for the worst-case channel error (that yields the minimum SNR at the target base station), and hence, for all the channel vectors in $\mathcal{A}$ [4], i.e.,

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \qquad \text{s.t.} \quad \min_{\tilde{\boldsymbol{n}} \in \mathcal{A}} \left| \boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}} \right| \geq 1. \tag{8}$$

Using the triangle inequality, we can write

$$\left| \boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}} \right| \geq \left| \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} \right| - \left| \boldsymbol{w}^H \boldsymbol{V} \boldsymbol{\Delta} \right|. \tag{9}$$

For the case of flat fading channels, we have

$$\left| \boldsymbol{w}^H \boldsymbol{V} \boldsymbol{\Delta} \right| \leq \sum_{m=1}^{M} \left| \boldsymbol{w}_m^H \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{\Delta}_m \right| \leq \sum_{m=1}^{M} \varepsilon_m \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\| \tag{10}$$

where $\boldsymbol{\Delta}_m = -\varepsilon_m e^{j\phi} \boldsymbol{R}_m^{\frac{H}{2}} \boldsymbol{w}_m / \| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \|$ satisfies (9) and (10) with equality, and $\phi = \arg\{\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}\}$. Combining (9) and (10), we can write the robust beamforming problem as

$$\min_{\boldsymbol{w}} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t.} \ \left| \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} \right| - \sum_{m=1}^{M} \varepsilon_m \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\| \geq 1. \tag{11}$$

The above optimization problem is nonconvex due to the absolute value operator in the constraint. However, we can always phase-rotate the vector $\boldsymbol{w}$ such that $\boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}}$ is real without changing the value of the cost function. Hence, we can write (11) as the following second-order cone program (SOCP)

$$\begin{aligned} \min_{\boldsymbol{w}, \alpha_m} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t.} \quad & \text{Imag}\left\{ \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} \right\} = 0 \\ & \left\| \boldsymbol{R}_m^{\frac{1}{2}} \boldsymbol{w}_m \right\| \leq \alpha_m \quad \forall m = 1, \ldots, M \\ & \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} - \sum_{m=1}^{M} \varepsilon_m \alpha_m \geq 1. \end{aligned} \tag{12}$$

Similarly, for the case of LOS propagation, we have

$$\left| \boldsymbol{w}^H \boldsymbol{V} \tilde{\boldsymbol{n}} \right| \geq \left| \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} \right| - \sum_{m=1}^{M} \varepsilon_m \left| \boldsymbol{a}_m^H \boldsymbol{w}_m \right| \tag{13}$$

with equality if $\Delta_m = -\frac{\varepsilon_m}{|\boldsymbol{a}_m^H \boldsymbol{w}_m|} e^{j\phi} \boldsymbol{a}_m^H \boldsymbol{w}_m$. Therefore, we can write the robust beamforming problem in (8) as the following SOCP

$$\begin{aligned} \min_{\boldsymbol{w}, \alpha_m} \boldsymbol{w}^H \boldsymbol{w} \quad \text{s.t.} \quad & \text{Imag}\left\{ \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} \right\} = 0 \\ & \left| \boldsymbol{a}_m^H \boldsymbol{w}_m \right| \leq \alpha_m \quad \forall m = 1, \ldots, M \\ & \boldsymbol{w}^H \boldsymbol{V} \hat{\boldsymbol{n}} - \sum_{m=1}^{M} \varepsilon_m \alpha_m \geq 1. \end{aligned} \tag{14}$$

The above SOCPs in (12) and (14) can be efficiently solved using interior point methods [8]. The computational complexity associated with solving an SOCP can be calculated as follows [7]. The number of iterations required to solve an SOCP problem is bounded by the square root of the number of constraints. The computational complexity associated with each iteration is of $\mathcal{O}(n_v^2 \sum_i q_i)$, where $n_v = 2K + M + 1$ is the number of design parameters and q_i is the dimension of the ith constraint. Therefore, the worst-case computational load of each of (12) and (14) is of $\mathcal{O}\big(\sqrt{M} K (M+K)^2\big)$.

Therefore, based on the propagation model, and given the matrix $\boldsymbol{V}$ and the presumed channel realization driving vector $\hat{\boldsymbol{n}}$, the base station can compute the robust uplink beamforming vector for each of the M relay terminals by solving the SOCP optimization problem in (12) or (14). The beamforming vector $\boldsymbol{w}_m$ is then fed back to the mth terminal to be used in subsequent transmissions.

One of the advantages of our proposed beamformer is that any additional convex constraints can be easily incorporated in the beamforming problem. We will provide examples of some possible additional constraints:

1-*Maximum Power Constraints*

Due to physical considerations, the maximum power transmitted by each terminal might be limited. This is equivalent to constraining the norm of the beamforming vector of each terminal, i.e., $\|\boldsymbol{w}_m\| \leq \sqrt{P_m}$ which is a convex second-order cone constraint of $2k_m + 1$ real dimensions.

