

# Iterative Joint Channel and Noise Variance Estimation and Primary User Signal Detection for Cognitive Radios

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**Abstract**—In this paper, we introduce a joint channel and noise variance estimation, and primary user (PU) signal detection scheme using the expectation-maximization (EM) algorithm for cognitive radios. In our investigation, we consider two scenarios: In the first scenario, the channel and noise variance are assumed to be perfectly known by the secondary user (SU). Here, we propose a maximum-likelihood (ML) solution of the PU signal detection as an upper bound on the performance of the proposed joint estimation and detection (JED) scheme. We also provide an iterative implementation of the ML-based detector using the EM algorithm. In the second case, we extend our work to the problem of channel and noise variance estimation in cognitive radios, where we propose an iterative JED scheme based on the EM algorithm. The simulation results show that the proposed JED scheme can iteratively attain a reliable performance with few iterations and modest computational complexity.

## I. INTRODUCTION

The current spectrum allocation policy divides the available radio frequency spectrum into fixed bands for specific applications. However, the frequency resources have become scarce as more wireless services/applications have emerged into the market. Furthermore, the Federal Communication Commission (FCC) in the U.S.A. has reported that the spectrum utilization varies greatly depending on the temporal and geographical location [1] with a nominal rate between 15% to 85% [2]. Hence, the need for a technology to overcome the spectrum scarcity and under-utilization has emerged in recent years. Cognitive radio (CR) has been proposed as the possible solution for the aforementioned shortcomings of the fixed spectrum allocation policy by providing opportunistic spectrum access over the licensed and unlicensed bands [3], [4].

The focus of this work will be on CR networks operating over the bands licensed to the primary users (PUs). In this context, CR is defined as an intelligent radio which can sense its surrounding environment to exploit the unoccupied spectrum bands without causing harmful interference to the PU's transmission. Spectrum sensing is the core of this operation as the mean to detect the spectrum holes and PU emergence. The most common spectrum sensing techniques in the literature are energy detection (ED) [5], [6], match filter detection [7], [8], and cyclostationary feature detection [9], [10]. The

choice of the appropriate technique is dictated by the *a priori* knowledge about the PU's signal and the receiver complexity. The matched-filter is the optimal detection technique when the PU's signal is known. The cyclostationary feature detector exploits the periodicity of the modulated signal to distinguish it from the stationary noise; however, it suffers from high computational complexity. The ED is the optimal detection scheme if the PU's signal is unknown. In this paper, we assume that there is no *a priori* knowledge about the PU's signal structure and modulation. Therefore, in this context, the ED is the most appropriate spectrum sensing technique.

Most works conducted in this area assume perfect knowledge of the channel and noise variance at the secondary user (SU) unit, and few researchers have investigated the effect of estimation errors on the performance of the detection process and possible estimation techniques [11]. Recently, there has been a growing interest in iterative joint estimation and detection (JED) techniques because of their ability to achieve accurate estimation without wasting the system resources [12]. In particular, the expectation-maximization (EM) algorithm has been proposed in iterative receivers due to its attractive features such as iteratively attaining the maximum-likelihood (ML) solution with reduced complexity [13] [14].

In this work, we first present a spectrum sensing technique based on the ML algorithm, assuming a perfect knowledge of the channel coefficients and noise variance by the SU. We also provide an iterative implementation of the ML-based detector using the EM algorithm, and we prove that the performance of the EM-based detector converges iteratively to the ML solution. Then, we extend our work to the problem of channel and noise variance estimation, where we propose an iterative JED scheme based on the EM algorithm. The results show that the proposed scheme enhances the quality of spectrum sensing with short processing time and modest computational complexity. The rest of the paper is organized as follows. The system model is described in Section II. In Section III, the ML-based spectrum sensing is presented. In Sections IV, we introduce the EM-based spectrum sensing and its extension to JED with unknown channel and noise variance. The initialization of the EM-based JED is discussed in Section V. Simulation results and discussions are then presented in Section VI. Finally, conclusions are drawn in Section VII.

