SIMPLE DESIGN OF FILTER BANKS FOR SUBBAND ADAPTIVE FILTERING

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ABSTRACT

Subband adaptive filtering is an important application of filter banks in which maximally decimated filter banks can not be used in general because of decimation aliasing effects. This leads to the use of oversampling schemes in the filter bank design wherein the perfect-reconstruction (PR) or near PR property is still required. In this work, a simple design technique for uniform DFT filter bank with near PR property is presented for the purpose of subband adaptive filtering. The prototype filter in the proposed filter banks can be obtained simply by performing an interpolation of a two-channel QMF filter. The filter bank design technique presented in this paper is of particular interest in engineering applications, as demonstrated by design examples.

1 INTRODUCTION

Subband adaptive filtering has received much attention in recent years especially in the context of acoustic echo cancellation (AEC) [1]-[2]. In a typical subband filtering scheme for AEC, both the input and the reference signals are split into subband components by analysis banks. Adaptive filters are applied in each subband at a decimated rate and the resulting outputs are recombined by a synthesis bank to create a full-band output signal at the original rate. In this application, a PR (perfect reconstruction) or near PR property for the filter banks is also needed because this subband system could also be used to transmit local speech signals during double talk periods.

Generally, maximally decimated filter banks, which have received considerable attention in the literature and are used in subband coding, can not be applied directly in a subband system for AEC due to aliasing effects [2]; accordingly, oversampling scheme (i.e, non-critical sampling) is typically used in this type of application of filter banks. Yet, despite the importance of providing simple and concrete ways to design filter banks satisfying certain requirements in engineering applications, specific design techniques of filter banks with arbitrary oversampling rates which are suitable for subband adaptive filtering do not appear to be available at the moment.

In this paper, we present a simple and systematic design technique for uniform DFT filter banks, which satisfy near-PR property in the oversampling scheme. With the proposed filter bank structure, design of the analysis/synthesis prototype filter simply amounts to performing an interpolation of the well-known two channel QMF filters, which have been tabulated in [3] and [4]. Weighted-overlap-add approaches [3] are used for the implementation of the filter banks such that an arbitrary sampling rate in subbands can be obtained efficiently.

2 A STRUCTURE FOR UNIFORM DFT FILTER BANKS

A structural diagram of the proposed K-channel filter banks is shown in Fig. 1, where $W_K = \exp[j2\pi/K]$ and $h(n)$ (analysis filter) and $g(n)$ (synthesis filter) are lowpass filters with cutoff frequency $\omega_c = \pi/K$. The ideal lowpass property for $H(z)$ should be

$$|H(e^{j\omega})| = \begin{cases} 1, & 0 \le |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases} \quad (1)$$

One can see that the analysis bank is a standard uniform DFT analyzer while the synthesis bank represents a generalized form of the DFT synthesizer found in [3]. In the analysis bank, after modulation and lowpass filtering, the subband signal is decimated by an integer factor M to produce the kth subband signal, $X_k(m)$. The latter can be expressed in the z-domain as

$$X_k(z) = \frac{1}{M}\sum_{m=0}^{M-1} H(z^{1/M}W_M^{-m})X(z^{1/M}W_M^{-m}W_K^k) \quad (2)$$

where $W_M = \exp[j2\pi/M]$.

In the synthesis bank, each subband signal is first upsampled by the factor M and then passed to the common synthesis filter $g(n)$. The output of each filter are demodulated and then added to produce the synthesized output $\hat{x}(n)$. If no modifications are made to the subband signals $X_k(z)$, the synthesizer output $\hat{x}(n)$ can be expressed in the z-domain as

$$\hat{X}(z) = \sum_{k=0}^{K-1} X_k(z^M W_K^{-kM})G(zW_K^{-k})W_K^k \quad (3)$$

Substituting (2) into (3), we have

$$\hat{X}(z) = \sum_{m=0}^{M-1} T_m(z) X(z W_M^{-m}) \quad (4)$$

where

$$T_m(z) = \frac{1}{M} \sum_{k=0}^{K-1} W_K^k H(z W_K^{-k} W_M^{-m}) G(z W_K^{-k}) \quad (5)$$