2-*Interference Suppression*

Another possible constraint is to completely suppress the interference caused at nearby base stations due to the cooperative transmission. This constraint can be written as

$$\boldsymbol{w}^H \hat{\boldsymbol{h}}^{(v)} = 0 \tag{15}$$

where the superscript $(\cdot)^{(v)}$ refers to the vth non-targeted base station. This constraint is a linear constraint that can be easily incorporated in the beamforming problem, e.g., by substituting $\boldsymbol{w} = \boldsymbol{N}_{\hat{\boldsymbol{h}}^{(v)}}^{\perp} \boldsymbol{v}$ where $\boldsymbol{N}_{\hat{\boldsymbol{h}}^{(v)}}^{\perp}$ is the $K \times (K-1)$ matrix spanning the subspace orthogonal to $\hat{\boldsymbol{h}}^{(v)}$ and $\boldsymbol{v}$ is the K–1 dimensional vector containing the new optimization variables.

3-*Robust Interference Reduction*

Each interference suppression constraint with the form of (15) reduces one of the degrees of freedom available for beamforming, and hence, reduces the received signal power at the target base station. An alternate solution is to limit the transmitted interference power in the directions of other base stations. Using the same signal model and notation discussed in Section II, we can write the stacked channel vector from the M terminals to the vth base station as

$$\boldsymbol{h}^{(v)} = \boldsymbol{V}^{(v)} \boldsymbol{n}^{(v)} = \boldsymbol{V}^{(v)} \left(\hat{\boldsymbol{n}}^{(v)} + \boldsymbol{\Delta} \right) \tag{16}$$

where the error vector Δ belongs to the uncertainty set

$$\mathcal{A}^{(v)} = \{ \boldsymbol{\Delta} = [\boldsymbol{\Delta}_1^T, \ldots, \boldsymbol{\Delta}_M^T]^T \mid \|\boldsymbol{\Delta}_m\| \leq \varepsilon_m \}. \tag{17}$$

The robust interference reduction constraint can be written as

$$\max_{\boldsymbol{\Delta} \in \mathcal{A}^{(v)}} \left| \boldsymbol{w}^H \boldsymbol{V}^{(v)} \left(\hat{\boldsymbol{n}}^{(v)} + \boldsymbol{\Delta} \right) \right| \leq \zeta^{(v)} \tag{18}$$

where $\zeta^{(v)}$ is a design parameter that controls the maximum admissible interference. Using the triangle inequality, we get

$$\left| \boldsymbol{w}^H \boldsymbol{V}^{(v)} \left(\hat{\boldsymbol{n}}^{(v)} + \boldsymbol{\Delta} \right) \right| \leq \left| \boldsymbol{w}^H \boldsymbol{V}^{(v)} \hat{\boldsymbol{n}}^{(v)} \right| + \left| \boldsymbol{w}^H \boldsymbol{V}^{(v)} \boldsymbol{\Delta} \right|. \tag{19}$$

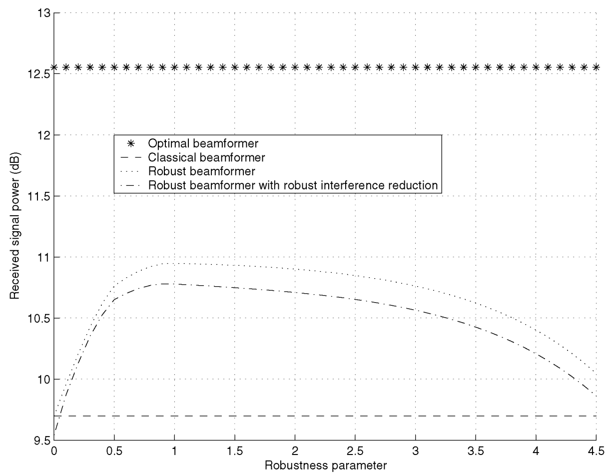

Fig. 1. Average received signal power versus ε.

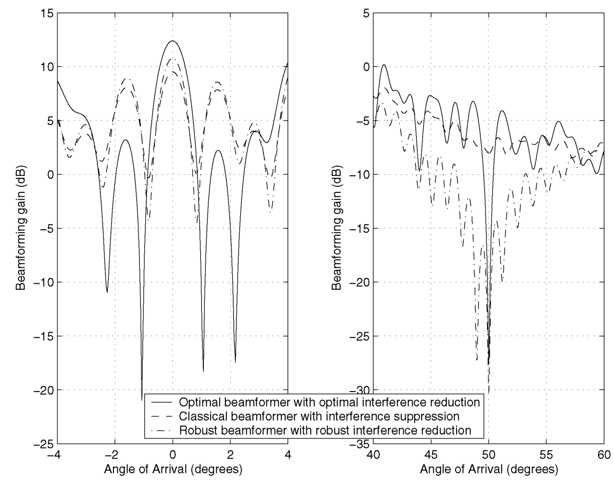

Fig. 2. Average beampattern.