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## II. SYSTEM MODEL

Consider that the SU detects the presence of the PU signal over a wideband frequency spectrum, which is divided into  $K$  frequency subbands. The channel between the PU and the SU experiences multipath fading with  $L$  resolvable paths. In this case, the discrete-time baseband complex envelope representation of the signal observed by the SU over the wideband frequency spectrum is defined as

$$r(n) = \sum_{l=0}^{L-1} h(l)s(n-l) + v(n), \quad (1)$$

where  $s(n)$  is the PU signal, and  $h(l)$  is the channel coefficient corresponding to the  $l$ -th path between the PU and the SU, and  $v(n)$  is a complex, zero-mean, additive white Gaussian noise term. Using a  $K$ -point discrete Fourier Transform (DFT) operation, successive frames of  $r(n)$  are decomposed into narrowband discrete frequency components as follows

$$R_k(m) = \sum_{n=0}^{K-1} r(mK+n)e^{-j2\pi nk/K}, \\ = H_k S_k(m) + V_k(m), \quad k = 0, 1, \dots, K-1, \quad (2)$$

where  $k$  is the frequency index,  $m \in \{0, 1, \dots, M-1\}$  is the frame index, and  $M$  is the number of frames available for detection. In (2),  $H_k$ ,  $S_k(m)$  and  $V_k(m)$  denote the  $k$ -th DFT coefficients of  $h(l)$ ,  $s(mK+n)$  and  $v(mK+n)$ , respectively. Also, the product of  $H_k$  and  $S_k(m)$  in (2) is an approximation of the corresponding convolution in (1) under the assumption that  $K > L$ .

Consider the following statistical model of  $\{S_k(m)\}$ ,  $\{H_k\}$  and  $\{V_k(m)\}$ , which is widely adopted in the literature (see, e.g., [15]). The PU signal samples,  $\{S_k(m)\}$ , and background noise samples,  $\{V_k(m)\}$ , are modeled as independent random processes, whereby, for any given state of occupancy of the wideband channel, samples from each process are independent across frequency and frame indices, and obey a zero-mean complex circular Gaussian distribution. We assume that the noise variance,  $E[|V_k(m)|^2] = \sigma_v^2$ , and the channel coefficients  $H_k$  remain constant during the processing interval of  $M$  frames. Without loss of generality, we set  $E[|S_k(m)|^2] = 1$  if the  $k$ -th subband is occupied while  $E[|S_k(m)|^2] = 0$  if the PU signal is absent. Using the formalism of subband occupancy proposed in [5], the occupancy of the  $k$ -th subband is modeled as a binary random variable (indicator),  $B_k$ , with realization  $b_k \in \{0, 1\}$ , where 0 represents a spectrum hole, while 1 indicates the presence of a PU signal in the  $k$ -th subband. Accordingly, the conditional signal power of the PU in the  $k$ th subband is expressed as  $E[|S_k(m)|^2 | B_k = b_k] = b_k$ . The mean and the variance of  $R_k(m)$  conditioned on  $B_k = b_k$  are given by

$$E[R_k(m) | B_k = b_k] = 0, \quad (3)$$

$$\text{Var}[R_k(m) | B_k = b_k] = b_k G_k + \sigma_v^2, \quad (4)$$

where  $E[\cdot]$  denotes the expected value,  $\text{Var}[\cdot]$  is the variance operator, and  $G_k = |H_k|^2$ . In the following sections, we propose a spectrum sensing technique based on the ML algorithm.

Initially, we assume perfect knowledge of the channel state information (CSI) and noise variance by the SU receiver. Then, we present an iterative implementation of the proposed ML-based spectrum sensing using the EM algorithm. Finally, we introduce a joint channel and noise variance estimation, and PU signal detection scheme based on the EM algorithm.

## III. ML-BASED SPECTRUM SENSING

In our work, we assume independent subband occupancy: the joint distribution of the occupancy vector,  $\mathbf{B} = [B_0, \dots, B_{K-1}]^T$ , is given by  $f_{\mathbf{B}}(\mathbf{b}) = \prod_{k=0}^{K-1} f_{B_k}(b_k)$ , where  $\mathbf{b} = [b_0, \dots, b_{K-1}]^T$ . Let  $\mathbf{R}$  represents the received samples over  $K$  subbands, i.e.,  $\mathbf{R} = [\mathbf{R}_0^T, \dots, \mathbf{R}_{K-1}^T]^T$ , where  $\mathbf{R}_k = [R_k(0), \dots, R_k(M-1)]^T$ , with corresponding realizations  $\mathbf{r} = [\mathbf{r}_0^T, \dots, \mathbf{r}_{K-1}^T]^T$ , where  $\mathbf{r}_k = [r_k(0), \dots, r_k(M-1)]^T$ . Then, the log-likelihood function of  $\mathbf{R}$  given  $\mathbf{B} = \mathbf{b}$  is defined as follows

