COMPARATIVE EVALUATION OF THE DUAL TRANSFORM DOMAIN ECHO CANCELLER FOR DMT-BASED SYSTEMS

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ABSTRACT

In DMT-based communication systems where full-duplex transmission is required, digital echo cancellers are employed to cancel echo by means of adaptive filters. In order to reduce the computational complexity of these cancellers, the structure of the Toeplitz matrix containing the transmitted signal is usually exploited to transform the time domain signals and perform the emulation and adaptive update in a more convenient domain (*e.g.* frequency domain). In this paper, we consider a recently proposed dual transform domain echo canceller, which is based on the general decomposition of the data Toeplitz matrix. A comprehensive comparative performance evaluation of the proposed method with the existing methods is provided. This evaluation includes the comparison of the convergence curves and computational cost of the algorithms. The comparison shows that the proposed canceller achieves a faster convergence with a low error floor with no increase in the complexity.

Index Terms— Echo cancellation, Discrete multitone, Transform domain adaptive filters, DSL systems.

1. INTRODUCTION

Echo cancellers are mainly used to accomplish full-duplex transmission on digital subscriber lines (DSL) with overlapping upstream and downstream bandwidths. These cancellers use adaptive filters to combat the interference of the transmitted signal on the collocated receiver, also known as echo. In DSL standards, discrete multitone (DMT) modulation is widely adopted because of its relative robustness against severe channel conditions. Various methods have been suggested for echo cancellation for DMT-based DSL systems where the computational cost of the canceller is reduced by exploiting the structure of the DMT signals.

In [1], Ho *et al.* introduced the circular echo synthesis (CES) canceller, in which the echo is emulated partially in the time domain and partially in the frequency domain, and the weights of the adaptive filter estimating the echo channel are updated in the frequency domain on a per tone basis. This approach reduces the computational complexity of the canceller, but suffers from slow convergence due to the lack of excitation on certain tones, which can be caused by power mask requirements or bit allocation algorithms in DSL systems. The convergence of the CES canceller is enhanced by the transmission of dummy data on the unused tones, at the expense of the extra interference.

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In [2], Ysebaert *et al.* suggested the circulant decomposition canceller (CDC). In this method, the echo is also emulated in the time and frequency domains, however, the tap-input vector used for the weight update usually has sufficient excitation on all tones. Later, in [3], Pisoni and Bonaventura introduced the symmetric decomposition canceller (SDC) which uses discrete cosine and sine transformations. They derived the relationship between the SDC canceller and CDC, and proposed a more efficient implementation of the latter based on the new decomposition. Eventually, in [4], Pisoni *et al.* introduced a canceller similar to the self-orthogonalizing filter, where the singular value decomposition (SVD) of the correlation matrix is used to improve the slow-modes.

In [5], we introduce the dual transform domain canceller for DMT-based DSL systems. In this approach, a pair of unitary transforms are used to map the received and transmitted signals into alternate domains where the echo emulation and weight adaptation can be performed more conveniently. This approach provides a unifying representation of the previous algorithms for echo cancellation and also provides a better understanding of the trade-off between the complexity reduction and improved convergence. In this paper, we study this algorithm in detail. We present the proof of the adaptive update formula used in this algorithm and show that it performs an exact adaptation in the dual domain. In addition, we expand this algorithm for the common practical case in which the length of the echo canceller is shorter than the symbol frame size. Finally, a comparative performance study is presented where the proposed canceller is compared with the existing cancellers, in terms of their convergence behaviour and computational cost. The results given show that the proposed canceller has a faster convergence with a low error floor. In addition, it is not affected by the lack of excitation on the unused tones and can be implemented with no increase in the computational complexity of the algorithm.

