2007 IEEE International Symposium
on Signal Processing and Information Technology

Linearly Constrained Adaptive Echo Cancellation for Discrete Multitone Systems

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***Abstract*—*Achieving the full duplex transmission in digital subscriber lines (DSL) is possible by using echo cancellation methods. Most of the methods available for echo cancellation, in DSL systems based on the discrete multitone (DMT) modulation, deploy partial echo cancellation in the time and frequency domain. In these systems, the weights of the FIR filter emulating the echo in the time domain are mapped from the frequency-domain adaptive echo weights. This mapping can be regarded as a linear constraint on an extended weight vector containing weights in the time and frequency domain, and echo cancellation can then be regarded as a linearly constrained optimization on this extended linear space. In this paper, linearly constrained adaptive echo cancellation for DMT-based DSL systems is proposed. Two approaches are introduced based on the adaptive algorithms by Frost and by Griffiths. The proposed structure provides a unifying and flexible framework for echo cancellation in different DSL systems. Furthermore, in the proposed method the proper choice of the constraint can address the drifting problem in the presence of the narrow band noise in the DSL systems.*

***Keywords*—*Echo cancellation, adaptive filtering, DMT modulation, DSL systems.*

I. Introduction

DSL systems are widely used for broadband communication over the existing telephone lines. In order to implement the simultaneous transmission in both directions (upstream and downstream) on a single pair of wires, a hybrid circuit (*i.e.* a four-wire to two-wire interface) must be used to separate the transmitter and receiver circuits from the line [1]. One of the elements that restrict the performance of DSL systems is echo, *i.e.* the leakage of the transmitting signal into the receiver caused by the impedance mismatch in the hybrid circuit. The echo problem can be avoided by using either the frequency division duplexing or an echo canceller.

In frequency division duplexing, there is a frequency gap between the two transmission directions and precise analog filters are used to separate them at the receiver. In the second method, an echo canceller removes the echo by subtracting an emulated echo from the received signal at the receiver. As a result, the two directions can overlap, so the frequency band is used more efficiently and higher data rates are achieved. In the echo canceller, an estimation of the echo is first generated by performing the linear convolution between the transmitted signal (causing the echo) and the estimated echo channel; the estimated echo is then subtracted from the received signal. In this method, the echo channel is modeled by a finite impulse response filter which is adaptively updated.

In DSL systems based on the Discrete Multitone (DMT) modulation, the channel is divided into orthogonal subchannels (or tones), and data are modulated and demodulated by the use of the inverse discrete Fourier transform (IDFT) and DFT operations. In DMT-based systems, where we have access to the presentation of the signal both in the time and frequency domain, the circular echo synthesis (CES) method, proposed by Ho *et. al* [2], is used for emulating the echo. In this method, the circular part of the echo is emulated and cancelled efficiently in the frequency domain and the residual echo is cancelled in the time domain. The filter weights modeling the echo is also updated in the frequency domain. Other methods for echo cancellation propose modifications to this echo canceller in order to lower the complexity and/or improve the performance of the system. For instance in [3], Jones has shown that the proper choice of the delay between the transmitted symbols and received symbols can reduce the complexity of the system. In [4], Ysebaert *et al.* have added double talk cancellation to the receiver, *i.e.* removing the effect of the far end signal, in order to improve the convergence of the adaptive weight update. In [5], they have also proposed an asynchronous echo canceller, which is used when there is a misalignment between the transmitted symbols and received symbols, integrated with double talk cancellation.

In most of the echo cancellation methods for DMT-based systems, the frequency-domain adaptive echo weights are mapped into the time-domain filtering weights used for the echo emulation by means of the inverse Fourier transform. This mapping can be regarded as a linear constraint on an extended set of weights, containing the time and frequency domain weights. Therefore, the echo cancellation problem can be expressed as a linearly constrained optimization on an extended linear space. In this paper, we introduce an extended linearly constrained adaptive (LCA) echo canceller structure for DMT-based systems. We examine two approaches for LCA echo cancellers. The first approach is based on the work by Frost in [6] and the second one is based on the generalized sidelobe canceller (GSC) structure based on the work by Griffiths in [7].