$$L_{\mathbf{R}|\mathbf{B}}(\mathbf{r}|\mathbf{b}) = \sum_{k=0}^{K-1} L_{\mathbf{R}_k|B_k}(\mathbf{r}_k|b_k) \\ = -M \sum_{k=0}^{K-1} \ln(G_k b_k + \sigma_v^2) - \sum_{k=0}^{K-1} \frac{1}{G_k b_k + \sigma_v^2} \sum_{m=0}^{M-1} |r_k(m)|^2. \quad (5)$$

Since the subband occupancies are independent of each other, the maximization process of (5) can be done independently for each  $k$ , i.e.,  $\hat{B}_k^{ML} = \arg \max_{b_k} L_{\mathbf{R}_k|B_k}(\mathbf{r}_k|b_k)$ . This leads to the "soft" occupancy estimation

$$\hat{B}_k^{ML} = \max \left\{ 0, \frac{1}{MG_k} \left( \sum_{m=0}^{M-1} |r_k(m)|^2 - M\sigma_v^2 \right) \right\}. \quad (6)$$

The right-hand side of (6) is indeed the ML estimate of the transmitted PU energy over the  $k$ -th subband. This value can be used to determine the occupancy of subband  $k$  by comparing  $\hat{B}_k^{ML}$  with a certain threshold,  $\gamma_k$ , resulting in a hard estimate of  $B_k$ , that is  $\hat{B}_k^{ML} \in \{0, 1\}$ , as follows

$$\hat{B}_k^{ML} \begin{cases} \hat{B}_k^{ML} = 1 \\ \geq \\ \gamma_k \\ \leq \\ \hat{B}_k^{ML} = 0 \end{cases} \quad (7)$$

### A. Performance Analysis

For a given probability of false alarm, the optimum threshold, which gives the maximum probability of detection, is derived using the Neyman-Pearson criterion as follows [16]. Let  $Z_k = \frac{1}{MG_k} \left( \sum_{m=0}^{M-1} |R_k(m)|^2 - M\sigma_v^2 \right)$ , and assume that  $M$  is sufficiently large. In this case, according to the central limit theorem,  $Z_k$  is approximately normally distributed under each hypothesis,  $B_k \in \{0, 1\}$ . Therefore, the probability of false alarm in the  $k$ -th subband,  $P_f^k$  is given by

$$P_f^k(\gamma_k) = Pr(\hat{B}_k^{ML} \geq \gamma_k | B_k = 0) \\ = Q \left( \frac{\gamma_k - E[Z_k | B_k = 0]}{\sqrt{\text{Var}[Z_k | B_k = 0]}} \right), \quad (8)$$

Using (3) and (4), and with some mathematical manipulations, the conditional mean and variance of  $Z_k$  given  $B_k = 0$  are obtained as

$$E[Z_k|B_k = 0] = 0, \quad (9)$$

$$\text{Var}(Z_k|B_k = 0) = \frac{\sigma_v^4}{MG_k^2}. \quad (10)$$

Under the constraint  $P_f^k(\gamma_k) = \epsilon_k$ , where  $0 < \epsilon_k \leq 1$ , the optimum threshold is given by

$$\gamma_k^{\text{opt}} = Q^{-1}(\epsilon_k) \sqrt{\frac{\sigma_v^4}{MG_k^2}}, \quad (11)$$

and consequently, the optimum probability of detection is derived as follows

$$\begin{aligned} P_d^k(\gamma_k^{\text{opt}}) &= \Pr(\hat{B}_k^{ML} \geq \gamma_k^{\text{opt}} | B_k = 1) \\ &= Q\left(\frac{\gamma_k^{\text{opt}} - E[Z_k|B_k = 1]}{\sqrt{\text{Var}[Z_k|B_k = 1]}}\right). \end{aligned} \quad (12)$$