In Section 2, current methods for echo cancellation for DMT-based systems are reviewed. In Section 3, the dual transform domain canceller is discussed, and in Section 4, the proof for the dual domain update is given and the expanded version is discussed. Finally, in Section 5, the comparative analysis is presented including the convergence curves and computational complexity figures. In this paper, the square identity matrix of size N is denoted by $\mathcal{I}_N$, and the all zero matrix of size $N \times M$ is denoted by $\mathbf{0}_{N\times M}$. The discrete Fourier transformation and its inverse are denoted by $\mathcal{F}_N$ and $\mathcal{F}_N^{-1}$, respectively. Finally, diag$\{\mathbf{v}\}$ indicates a diagonal matrix whose diagonal elements are given by vector $\mathbf{v}$.

2. BACKGROUND

We assume a symmetric DMT transceiver with symbol frame size of N and cyclic prefix size of υ. For this setting, the emulated echo at each symbol period is expressed by

$$\mathbf{y}_e^k = \mathcal{U}^k\, \mathbf{w}^k \tag{1}$$

where $\mathbf{w}^k$ is the weight vector of the echo canceller. For the general asynchronous case with a delay of Δ samples between the echo frames and received far-end frames, the matrix $\mathcal{U}^k$ in (1) is an $N \times N$ Toeplitz matrix consisting of elements from symbols, $\mathbf{u}^{k-1}$, $\mathbf{u}^k$ and $\mathbf{u}^{k+1}$. The first row of this matrix is $[u_\Delta^k, \cdots, u_0^k, u_{N-1}^k, \cdots, u_{N-\upsilon}^k, u_{N-1}^{k-1}, \cdots, u_{\Delta+\upsilon+1}^{k-1}]$ and its first column is $[u_\Delta^k, \cdots, u_{N-1}^k, u_{N-\upsilon}^{k+1}, \cdots, u_{N-1}^{k+1}, u_0^{k+1}, \cdots, u_{\Delta-\upsilon-1}^{k+1}]^T$, where u_i^k $(i = 0, \cdots, N-1)$ denotes the i^{th} sample of the k^{th} symbol. Using the received symbol $\mathbf{y}^k$, the error signal, $\mathbf{e}^k = \mathbf{y}^k - \mathbf{y}_e^k$, can then be used to adaptively update the weights. The least mean square (LMS) update is given by

$$\mathbf{w}^{k+1} = \mathbf{w}^k + \mu\, \mathcal{U}^{k^T} \mathbf{e}^k, \tag{2}$$

where μ denotes the step size.

In order to avoid the matrix multiplication in (1) and (2), in [1], Ho *et al.* introduced the CES canceller, where the matrix $\mathcal{U}^k$ is decomposed as a sum:

$$\mathcal{U}^k = \mathcal{X}^k + \mathcal{L}^k. \tag{3}$$

In (3), $\mathcal{L}^k$ is a circulant matrix with the first column given by $[u_\Delta^k, \cdots, u_{N-1}^k, u_0^k, \cdots, u_{\Delta-1}^k]^T$ and $\mathcal{X}^k = \mathcal{U}^k - \mathcal{L}^k$ is a residual component. The circulant matrix $\mathcal{L}^k$ can be diagonalized using the Fourier transform, where the diagonal elements $\mathbf{V}^k$ are obtained by the Fourier transform of the first column of $\mathcal{L}^k$. Therefore, the echo emulation can be done partially in the time and partially in the frequency domain with a lower cost, *i.e.*,

$$\mathbf{E}^k = \mathcal{F}_N \left(\mathbf{y}^k - \mathcal{X}^k\, \mathbf{w}^k\right) - \text{diag}\{\mathbf{V}^k\}\, \mathbf{W}^k \tag{4}$$

where $\mathbf{W}^k$ and $\mathbf{E}^k$ are the Fourier transforms of the echo channel weights and error signals, respectively. Also, the complexity is reduced in the weight update step by using an approximation, where the data matrix $\mathcal{U}^k$ is substituted by $\mathcal{L}^k$. In the frequency domain, this approximation results into a per tone LMS update, given by

$$\mathbf{W}^{k+1} = \mathbf{W}^k \;+\; \tilde{\mu}\, \text{diag}\{\mathbf{V}^{k^*}\}\, \mathbf{E}^k. \tag{5}$$