We now make the following assumptions: (a) the downsampling rate M is less than the number of the subband K, i.e., $M < K$; (b) the analysis filter $h(n)$ is a FIR filter, whose length L can be expressed as a multiple of the subband number K; (c) The synthesis filter $g(n)$ is obtained by flipping $h(n)$ in the time domain, i.e., $g(n) = h(L-n-1),\ n = 0, \ldots, L-1$.

Under these assumptions, it can be shown that aliasing components in the synthesis bank output $\hat{x}(n)$ will be eliminated approximately, i.e., $T_m(z) \approx 0$ for $m = 1, \ldots, M-1$, and that the equivalent transfer function between $x(n)$ and $\hat{x}(n)$ can be expressed as:

$$T_0(e^{j\omega}) = e^{-j(L-1)\omega} \frac{1}{M} \sum_{k=0}^{K-1} \left| H(e^{j\omega} W_K^{-k}) \right|^2 \quad (6)$$

Clearly, $T_0(e^{j\omega})$ has a linear phase, which is one of the desirable property of the filter bank.

Then, it remains only to design the lowpass filter $H(z)$ to minimize the amplitude distortion of $T_0(e^{j\omega})$. According to (6), an ideal flatness requirement for $|T_0(e^{j\omega})|$ can now be expressed as

$$\sum_{k=0}^{K-1} \left| H(e^{j\omega} W_K^{-k}) \right|^2 = 1, \quad \text{for all } \omega \quad (7)$$

In the next section, a simple approach is given to design the prototype filter $H(z)$ such that $|T_0(e^{j\omega})|$ is acceptably flat.

3 PROTOTYPE FILTER DESIGN

The requirements (1) and (7) can not be satisfied exactly for any FIR filter $H(z)$. The design problem for $H(z)$ thus converts to approximate these requirements under a certain criterion. A commonly used criterion [3], [4] is to minimize an error function E, which is defined as

$$E = \alpha E_s + E_r \quad (8)$$

where α is a real positive weighting factor while E_S and E_r express the errors in approximating the conditions (1) and (7), respectively, and are given by

$$E_s = \int_{\omega_s}^{\pi} \left| H(e^{j\omega}) \right|^2 d\omega \quad (9)$$

$$E_r = \int_0^{2\pi} \left| \sum_{k=0}^{K-1} \left| H(e^{j\omega} W_K^{-k}) \right|^2 - 1 \right| d\omega \quad (10)$$

In (9), ω_s ($\omega_s > \omega_c$) is the stopband edge which determines the transition band of $H(e^{j\omega})$. The basic idea of the technique that we propose below for the design of the prototype $h(n)$ is to transform the K-channel problem (8)-(10) into an equivalent problem for a 2-channel QMF bank. Solutions for the latter can then be obtained easily by computation or table look-up. The details of this approach are now exposed.

Suppose K is even, i.e., $K = 2I$, where I is a positive integer. When $K = 2$ (i.e., $I = 1$) and $h(n)$ is symmetrical, the cost function in (8) is identical to that used in Johnston's scheme [4]. As a result, minimizing (8) will lead to solutions for the well-known two-channel QMF bank. Let $h_0(n)$ denote the resulting QMF prototype. The corresponding transfer function $\tilde{T}_0(e^{j\omega})$, obtained from (6) with $K = 2$, will be

$$\tilde{T}_0(e^{j\omega}) = e^{-j(L-1)\omega} \frac{1}{2} \{ \left| H_0(e^{j\omega}) \right|^2 + \left| H_0(-e^{j\omega}) \right|^2 \} \quad (11)$$

We recall that QMF filters have been widely used in two-channel filter banks and their coefficients have been tabulated [3], [4] for various choices of the parameters L, α and ω_S.