Using (10), we can write the constraint in (18) in the case of flat fading channels as

$$\left|\boldsymbol{w}^H \boldsymbol{V}^{(v)} \hat{\boldsymbol{n}}^{(v)}\right| + \sum_{m=1}^{M} \varepsilon_m \left\|\boldsymbol{R}_m^{(v)\frac{1}{2}} \boldsymbol{w}_m\right\| \leq \zeta^{(v)}, \qquad (20)$$

which is equivalent to the M+1 second-order cone constraints:

$$\left|\boldsymbol{w}^H \boldsymbol{V}^{(v)} \hat{\boldsymbol{n}}^{(v)}\right| \leq \zeta^{(v)} - \sum_{m=1}^{M} \varepsilon_m \alpha_m^{(v)} \qquad (21)$$

$$\left\|\boldsymbol{R}_m^{(v)\frac{1}{2}} \boldsymbol{w}_m\right\| \leq \alpha_m^{(v)} \qquad \forall m = 1, \ldots, M. \qquad (22)$$

Similarly, for LOS propagation we can write (18) as

$$\left|\boldsymbol{w}^H \boldsymbol{V}^{(v)} \hat{\boldsymbol{n}}^{(v)}\right| + \sum_{m=1}^{M} \varepsilon_m \left\|\boldsymbol{a}_m^{(v)H} \boldsymbol{w}_m\right\| \leq \zeta^{(v)} \qquad (23)$$

which is equivalent to $M+1$ second-order cone constraints.

4. NUMERICAL SIMULATIONS

Simulation 1: Line-of-sight propagation environment

We consider the uplink of a wireless communication system with $M = 5$ cooperating terminals. Each terminal is equipped with an antenna array of $k_1 = 4$, $k_2 = 3$, $k_3 = 2$, $k_4 = 4$, and $k_5 = 5$ elements with half-wavelength spacing. The antenna arrays of the first, third, and fourth terminals are located parallel to the X-axis with the center of the arrays presumed to be at $[50.75\lambda, 25\lambda]$, $[75.25\lambda, 0]$, and $[60.75\lambda, -15\lambda]$, respectively. The arrays of the second and fifth terminals are located parallel to the Y-axis with the center of the arrays presumed to be at $[75\lambda, 25.5\lambda]$ and $[90\lambda, \lambda]$, respectively. The actual location of the mth terminal is displaced along the X- and Y-axes from its nominal location by independent random displacements that are uniformly distributed between $[-0.5\lambda\delta_m, 0.5\lambda\delta_m]$ where $\delta_1 = 0.1$, $\delta_2 = 1$, $\delta_3 = 2$, $\delta_4 = 0.2$, and $\delta_5 = 0.1$. The uncertainty sets $\mathcal{A}$ and $\mathcal{A}^{(v)}$ are formed using the values $\{\varepsilon_m = \varepsilon\delta_m\}$. The desired base station is located along $\theta = 0°$, where θ is measured relative to the X-axis, and the wave propagation is planar. A second (non-targeted) base station is located at $\theta = 50°$. All the beamforming vectors are normalized to have unit norm. Simulation results are averaged over 10^4 Monte Carlo runs.

Fig. 1 shows the average received power by the target base station using our robust beamformer in (14) and that with the additional robust interference reduction constraints in (23) where $\zeta^{(v)}$ is selected as 10^{-2}. The performance of the two beamformers is tested for different values of the parameter ε. It also shows the average received power using the classical non-robust beamformer and the maximum received power using the optimal beamformer (with perfect channel knowledge). We can clearly see the SNR improvements achieved by our beamformers compared to the classical beamformer. Moreover, they are not very sensitive to the exact size of the uncertainty sets $\mathcal{A}$ and $\mathcal{A}^{(v)}$ and perform well over a wide range of the parameter ε. We can also notice that the additional constraints in (23) do not considerably degrade the received signal power at the target base station.

Fig. 2 shows the average beampattern versus the angle of transmission, i.e., the received power at different directions. We compare the performance of our robust beamformer (with $\varepsilon = 1$) with the additional robust interference reduction constraints, the classical non-robust beamformer with the interference suppression constraint in (15), and the optimal beamformer with the interference reduction constraint $|\boldsymbol{w}^H \boldsymbol{h}^{(v)}| \leq \zeta^{(v)}$. We can clearly see the effect of the robust interference reduction constraint in widening and deepening the null in the direction of the non-desired base station. We can also notice that the robustness constraint provides high gain at $\theta = 0°$ compared to the classical beamformer.