The proposed structure provides a unified framework of existing methods for echo cancellation for DMT-based DSL systems and opens the door to a realm of new possibilities by employing various techniques available for constrained adaptation. In addition, the proper choice of the linear constraint in the proposed method can improve the performance of the system. For example, in this paper we shown that by properly modifying the constraint on the extended weight vector the drifting problem in the presence of the narrow band noise in the DSL systems can be improved. In this paper, because of the space limitation, we only discuss the LCA echo cancellers in the case where the data rates at the transmitter and receiver are equal and the transmitted and received symbols are properly aligned (synchronous system). The extension of this work to multirate and asynchronous systems is possible which will be considered in a separate contribution.

The following notation is used throughout the paper. An $N \times N$ Toeplitz matrix, with the first column given by $[c_0, c_1, \cdots, c_{N-1}]^T$ and the first row given by $[c_0, r_1, \cdots, r_{N-1}]$, is denoted by:

$$T(c_{N-1}, \cdots, c_1, \underline{c_0}, r_1, \cdots, r_{N-1}).$$

In addition, an $N \times N$ circulant matrix, with the first column given by $[c_0, c_1, \cdots, c_{N-1}]^T$, is denoted by $C(c_0, c_1, \cdots, c_{N-1})$.

II. Background

In this paper, we examine the echo canceller using circular echo synthesis (CES) introduced by Ho *et al.* for DMT-based DSL systems [8]. In this method, echo cancellation is performed by subtracting the emulated echo from the received signal. If the echo is periodic (*i.e.* generated by a circular convolution in the time domain) it can be regenerated with reduced complexity in the frequency domain. Therefore, in the echo canceller with CES the only process performed in the time domain is to make the echo periodic by canceling the echo generated by the head and tail of the transmitted signal, and the remaining echo is then removed in the frequency domain with one complex multiplication per tone. Following, the mathematical presentation of this method is discussed.

An ADSL setup is assumed where DMT modulation/demodulation is performed by the use of IDFT/DFT of equal length N both at the transmitter and receiver (see Fig. 1). The transmitted symbol at symbol period k is denoted by $\mathbf{u}(k) = [u_o^k, \cdots, u_{N-1}^k]$. Vector $\mathbf{u}(k)$ is the N-point IDFT of the vector $\mathbf{U}(k)$ which contains the QAM modulated data of the transmitter in the frequency domain. Later, a cyclic prefix with the length υ (which is larger than the assumed length of the far end channel) is added to each symbol. This setup ensures that the channel is divided into a bank of parallel independent subchannels. The true echo channel is represented by the vector $\mathbf{h}$ of size $N \times 1$, modeling the effect of the hybrid circuit, the digital and analog front end filters and the time domain equalizer (TEQ).

In the case where received frames and echo frames are aligned, the echo symbol effecting the received symbol at symbol period k can be described completely by two consecutive transmitted symbols $\mathbf{u}(k-1)$ and $\mathbf{u}(k)$ (two symbols are needed because the length of the echo channel is longer than the cyclic prefix). For clarity, we refer to the transmitted symbols generating echo as echo reference symbols. The echo symbol generated by the linear convolution of the echo reference symbols and the echo channel is given by

$$\mathbf{y}(k) = \mathcal{U}(k)\ \mathbf{h} \quad (1)$$

where $\mathcal{U}(k)$ is an $N \times N$ Toeplitz matrix consisting of the elements from symbols $\mathbf{u}(k)$ and $\mathbf{u}(k-1)$, defined by

$$\mathcal{U}(k) = T(u_{N-1}^k, \cdots, \underline{u_0^k}, u_{N-1}^k, \\ \cdots, u_{N-\upsilon}^k, u_{N-1}^{k-1}, \cdots, u_{\upsilon+1}^{k-1}).$$