For $I > 1$, solutions for the filter design based on minimizing the cost function (8) can be obtained with the use of computer-aided optimization techniques. For example, the Hooke and Jeaves search algorithm used in Johnston's scheme [4] can also be applied to (8) for designing $h(n)$. Instead of using complex optimization programs, however, we will show that a simple way to design $h(n)$ is just to perform an I-point interpolation of a two-channel QMF prototype $h_0(n)$. The simplicity of this design procedure will bring great convenience in practical engineering applications.

To this end, let $h(n)$ be obtained from the I-point interpolation of $h_0(n)$ followed by anti-imaging lowpass filtering, i.e.

$$H(e^{j\omega}) = H_0(e^{jI\omega}) F(e^{j\omega}) \quad (12)$$

where $F(e^{j\omega})$ is a low-pass filter with cutoff frequency at π/I. The amplitude responses for $H_0(e^{jI\omega})$ and $F(e^{j\omega})$ are illustrated in Fig. 2 for the case $K = 8$. Since $H_0(e^{j\omega})$ itself is lowpass with cutoff frequency at $\pi/2$, it is not difficult to design the anti-imaging filter $F(e^{j\omega})$ such that (see Fig. 2)

$$|H(e^{j\omega})| = \begin{cases} |H_0(e^{j\omega I})|, & |\omega| \le \frac{\pi}{I} \\ 0, & \frac{\pi}{I} < |\omega| < \pi \end{cases} \quad (13)$$

A simple way to perform such an I-point interpolation, for example, is to use the Matlab interpolation function interp(), such as h = interp(h0, I), to obtain $h(n)$ satisfying the property (13) with a good approximation.

Substituting (13) into (6), one can show that

$$T_0(e^{j\omega}) = e^{-j(L-1)\omega}\frac{1}{M}\{|H_0(e^{j(\omega I)})|^2 + |H_0(-e^{j(\omega I)})|^2\} \quad (14)$$

Comparing (14) with (11), we see that $|T_0(e^{j\omega})|$ has the same flatness as $|\tilde{T}_0(e^{j\omega})|$ does. Thus, the reconstruction error for the filter bank with $H(e^{j\omega})$ should be consistent to that for the corresponding two-channel QMF bank based on the $K = 2$ prototype $h_0(n)$.

In summary, the prototype filter $h(n)$ used for the K-channel filter bank in Fig.1 can be obtained by performing a $K/2$-point interpolation on a two-channel QMF prototype $h_0(n)$. The reconstruction error is determined by the corresponding two-channel QMF bank. As mentioned earlier, QMF prototypes can be found conveniently from tables in [3] and [4] and a Matlab interpolation function can be used to perform the corresponding interpolation. In this way, the usual filter design procedure which needs complex optimization programing is avoided. This characteristic is of particular significance in engineering applications.

4 REALIZATION

It is well-known that the uniform DFT filter bank is the most efficient one in terms of realization. There are basically two different approaches for the efficient realization of uniform DFT filter banks [3]: one is based on the polyphase structure and the other one is based on the weighted-overlap-add (WOA) structure. The polyphase structure is only suitable for a decimation factor M satisfying $K = M\ i$, where i is a positive integer. The WOA structure is more general and can be used with an arbitrary value of the decimation factor M. In the context of subband adaptive filtering, the WOA structure is thus preferred since it offers more flexibility in the selection of the decimation factor M.

The WOA method for the realization of the uniform DFT analysis bank can be found in [3]. Since the synthesis bank in Fig. 1 differs from the conventional uniform DFT synthesis bank [3] in the choice of the demodulation function, corresponding differences will appear in their WOA-based realization. Due to space limitation, details about this realization will not be provided here.

5 DESIGN EXAMPLES

In this section, we present one filter bank design example that demonstrates the simplicity of application of the new technique that we propose in this work as well as the quality of the resulting filter banks.