Simulation 2: Flat fading environment

We consider the same collaborative transmission scenario described in the previous simulation. The propagation environment for each of the 5 terminals is modeled as a Ricean flat fading channel with Ricean K-factor equal to 0.1 and random

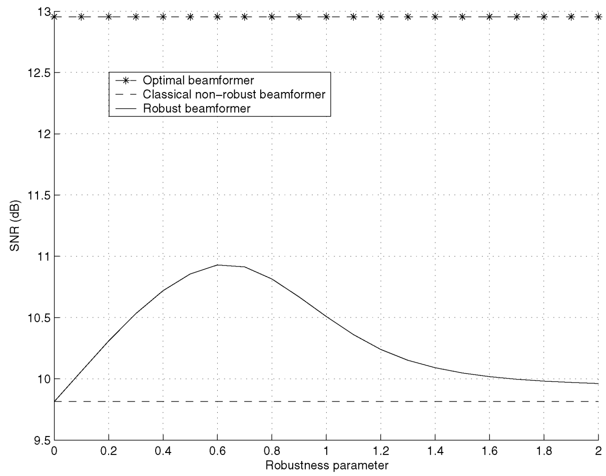

Fig. 3. Average received signal power versus ε.

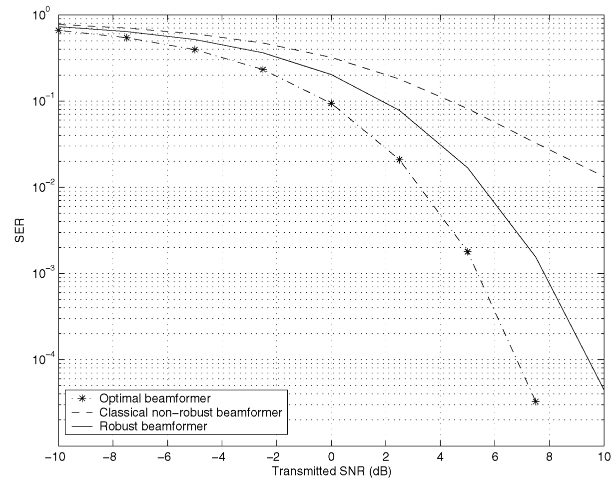

Fig. 4. Average SER versus transmitted SNR.

LOS arrival angles uniformly distributed between $[0, 2\pi]$. The scattered component of the received signal due to each of the 5 terminals has a Laplacian power-angle-profile with random mean angle of arrival uniformly distributed between $[0, 2\pi]$ and angular spread $8°$, $3°$, $2°$, $2°$, and $10°$ for the 1st to 5th terminal, respectively. We generate 100 independent channel realizations. For each channel realization, the estimate of the channel realization vector of the mth terminal is obtained as

$$\hat{\boldsymbol{n}}_m = \boldsymbol{n}_m + \frac{\delta_m}{\|\boldsymbol{\Delta}_m\|}\boldsymbol{\Delta_m} \tag{24}$$

where $\boldsymbol{\Delta}_m$ is modeled as a standard circular Gaussian vector with independent components, and δ_m is the relative magnitude of the error in the channel vector estimate. The values of δ_m are given by 0.2, 3, 2, 4, and 0.1 for $m = 1$ to $m = 5$, respectively. The uncertainty set $\mathcal{A}$ is formed using the values $\{\varepsilon_m = \varepsilon\delta_m\}$. Simulation results are averaged over 50 realizations of $\{\boldsymbol{\Delta}_m\}$ for each of the 100 independent channel realizations. Fig. 3 shows the average received signal power at the base station versus different values of the parameter ε. We can clearly see that our robust beamforming technique can improve the received signal power by more than 1 dB compared to the classical non-robust beamformer. We can also notice that the received signal power does not degrade severely over a wide range of the size of the robustness set. Fig. 4 shows the average symbol error rate (SER) versus the transmitted SNR for different beamformers using a QAM-16 constellation. For our robust beamformer, we have selected the value of ε that yields the highest received SNR. From Fig. 4, it is clear that the power gain offered by our beamformer is translated into a corresponding gain in the SER.

5. CONCLUSION

In this paper, we have presented a framework for collaborative transmit beamforming with robustness against mismatches in the channel state information. Our technique is applicable to both LOS propagation and fading environments. The beamforming vector is derived by minimizing the transmitted power while preserving the received SNR at the target base station for a predefined set of channel realizations centered around the current estimate. The base station calculates the beamforming coefficients using the uplink measurements by solving an SOCP optimization problem. These coefficients are then fed back to the collaborating relay terminals to be used in uplink transmit beamforming. Simulation results have been presented showing the improved performance of our proposed algorithms compared to classical beamforming techniques.

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