In echo cancellers, the true echo channel is unknown, so the emulated echo is estimated by replacing the true echo channel by $\mathbf{w}(k)$ which is the weight vector of the adaptive filter modeling the echo channel. Using (1), the emulated echo is given by

$$\mathbf{y}_e(k) = \mathcal{U}(k)\ \mathbf{w}(k). \quad (2)$$

The echo weights are obtained adaptively using methods such as least mean square (LMS) method, where an error signal is used to update the weights iteratively [9]. The error signal is the difference of the received signal and the emulated echo, given by

$$\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{y}_e(k).$$

As proposed in [8], in order to avoid the matrix multiplication in (2), the echo emulation is performed partially in the time and frequency domain. This can be achieved by rewriting the matrix $\mathcal{U}(k)$ as a sum of a circulant matrix $\mathcal{L}(k)$ and a correction matrix $\mathcal{X}(k)$, as following

$$\mathcal{U}(k) = \mathcal{X}(k)\ +\ \mathcal{L}(k) \quad (3)$$

where matrix $\mathcal{L}(k)$ is the periodic part of $\mathcal{U}(k)$, and contains only the elements of symbol $\mathbf{u}(k)$, defined as

$$\mathcal{L}(k) = C(u_0^k, u_{N-1}^k, \cdots, u_1^k).$$

The matrix $\mathcal{X}(k)$ is a sparse upper triangular matrix equal to $\mathcal{X}(k) = \mathcal{U}(k)\ -\ \mathcal{L}(k)$. Using (3), the emulated echo can be expanded as

$$\mathbf{y}_e(k) = \mathcal{X}(k)\mathbf{w}(k)\ +\ \mathcal{L}(k)\mathbf{w}(k). \quad (4)$$

The circular term $\mathcal{L}(k)\mathbf{w}(k)$ can be implemented in the frequency domain with less complexity, by diagonalizing the circulant matrix $\mathcal{L}(k)$. This can be achieved by using DFT and IDFT matrices, resulting into the decomposition

$$\mathcal{L}(k) = \mathcal{F}_N^{-1}\ \text{diag}(\mathbf{U}(k))\ \mathcal{F}_N$$

where $\mathcal{F}_N$ and $\mathcal{F}_N^{-1}$ are DFT and IDFT matrices of size N such that $\mathcal{F}_N^{-1} = \mathcal{F}_N^H$, and $\mathbf{U}(k) = \mathcal{F}_N\mathbf{u}(k)$. Therefore, the error signal in the frequency domain (*i.e.* $\mathbf{E}(k)$=$\mathcal{F}_N\ \mathbf{e}(k)$) can be written as

$$\mathbf{E}(k) = \mathcal{F}_N\{\mathbf{y}(k) - \mathcal{X}(k)\,\mathbf{w}(k)\}\ -\ \text{diag}(\mathbf{U}(k))\mathbf{W}(k) \quad (5)$$

where $\mathbf{W}(k) = \mathcal{F}_N\mathbf{w}(k)$ is the echo channel weights in the frequency domain. As shown in Fig. 1, partial echo cancellation is performed in the time domain, and the residual

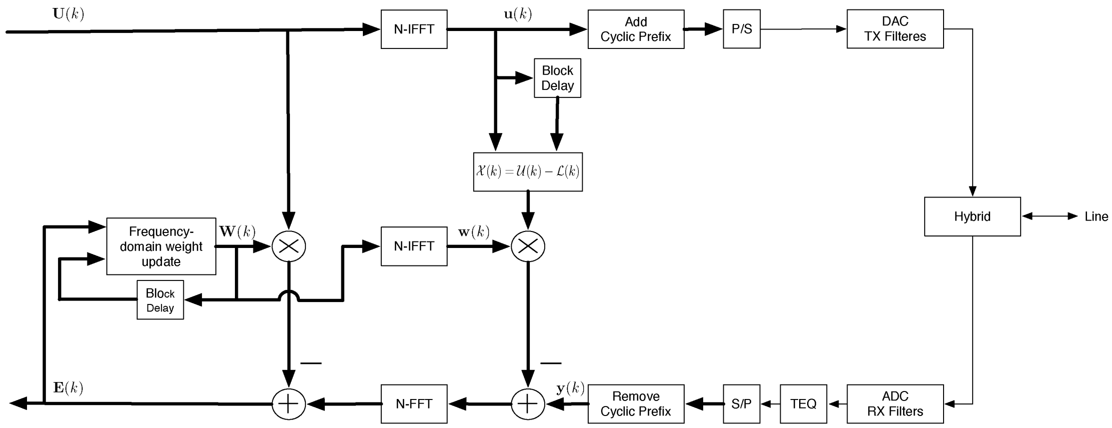

Fig. 1. Block diagram for CES echo canceller

signal is transferred into the frequency domain, where the remaining echo is emulated and cancelled per tone.