In this example, a $K = 8$ channel filter bank is designed. We first selected a QMF prototype 32D tabulated in [3]-[4] and denoted it as $h_0(n)$. This filter has a total of $L_0 = 32$ symmetrical coefficients and its stopband attenuation is 38 dB; when used in a two-channel QMF bank, the reconstruction error is 0.025 dB. To design a $K = 8$ channel filter bank, $h(n)$ is obtained from $h_0(n)$ by using the Matlab function h = interp(h0,4) to perform an $I = 4$ point interpolation. The resulting prototype filter $h(n)$ has $L = 128$ coefficients. The cut-off frequency of $H(e^{j\omega})$ is at 0.5/8 = 0.0625. Fig. 3 shows the magnitude response of $H(e^{j\omega})$.

The amplitude distortion of the filter bank can be seen from the plot of $M|T_0(e^{j\omega})|$ appearing in Fig. 4. The amplitude distortion is within ±0.035 dB . The aliasing distortions are shown in Fig. 5 for $M = 4$, 7 and 8 (critical sampling rate).

As can be seen from this figure, high aliasing occurs only for critical sampling rate; when $M = 7$ (or less), aliasing errors are acceptably small (less than -56dB). Thus, in this example, a filter bank which can be used appropriately for subband adaptive filtering can be obtained by choosing $M = 7$ in the designed filter bank.

6 CONCLUSIONS

Filter banks with oversampling rates in the subbands are typically used in subband adaptive filtering. In this work, a simple design technique for a modified uniform DFT filter bank was presented. The proposed filter bank exhibits the following properties: (a) near-PR property in the oversampling scheme; (b) simple design procedure based on an interpolation of a 2-channel QMF prototype; (c) efficient weighted-overlap-add realization. These characteristics are particularly interesting for engineering applications.

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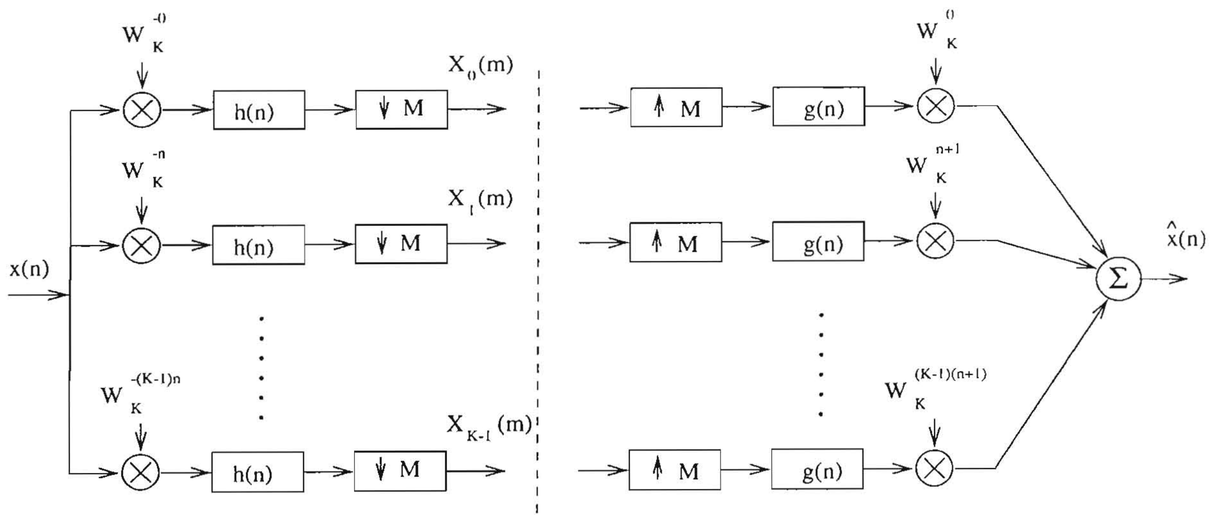

Fig. 1. Uniform DFT filter banks: analyzer (left) and synthesizer (right)