In [2], Ysebaert *et al.* proposed the CDC method to ameliorate the convergence of the CES canceller. In this approach, the Toeplitz matrix $\mathcal{U}^k$ is decomposed into a sum of a circulant matrix and a skew-circulant matrix. Then, by transforming the latter into a complex valued circulant matrix, the two circulant matrices are diagonalized using the Fourier transform (for more details see [2]). Consequently, the error signal in the frequency domain is given by

$$\mathbf{E}^k = \mathcal{F}_N \left(\mathbf{y}^k - \frac{1}{2}\mathcal{Q}^H \mathcal{F}_N^{-1} \text{diag}\{\tilde{\mathbf{V}}_{\text{odd}}^k\} \mathcal{F}_N\, \mathcal{Q}\, \mathbf{w}^k\right) - \frac{1}{2}\text{diag}\{\tilde{\mathbf{V}}_{\text{even}}^k\} \mathbf{W}^k \tag{6}$$

where $\tilde{\mathbf{V}}_{\text{even}}^k$ and $\tilde{\mathbf{V}}_{\text{odd}}^k$ are the elements obtained from the diagonalization of the circulant and skew-circulant parts. Matrix $\mathcal{Q} = \text{diag}\{[1, \cdots, e^{-j\pi(i-1)/N}, \cdots, e^{-j\pi(N-1)/N}]^T\}$, and $\mathbf{W}^k$ is the echo weight vector in the frequency domain. The per tone approximate weight update is performed in the frequency domain as

$$\mathbf{W}^{k+1} = \mathbf{W}^k \;+\; \hat{\mu}\, \text{diag}\{\tilde{\mathbf{V}}_{\text{even}}^{k^*}\}\, \mathbf{E}^k. \tag{7}$$

As shown in [2], if $\Delta \neq -\upsilon$, then the CDC algorithm provides an acceptable convergence since the elements of $\tilde{\mathbf{V}}_{\text{even}}^k$ are nonzero and so the transmission of dummy data on the unused tones is not needed.

In [3], Pisoni and Bonaventura proposed the SDC method, where the Toeplitz matrix $\mathcal{U}^k$ is decomposed as a sum of symmetric/anti-symmetric Toeplitz and Hankel matrices. Each of these matrices is then diagonalized individually, using discrete cosine and sine transforms (DCT and DST). Consider the column vector $\mathbf{a}^k$, $a^k(i) = \mathcal{U}^k(0,i) + \mathcal{U}^k(i,0)$ for $(i = 0, \cdots, N-1)$ and the vector $\mathbf{b}^k$, with $b^k(i) = \mathcal{U}^k(0,i) - \mathcal{U}^k(i,0)$ for $(i = 0, \cdots, N-1)$, where $\mathcal{U}^k(i,j)$ denotes the entry in row i, column j of the matrix $\mathcal{U}^k$. Hence, the matrix $\mathcal{U}^k$ can be written as the sum

$$\begin{aligned}\mathcal{U}^k = {} & \mathcal{C}_{\text{II}}^T\, \tilde{\mathcal{Z}}^T \mathcal{D}^k\, \tilde{\mathcal{Z}} \mathcal{C}_{\text{II}} + \mathcal{S}_{\text{II}}^T\, \mathcal{Z}^T \mathcal{D}^k\, \mathcal{Z} \mathcal{S}_{\text{II}} \\ & + \mathcal{C}_{\text{II}}^T\, \tilde{\mathcal{Z}}^T \tilde{\mathcal{D}}^k\, \mathcal{Z} \mathcal{S}_{\text{II}} - \mathcal{S}_{\text{II}}^T\, \mathcal{Z}^T \tilde{\mathcal{D}}^k\, \tilde{\mathcal{Z}} \mathcal{C}_{\text{II}}\end{aligned} \tag{8}$$