Finally, the error signal is used to update the echo weights in the frequency domain, using the LMS method, given by

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \mu \, \text{diag}(\mathbf{U}^*(k))\mathbf{E}(k) \tag{6}$$

where μ is the step size of the adaptive algorithm. The inverse Fourier transform is then used to map the updated weights into the time domain. The complete block diagram for this echo canceller is shown in Fig. 1.

III. Linearly Constrained Adaptive Echo Cancellation

As discussed in the previous section, the echo cancellation in DMT-based DSL systems is based on the dual filtering in the time and frequency domain using the filtering weights mapped from the frequency domain into the time domain. This mapping can be considered as a constraint between the frequency-domain and time-domain weights. Therefore, the adaptive echo cancellation can be regarded as a linearly constrained optimization on an extended linear space containing both the time and frequency domain weights. In the following, we introduce linearly constrained adaptive (LCA) echo cancellers that can be employed in DMT-based systems. Two approaches are discussed for the proposed echo cancellers, the one based on the approach by Frost in [6] and other based on the generalized sidelobe canceller (GSC) by Griffiths in [7]. In this paper, the synchronous case with equal rates is examined, however, these methods are also applicable to the asymmetric and multirate systems.

A. Extended Constrained Optimization Formulation

Similar to an optimization problem where the error is minimized based on a constraint, the echo cancellation for DMT-based systems can also be expressed as the minimization of the error power over an extended linear space. This extended space is spanned by the extended weight vector, containing the weights both in the time and frequency domain, given by

$$\boldsymbol{\omega}(k) = \begin{bmatrix} \mathbf{w}(k) \\ \mathbf{W}(k) \end{bmatrix}$$

where $\boldsymbol{\omega}(k)$ is a vector of size $2N \times 1$. The error signal in (5) can be rewritten in terms of the extended weight vector, *i.e.*

$$\mathbf{E}(k) = \mathcal{F}_N \mathbf{y}(k) - [\mathcal{F}_N \mathcal{X}(k), \ \text{diag}(\mathbf{U}(k))] \, \boldsymbol{\omega}(k) \tag{7}$$

To simplify the notation, we define an extended echo reference matrix $\Phi(k)$ of size $2N \times N$ as

$$\Phi(k) = \begin{bmatrix} \mathcal{X}^H(k) \, \mathcal{F}_N^{-1} \\ \text{diag}(\mathbf{U}^*(k)) \end{bmatrix}$$

and also note that the received signal in the frequency domain is denoted by $\mathbf{Y}(k) = \mathcal{F}_N \mathbf{y}(k)$. Using the above definitions, the error signal in equation (7) can be rewritten as

$$\mathbf{E}(k) = \mathbf{Y}(k) - \Phi^H(k) \, \boldsymbol{\omega}(k). \tag{8}$$

Using this extended notation, the echo cancellation can be expressed as a linearly constrained optimization, where the error power is minimized subject to a liner constraint on $\boldsymbol{\omega}(k)$. This can be formulated as

$$\min \ \text{E}[\|\mathbf{E}(k)\|^2] \quad \text{s.t.} \ \mathcal{C}^H \boldsymbol{\omega}(k) = \mathbf{g} \tag{9}$$

where a linear constraint is assumed, described by the constraint matrix $\mathcal{C}$ of size $2N \times 2N$ and vector $\mathbf{g}$ of size $2N \times 1$.

Current echo cancellers can be interpreted as a LCA echo canceller, where the constraint represents the Fourier transform relation between the weights in the time and frequency domain. This relation is presented in the matrix form as

$$[-\mathcal{F}_N \ \ \mathcal{I}_N] \, \boldsymbol{\omega}(k) = \mathbf{0}.$$

where $\mathcal{F}_N$ is the N-point Fourier transform matrix and $\mathcal{I}_N$ is the identity matrix of size $N \times N$. Therefore, current methods for echo cancellation can be represented as a special case of