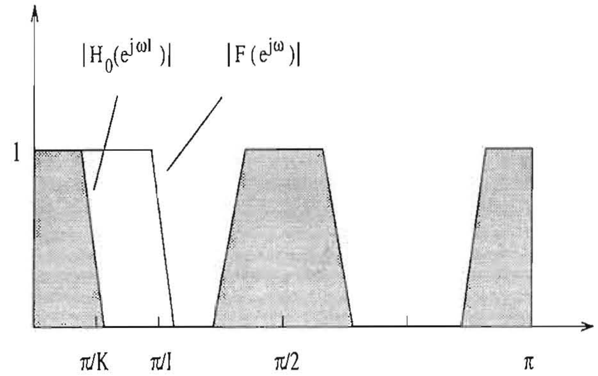

Fig. 2. Amplitude responses of $H_0(e^{j\omega l})$ and $F(e^{j\omega})$ for the case $K = 8$

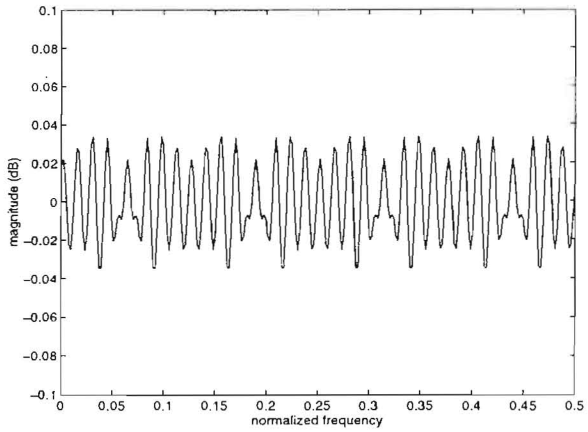

Fig. 4. Magnitude distortion $M\,|T_0(e^{jw})|$

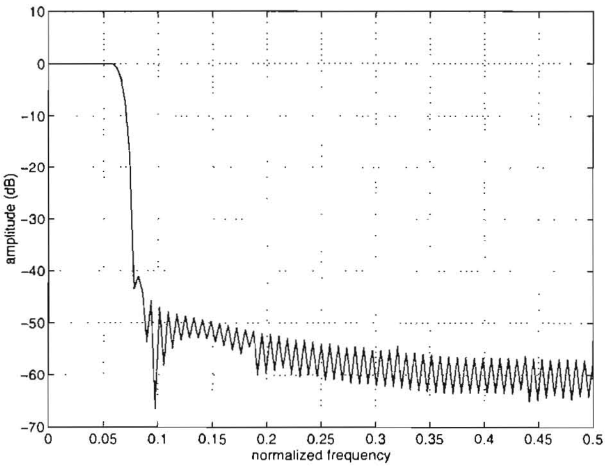

Fig. 3. Magnitude response of a $K = 8$ channel prototype filter with length $L = 128$

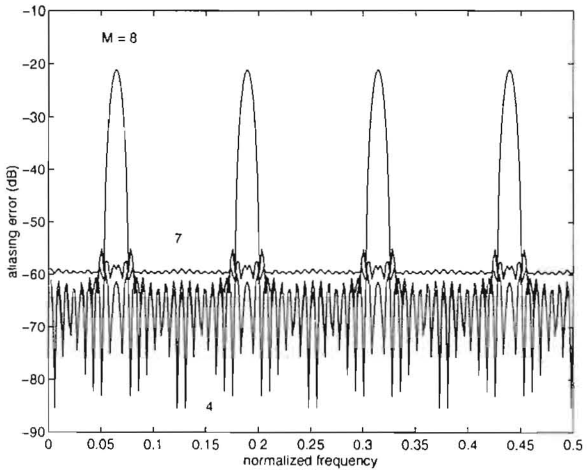

Fig. 5. Aliasing distortion of the filter bank for different downsampling rate M