Similar to (9) and (10), the mean and variance of  $Z_k$  conditioned on  $B_k = 1$  are given by

$$E[Z_k|B_k = 1] = 1, \quad (13)$$

$$\text{Var}(Z_k|B_k = 1) = \frac{(G_k + \sigma_v^2)^2}{MG_k^2}. \quad (14)$$

Fig. 1 presents the receiver operating characteristic (ROC) curve of the proposed ML-based detector and the traditional energy detector [6] for  $M = 100$  and  $G_k = 0.25$  for all  $k$ . The detection threshold of both detectors are derived based on Neyman-Pearson condition. The results show that the ROC curve of the ML detector achieves a perfect match with the performance of the energy detector. We also note that the ML-based detection amounts to energy detection based on the ML estimate of the transmitted PU energy over each subband.

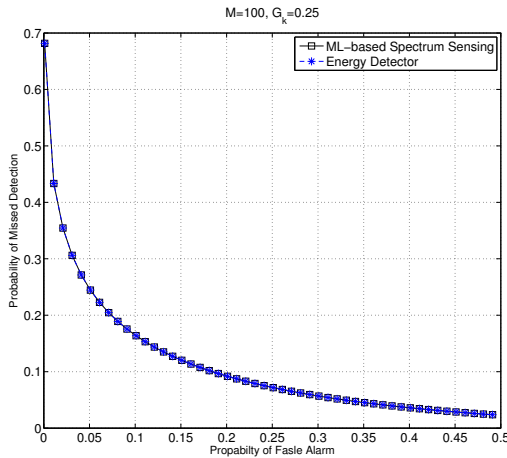


Fig. 1. ROC curve of ML-based spectrum sensing

#### IV. EM-BASED SPECTRUM SENSING

In this part, we propose an iterative spectrum sensing technique based on the EM algorithm. In our investigation, we consider two scenarios: in the first scenario, we assume a perfect knowledge of the channel between the PU and the SU, and the noise variance at the SU side. In this case, the EM-based detector provides an iterative estimation of the transmitted PU energy. We also prove that the EM estimate of this energy converges to the ML solution in (6) with few iterations. In the second scenario, the SU has no prior knowledge of the channel coefficients and noise variance. This case results in a multidimensional optimization problem, where the ML solution is not feasible because of its high computational complexity. In the literature, the EM algorithm is known to achieve the ML solution with low computation complexity. Therefore, we propose an iterative joint channel and noise variance estimation, and PU signal detection based on the EM algorithm. Since we assume independent subband occupancy over the wideband spectrum, the estimation process is only presented for one subband. In other words, the same procedure of estimating the unknown parameters is applied in each subband.

##### A. Both $H_k$ and $\sigma_v^2$ are known

In this case, the only unknown parameter is  $B_k$ , and vector  $\mathbf{R}_k$  is defined as the incomplete data according to the EM terminology [14]. Let  $\mathbf{S}_k = [S_k(0), \dots, S_k(M-1)]^T$ , then the so-called complete data are defined as  $\mathbf{Y}_k = [\mathbf{R}_k^T, \mathbf{S}_k^T]^T$ . The complete data log-likelihood function is given by

$$\begin{aligned} L_{\mathbf{Y}_k|B_k}(\mathbf{y}_k|b_k) &= L_{\mathbf{R}_k|\mathbf{S}_k, B_k}(\mathbf{r}_k|\mathbf{s}_k, b_k) + L_{\mathbf{S}_k|B_k}(\mathbf{s}_k|b_k) \\ &= -M \ln(\sigma_v^2) - M \ln(b_k) - \frac{1}{b_k} \sum_{m=0}^{M-1} |s_k(m)|^2 \\ &\quad - \frac{1}{\sigma_v^2} \sum_{m=0}^{M-1} |r_k(m) - H_k s_k(m)|^2 \end{aligned} \quad (15)$$

where  $\mathbf{s}_k = [s_k(0), \dots, s_k(M-1)]^T$ ,  $\mathbf{y}_k = [y_k(0), \dots, y_k(M-1)]^T$ , and  $y_k(m) = [r_k(m), s_k(m)]^T$ . In the E-step of the EM algorithm, we estimate the conditional expectation of (15) given  $\mathbf{R}_k = \mathbf{r}_k$  and  $B_k = \hat{b}_k^{(i)}$ , where  $\hat{b}_k^{(i)}$  is the EM estimate of  $b_k$  at the  $i$ -th iteration. By neglecting the terms independent of  $b_k$ , we obtain