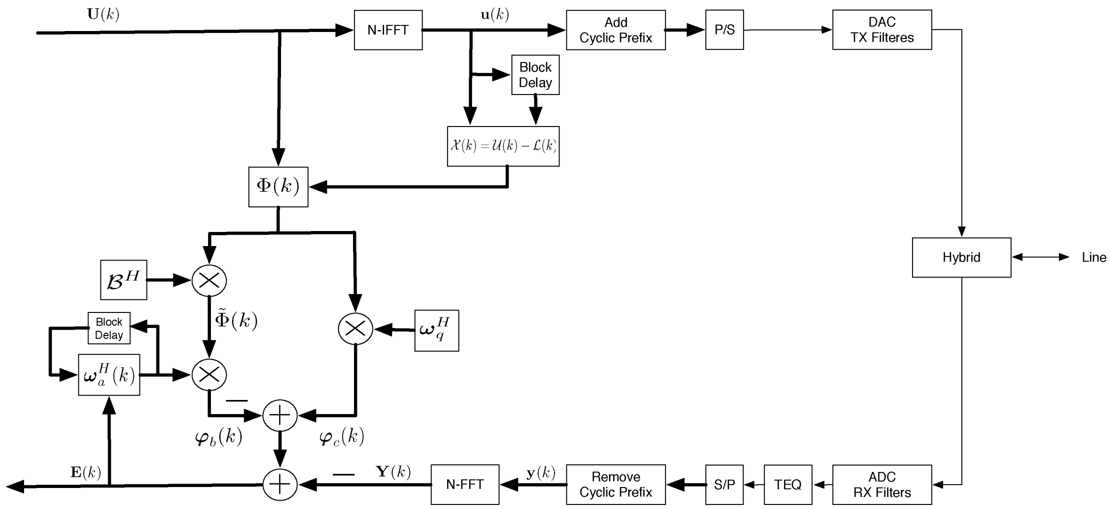

Fig. 2. Block diagram for GSC-based LCA echo canceller

the LCA echo cancellation where the constraint matrix is given by

$$\mathcal{C} = [-\mathcal{F}_N \ , \ \mathcal{I}_N]^H. \tag{10}$$

In the next sections, we introduce the adaptive methods to implement the linearly constrained optimization presented in (9). In our derivation of LCA echo cancellers we assume a general linear constraint on $\boldsymbol{\omega}(k)$, unless otherwise stated.

B. Linearly Constrained Adaptive Echo Canceller

In [6], Frost has proposed an algorithm for implementing linearly constrained adaptive processors, where the Lagrange multiplier method is used to transform the constrained optimization into an unconstrained one. This method can be applied to the constrained optimization defined in (9), where the cost function for the unconstrained problem can then be written as

$$J = \mathrm{E}[\|\mathbf{E}(k)\|^2] + (\boldsymbol{\omega}^H(k)\,\mathcal{C} - \mathbf{g}^H)\boldsymbol{\lambda} + \boldsymbol{\lambda}^H(\mathcal{C}^H\boldsymbol{\omega}(k) - \mathbf{g}). \tag{11}$$

where $\boldsymbol{\lambda}$ a vector of size $2N \times 1$ is the Lagrange multiplier.

Using the steepest decent method [9], the gradient of the cost function can be used to adaptively update the extended weight vector (the calculation of the gradient is based on [10]) given by,

$$\begin{aligned}\boldsymbol{\omega}(k+1) &= \boldsymbol{\omega}(k) - \mu\nabla_{\boldsymbol{\omega}^H} J \\ &= \boldsymbol{\omega}_q + \mathcal{P}_c^{\perp}[\boldsymbol{\omega}(k) - \mu(\mathcal{R}\,\boldsymbol{\omega}(k) - \mathbf{P})]\end{aligned} \tag{12}$$

where $\boldsymbol{\omega}_q = \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathbf{g}$ is the quiescent term and depends only on the constraint. The projection matrix, $\mathcal{P}_c^{\perp} = \mathcal{I}_N - \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathcal{C}^H$ is the projection onto the subspace orthogonal to the subspace spanned by the constraint matrix ($\mathcal{I}_N$ is the identity matrix). The matrix $\mathcal{R}$ and the vector $\mathbf{P}$ are the correlation matrix and cross correlation vector, respectively, defined by

$$\mathcal{R} = \mathrm{E}[\Phi(k)\,\Phi^H(k)], \tag{13}$$

$$\mathbf{P} = \mathrm{E}[\Phi(k)\,\mathbf{Y}(k)]. \tag{14}$$

In the LMS adaptive method these correlations are replaced by their instantaneous estimates given by

$$\mathcal{R}(k) = \Phi(k)\,\Phi^H(k),$$

$$\mathbf{P}(k) = \Phi(k)\,\mathbf{Y}(k).$$

Equation (12) provides the adaptive update for the extended weight vector which is used to emulate and cancel the echo. In the next section, a more efficient implementation of the LCA echo canceller is introduced based on the approach in [7].