where $\mathcal{C}_{\text{II}}$ and $\mathcal{S}_{\text{II}}$ are the $N \times N$ Type-II DCT and DST matrices, respectively [6]. The $(N+1) \times N$ matrices $\mathcal{Z} = [\mathbf{0}_{N\times 1} | \mathcal{I}_N]^T$ and $\tilde{\mathcal{Z}} = [\mathcal{I}_N | \mathbf{0}_{N\times 1}]^T$. The $(N+1) \times (N+1)$ matrices $\mathcal{D} = \frac{1}{2}\text{diag}\{[\tilde{\mathcal{C}}_I[a(0), \cdots, a(N-1)], 0]^T\}$ and $\tilde{\mathcal{D}} = \frac{1}{2}\text{diag}\{[0, \tilde{\mathcal{S}}_I[b(1), \cdots, b(N-1)]^T, 0]\}$, where $\tilde{\mathcal{C}}_I$ and $\tilde{\mathcal{S}}_I$ are the non-normalized DCT-I and DST-I matrices, respectively [6]. In this algorithm, the echo emulation is performed using the decomposition in (8), and (7) is used to update the weights. Since the elements of the vector $\tilde{\mathbf{V}}_{\text{even}}^k$, used in the CDC update, can be obtained from the elements of the matrices $\mathcal{D}$ and $\tilde{\mathcal{D}}$. The use of the trigonometric transformations in the SDC algorithm provides a more cost efficient implementation of the CDC algorithm than the one presented in [2]. However, both algorithms have a similar convergence, since they use the same weight update formula in (7).

3. DUAL TRANSFORM DOMAIN CANCELLER

In [5], we introduce a dual transform domain canceller (DTDC), where time domain signals and filter weights are transformed by the unitary matrices obtained from the decomposition of the matrix $\mathcal{U}^k$. In general, this matrix can be decomposed as follows

$$\mathcal{U}^k = [\mathcal{G}_1^H \quad \mathcal{G}_2^H] \begin{bmatrix} \mathcal{S}_1^k & \mathcal{S}_2^k \\ \mathcal{S}_3^k & \mathcal{S}_4^k \end{bmatrix} \begin{bmatrix} \mathcal{G}_1 \\ \mathcal{G}_2 \end{bmatrix} \tag{9}$$

where $\mathcal{G}_i$ $(i = 1, 2)$ are constant $N \times N$ unitary matrices, and $\mathcal{S}_i^k$ $(i = 1, \cdots, 4)$ are $N \times N$ matrices with elements depending on the transmitted symbols. For brevity, the above decomposition is represented by

$$\mathcal{U}^k = \mathcal{G}^H \mathcal{S}^k\, \mathcal{G}. \tag{10}$$

As shown in [5], previous decompositions for the matrix $\mathcal{U}^k$ can be formulated as special cases of the form in (9). For example, the symmetric decomposition used in the SDC algorithm can be expressed as

$$\mathcal{U}^k = \begin{bmatrix} \mathcal{C}_{\text{II}}^T & \mathcal{S}_{\text{II}}^T \end{bmatrix} \begin{bmatrix} \tilde{\mathcal{Z}}^T \mathcal{D}^k \tilde{\mathcal{Z}} & \tilde{\mathcal{Z}}^T \tilde{\mathcal{D}}^k \mathcal{Z} \\ -\mathcal{Z}^T \tilde{\mathcal{D}}^k \tilde{\mathcal{Z}} & \mathcal{Z}^T \mathcal{D}^k \mathcal{Z} \end{bmatrix} \begin{bmatrix} \mathcal{C}_{\text{II}} \\ \mathcal{S}_{\text{II}} \end{bmatrix} \tag{11}$$

where the matrices involved are introduced in Section 2.