$$\begin{aligned} \Delta(b_k|\hat{b}_k^{(i)}) &= E[L_{\mathbf{Y}_k|B_k}(\mathbf{Y}_k|b_k)|\mathbf{R}_k = \mathbf{r}_k, B_k = \hat{b}_k^{(i)}] \\ &= -M \ln(b_k) - \frac{1}{b_k} \sum_{m=0}^{M-1} E[|S_k(m)|^2|\mathbf{r}_k, \hat{b}_k^{(i)}], \end{aligned} \quad (16)$$

and  $E[|S_k(m)|^2|\mathbf{r}_k, \hat{b}_k^{(i)}] = |E[S_k(m)|\mathbf{r}_k, \hat{b}_k^{(i)}]|^2 + \text{Var}[S_k(m)|\mathbf{r}_k, \hat{b}_k^{(i)}]$ . Since  $\mathbf{R}_k$  and  $\mathbf{S}_k$  are jointly gaussian, the conditional mean and variance of  $S_k(m)$  given  $\mathbf{r}_k$  and  $\hat{b}_k^{(i)}$  are given respectively by [16]

$$E[S_k(m)|\mathbf{r}_k, \hat{b}_k^{(i)}] = \frac{\hat{b}_k^{(i)} H_k^*}{\hat{b}_k^{(i)} G_k + \sigma_v^2} r_k(m), \quad (17)$$

$$\text{Var}[S_k(m)|\mathbf{r}_k, \hat{b}_k^{(i)}] = \frac{\hat{b}_k^{(i)} \sigma_v^2}{\hat{b}_k^{(i)} G_k + \sigma_v^2}, \quad (18)$$

where \* denotes the complex conjugate operation. Now, the M-step of the EM algorithm is performed by maximizing (16) with respect to  $b_k$ , which results in

$$\hat{b}_k^{(i+1)} = \frac{1}{M} \sum_{m=0}^{M-1} E[|S_k(m)|^2 | \mathbf{r}_k, \hat{b}_k^{(i)}]. \quad (19)$$

The convergence of the EM estimate of  $b_k$ ,  $\hat{b}_k^{(i+1)}$ , to the ML solution is proven as follows. Let  $\hat{b}_k^{\text{inf}} = \lim_{i \rightarrow \infty} \hat{b}_k^{(i)}$ . Then, by substituting  $\hat{b}_k^{\text{inf}} = \hat{b}_k^{(i+1)} = \hat{b}_k^{(i)}$  in (17)-(19), we have

$$\begin{aligned} \hat{b}_k^{\text{inf}} &= \left| \frac{\hat{b}_k^{\text{inf}} H_k^*}{\hat{b}_k^{\text{inf}} G_k + \sigma_v^2} \right|^2 \frac{1}{M} \sum_{m=0}^{M-1} |r_k(m)|^2 \\ &+ \frac{\hat{b}_k^{\text{inf}} \sigma_v^2}{\hat{b}_k^{\text{inf}} G_k + \sigma_v^2}. \end{aligned} \quad (20)$$

By solving this equation, we obtain  $\hat{b}_k^{\text{inf}} \in \left\{ 0, \frac{1}{MG_k} \left( \sum_{m=0}^{M-1} |r_k(m)|^2 - M\sigma_v^2 \right) \right\}$ , which is the ML estimate of  $B_k$ .

### B. Both $H_k$ and $\sigma_v^2$ are unknown

In this case, the unknown parameter vector is define as  $\xi_k = [b_k, \sigma_v^2, H_k]$ . Here, we use the same definition of the complete data in Section IV-A. Assume that  $\hat{\sigma}_v^{2(i)}$  and  $\hat{H}_k^{(i)}$  are the EM estimates of  $\sigma_v^2$  and  $H_k$  at the  $i$ -th iteration respectively. By taking the conditional expectation of (15) given  $\mathbf{R}_k = \mathbf{r}_k$  and  $\xi_k = \hat{\xi}_k^{(i)}$ , where  $\hat{\xi}_k^{(i)} = [\hat{b}_k^{(i)}, \hat{\sigma}_v^{2(i)}, \hat{H}_k^{(i)}]$ , we obtain

$$\begin{aligned} \Delta(\xi_k | \hat{\xi}_k^{(i)}) &= -M \ln(b_k) - \frac{1}{b_k} \sum_{m=0}^{M-1} E[|S_k(m)|^2 | \mathbf{r}_k, \hat{\xi}_k^{(i)}] \\ &- M \ln(\sigma_v^2) - \frac{1}{\sigma_v^2} \sum_{m=0}^{M-1} E[|r_k(m) - H_k S_k(m)|^2 | \mathbf{r}_k, \hat{\xi}_k^{(i)}]. \end{aligned} \quad (21)$$