C. LCA Echo Canceller based on Generalized Sidelobe Canceller

One of the efficient implementations for adaptive constrained optimization is proposed by Griffiths in [7]. In this method, known as generalized sidelobe canceller (GSC), the $2N$-dimensional space is divided into two subspaces, the constraint subspace spanned by the constraint matrix and the subspace orthogonal to this one spanned by the blocking matrix $\mathcal{B}$, where each column of the $\mathcal{B}$ is orthogonal to each column of $\mathcal{C}$ (*i.e.* $\mathcal{B}^H\mathcal{C} = \mathbf{0}$). Originally, this adaptive processor was introduced in the beamforming applications, where the processor is partitioned into two processing paths. One path processes the desired-signal and the other one, filtered by the blocking matrix, processes the interference and the noise.

Employing this method, the extended weight vector can be written as two components resulted from its projection onto the two subspaces, given by

$$\boldsymbol{\omega}(k) = \boldsymbol{\omega}_q - \mathcal{B}\,\boldsymbol{\omega}_a(k) \tag{15}$$

where $\boldsymbol{\omega}_q$ is its projection onto the constraint subspace and $\mathcal{B}\,\boldsymbol{\omega}_a(k)$ is its projection onto the orthogonal subspace [11].

As noted in the previous section, $\boldsymbol{\omega}_q$ depends only on the constraint and can be calculated once. Using some arithmetic and the assumption that the blocking matrix is unitary, the adaptive update for the extended weight vector for the GSC-based LCA echo canceller can be written as

$$\boldsymbol{\omega}_a(k+1) = \boldsymbol{\omega}_a(k) + \mu\,\mathcal{B}^H[\mathcal{R}\,\boldsymbol{\omega}_q - \mathbf{P} - \mathcal{R}\,\mathcal{B}\,\boldsymbol{\omega}_a(k)] \quad (16)$$

where the matrix $\mathcal{R}$ and the vector $\mathbf{P}$ are defined in (13) and (14), respectively.

It is notable that if the rank of the constraint matrix is M_c, the rank of the blocking matrix is equal to $2N - M_c$. Therefore, the size of the matrices in the update equation for the GSC-based LCA echo canceller (16) is $2N - M_c$ compared to the $2N$ of the previous LCA method in (12).

This echo canceller can be implemented efficiently, where the echo emulation and cancellation is performed as a part of the adaptive algorithm. The adaptive equation can be rewritten as

$$\boldsymbol{\omega}_a(k+1) = \boldsymbol{\omega}_a(k) + \mu\tilde{\Phi}(k)\ \left[\boldsymbol{\varphi}_c(k) - \mathbf{Y}^H(k) - \boldsymbol{\varphi}_b(k)\right]^H \quad (17)$$

where $\tilde{\Phi}(k) = \mathcal{B}^H\,\Phi(k)$ and $\boldsymbol{\varphi}_c(k) = \boldsymbol{\omega}_q^H\,\Phi(k)$ and $\boldsymbol{\varphi}_b(k) = \boldsymbol{\omega}_a^H(k)\tilde{\Phi}(k)$. The block diagram for GSC-based LCA echo canceller is shown in Fig. 2.

IV. Simulation Results

In this section, we compare the performance of the proposed LCA echo cancellers with the current echo cancellers. In addition, we examine an application of the proposed structure where by modifying the constraint the robustness of the system is improved in the presence of the narrow band noise.