By taking the derivative of (21) with respect to  $b_k$  and solving the resultant equation, we obtain

$$\hat{b}_k^{(i+1)} = \frac{1}{M} \sum_{m=0}^{M-1} E[|S_k(m)|^2 | \mathbf{r}_k, \hat{\xi}_k^{(i)}], \quad (22)$$

where  $E[|S_k(m)|^2 | \mathbf{r}_k, \hat{\xi}_k^{(i)}] = |E[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)}]|^2 + \text{Var}[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)}]$ . Similar to (17) and (18), the conditional mean and variance of  $S_k(m)$  given  $\mathbf{r}_k$  and  $\hat{\xi}_k^{(i)}$  are given by

$$\begin{aligned} E[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)}] &= \frac{\hat{b}_k^{(i)} \hat{H}_k^{(i)*}}{\hat{b}_k^{(i)} \hat{G}_k^{(i)} + \hat{\sigma}_v^{2(i)}} r_k(m), \\ \text{Var}[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)}] &= \frac{\hat{b}_k^{(i)} \hat{\sigma}_v^{2(i)}}{\hat{b}_k^{(i)} \hat{G}_k^{(i)} + \hat{\sigma}_v^{2(i)}}. \end{aligned} \quad (23)$$

Following the same procedure as above, we can derive the EM estimates of  $H_k$  and  $\sigma_v^2$  at the  $(i+1)$ -iteration. By maximizing (21) with respect to  $H_k$ , we obtain

$$\hat{H}_k^{(i+1)} = \frac{\sum_{m=0}^{M-1} r_k(m) E[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)*}]}{\sum_{m=0}^{M-1} E[|S_k(m)|^2 | \mathbf{r}_k, \hat{\xi}_k^{(i)}]}, \quad (24)$$

Subsequently, we substitute  $H_k$  by  $\hat{H}_k^{(i+1)}$  in (21), which is maximized with respect to  $\sigma_v^2$ , yielding

$$\hat{\sigma}_v^{2(i+1)} = \frac{1}{M} \sum_{m=0}^{M-1} \hat{X}_k(m) \quad (25)$$

where

$$\begin{aligned} \hat{X}_k(m) &= |r_k(m)|^2 - r_k^*(m) \hat{H}_k^{(i+1)} E[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)}] \\ &- r_k(m) (\hat{H}_k^{(i+1)})^* E[S_k(m) | \mathbf{r}_k, \hat{\xi}_k^{(i)*}] + |\hat{H}_k^{(i+1)}|^2 \\ &\times E[|S_k(m)|^2 | \mathbf{r}_k, \hat{\xi}_k^{(i)}]. \end{aligned} \quad (26)$$

## V. INITIALIZATION

Since the EM algorithm is sensitive to the initialization of the parameters to be estimated [17], we assume that our proposed EM-based JED is initialized by reliable estimates of the unknown parameters. This guarantees that the performance of our proposed scheme converges to the ML solution with few iterations. In our case, we assume that the SU receives a short training sequence from a CR in the vicinity of the PU, which is used to give an initial estimate of  $H_k$ ,  $\hat{H}_k^{(0)}$ , e.g., based on the minimum-mean square error (MMSE) estimation technique. Assuming that the SU has *a priori* knowledge about the history of the PU activities, the initialization of  $\sigma_v^2$ ,  $\hat{\sigma}_v^{2(0)}$ , is performed by estimating the sample variance of the observations when the PU is absent, i.e.,  $R_k(m) = V_k(m)$ . Finally, each  $b_k$  is initialized by  $\gamma_k$ , i.e.,  $\hat{b}_k^{(0)} = \gamma_k$ .

## VI. SIMULATION RESULTS

In this part, the performance of the proposed spectrum sensing scheme based on the EM algorithm is evaluated through its ROC curve, considering the two scenarios introduced in Section IV-A and IV-B. Throughout our simulations, we assume that  $M = 150$  and  $\sigma_v^2 = 1$ . Also,  $10^5$  trials are performed for each choice of  $\gamma_k$ , and the performance of the EM-based JED is evaluated after 3 iterations. Since the estimation of the unknown parameters of each subband is performed independently from other subbands, our results are presented only for one subband.

Fig. 2 plots the ROC for both the ML-based spectrum sensing and the EM-based spectrum sensing schemes assuming that  $H_k$  and  $\sigma_v^2$  are perfectly known by the SU. The simulations are performed for a time-invariant channel with  $G_k=0.25$ . The results show that the performance of the EM-based detector achieves a perfect match with the ML solution after 3 iterations.

In Fig. 3, we evaluate the performance of the proposed EM-based JED over a Rayleigh fading channel, where  $H_k$  or  $\sigma_v^2$  is assumed unknown by the SU. In the simulations,

$H_k$  is modeled as a complex Gaussian random variable with zero mean and variance 1. We also assume that  $H_k$  is constant during a block interval of  $M$  frames and changes independently from one block to another. As a reference, we plot the performance of the EM-based detector assuming a perfect estimation of  $H_k$  and  $\sigma_v^2$ . The estimation quality of the proposed JED is examined assuming different scenarios: (i)  $H_k$  is unknown while  $\sigma_v^2$  is known (ii)  $H_k$  is known while  $\sigma_v^2$  is unknown (iii) Both  $H_k$  and  $\sigma_v^2$  are unknown. Compared to the perfect estimation case, the EM-based JED achieves a reliable estimation of the channel while it is sensitive to the error in the noise variance estimation.

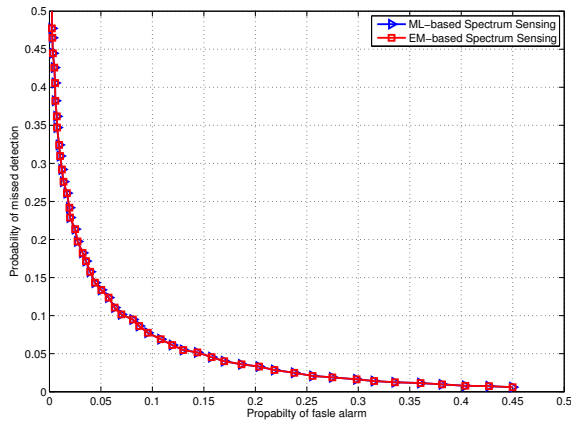


Fig. 2. ROC of ML and EM-based spectrum sensing schemes ( $M=150$ , 3-iteration)

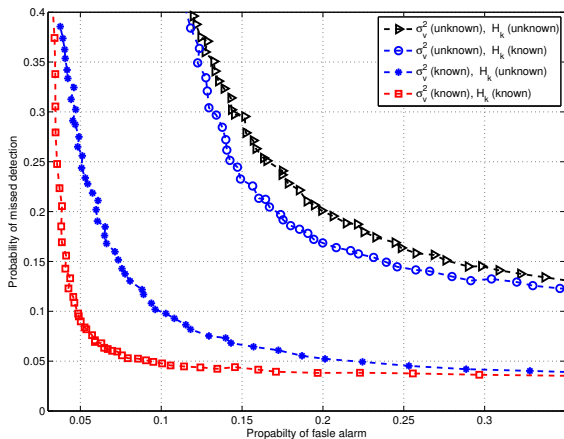


Fig. 3. ROC curve of EM-based JED over Rayleigh fading channels ( $M=150$ , 3-iteration)

## VII. CONCLUSION

We developed an iterative EM-based spectrum sensing scheme for cognitive radios. Assuming perfect knowledge of the channel coefficients and noise variance, we showed that

the performance of the EM-based spectrum sensing scheme converged iteratively to the ML solution within few iterations. Then, we extended our work to the problem of channel and noise variance estimation, where we proposed an iterative JED scheme based on the EM algorithm. The simulation results showed that the joint estimation of the unknown parameters using the EM algorithm enhanced the detection process.

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