A. Comparison of LCA echo canceller with current echo Cancellrs

In order to examine the performance of the proposed methods, the convergence of the LCA echo cancellers are compared with the current echo cancellation methods. In the simulations an ADSL system with the carrier serving area (CSA) loop #1 setup is used. In addition DMT modulation is used where for upstream tones 7-31 and for downstream tones 33-255 are used, and each tone transmits a 4-QAM signal constellation. The downstream and upstream signal transmit with -40dbm/Hz, and the echo reference signal contains 20 dB lower power on the unused tones to ensure convergence. The external additive noise is white Gaussian noise at -140 dBm/Hz. The transmit block length is 64 and the receive block length is 512. The true echo channel contains 512 samples at 2.2MHz, while the number of echo canceller taps used is 220 and the taps are initialized with all zero. The results for echo cancellation is given in Fig. 3, where the convergence of the echo cancellers using CES and LCA methods are compared. As expected, the performance of the proposed methods with the constraint given by (10) is the same as the performance of the CES echo canceller.

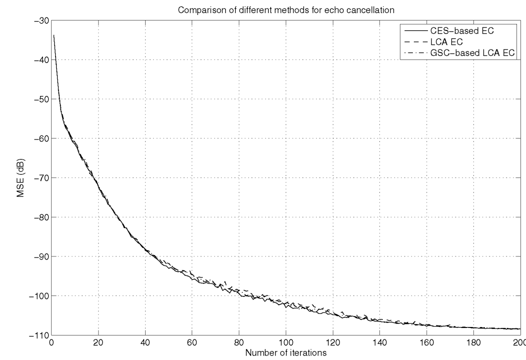

Fig. 3. Convergence comparison for different echo cancellation methods

B. LCA echo canceller in the presence of narrow band noise

One of the impairments in the DSL systems is the narrow band noise (NBN), which is usually a narrow band and high amplitude noise over a short period of time. In the presence of NBN, the weights corresponding to the tones where the noise is present increase substantially. Therefore, even after the removal of the NBN the echo canceller needs to offset for the large error on these tones. This phenomenon is known as the drifting problem.

The LCA echo canceller can be deployed to improve the robustness of the system in the presence of the NBN. If the NBN is detected on some tones [12], the constraint can be modified to zero the weights corresponding to these tones in the frequency domain. Therefore, after the removal of the NBN, these tones do not suffer from the large increase on their corresponding weights. It is notable that in this case the constraint includes the Fourier transform relation and the linear equations to ensure that the effected tones are forced to zero. This modified constraint is only used during the presence of NBN, and in the absence of the noise the usual constraint is used.

In order to examine the convergence of the echo canceller in the presence of the NBN, the noise is introduced to the system after the echo canceller has converged initially (iteration 200) and removed after iteration 300. The error for echo cancellation in the presence of NBN around tone 120 is shown in Fig. 4. For the echo canceller with the modified constraint, the constraint is modified to force 7 tones around this tone to zero for the duration of the NBN. While the noise is present, for the echo canceller with the regular constraint the weights corresponding to the tones around the noise are augmented drastically to compensate for the large error on these tones. Therefore, even after the removal of the NBN source, the system experiences high error because of the drifting problem. However, for the echo canceller with modified constraint the convergence to the true echo channel is achieved in several iteration after the NBN source is removed.

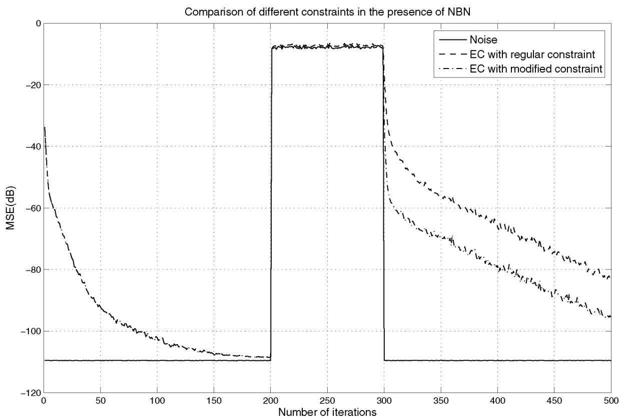

Fig. 4. Comparison of different echo cancellers in the presence of NBN

V. Conclusion

In this paper we have introduced a linearly constrained adaptive echo canceller for DMT-based DSL systems where echo is cancelled partially in the frequency and time domain. Two approaches have been discussed based on [6] and [7]. LCA echo canceller can be used to implement various methods for constrained optimization in order to improve the convergence of the system. In addition, the proper choice of the constraint can improve the robustness of the system. For instance, the modified constraint is used to improve the robustness of the system in the presence of the NBN.

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