In the dual transform domain canceller, the unitary matrices $\mathcal{G}_1$ and $\mathcal{G}_2$, are used to transform the time domain signals and filter weights. Therefore, the transformed emulated echo vector is given by

$$\mathbf{Y}_e^k = \Phi^k \boldsymbol{\omega}^k \tag{12}$$

where Φ^k consists of the transformed input samples, given by

$$\Phi^k = \mathcal{G}\, \mathcal{G}^H \mathcal{S}^k, \tag{13}$$

and the transformed weigh vectort is defined as $\boldsymbol{\omega}^k = \mathcal{G}\,\mathbf{w}^k$. Using the above definitions, the transformed error signal for the proposed DTDC is given by

$$\mathbf{E}^k = \mathbf{Y}^k - \Phi^k \boldsymbol{\omega}^k \tag{14}$$

where $\mathbf{Y}^k$ is the transformed received signal $\mathbf{Y}^k = \mathcal{G}\mathbf{y}^k$. Using the LMS algorithm, the echo weights are updated by

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu\,\Phi^{k^H}\,\mathbf{E}^k. \tag{15}$$

The above equation offers a complete and *exact* adaptation for the transform domain canceller. In echo cancellers discussed in Section 2, the dual transformation is partially used in the emulation part but avoided in the weight update part, where an approximate weight update results in a reduced computational complexity. However, as shown in the next section, an exact and computationally efficient weight update is achievable by using the general decomposition representation.

4. DUAL DOMAIN ADAPTIVE WEIGHT UPDATE

In current echo cancellers, the adaptive weight update is done mostly in the frequency domain, using an approximate representation of the data matrix. The use of this approximation deteriorates the convergence of the adaptive algorithm and in some cancellers, *e.g.*, CES makes it sensitive to cases where there is a lack of excitation on some of the tones. However, the above DTDC yields an exact and computationally efficient implementation of the adaptive weight updating for the dual domain canceller.

Considering (15), the value of Φ^k is substituted from (13), resulting in

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \mu\,\mathcal{S}^{k^H}\mathcal{G}\,\mathcal{G}^H\left(\mathcal{G}\,\mathbf{y}^k - \mathcal{G}\,\mathcal{G}^H\,\mathcal{S}^k\,\boldsymbol{\omega}^k\right). \tag{16}$$

Since, $\mathcal{G}_1$ and $\mathcal{G}_2$ are unitary matrices, $\mathcal{G}^H\mathcal{G} = 2\mathcal{I}_N$. Therefore, the weight update can be written as

$$\begin{aligned}\boldsymbol{\omega}^{k+1} &= \boldsymbol{\omega}^k + \bar{\mu}\,\mathcal{S}^{k^H}\mathcal{G}\,\left(\mathbf{y}^k - \mathcal{G}^H\,\mathcal{S}^k\,\boldsymbol{\omega}^k\right) \qquad (17)\\ &= \boldsymbol{\omega}^k + \bar{\mu}\,\mathcal{S}^{k^H}\mathbf{E}^k.\end{aligned}$$

Therefore, for decompositions in which the matrices $\mathcal{S}_i^k$ $(i = 1, \cdots, 4)$ are diagonal or at least tridiagonal, the echo canceller weights can be updated using no approximation and with low computational complexity. In the implementation of the DTDC using the symmetric decomposition, where the data matrix decomposition in (11) is used, the $\mathcal{S}^k$'s are either diagonal or with nonzero elements only on the sub-diagonal or super-diagonal.

In the derivation above, we assume that the FIR filter of the echo canceller has the same length as the symbol frame. However, in practice, the echo canceller has T_e taps which is typically shorter than the symbol frame size N. Echo cancellation methods have exploited this fact and improved the performance of the canceller by forcing the last $N - T_e$ weights to zero. For the DTDC algorithm, the zeroing of the tail weights can be implemented by using the following modifications. The transformed weight vector is redefined as $\boldsymbol{\omega}^k = \mathcal{G}\,\mathcal{M}\mathbf{w}^k$, where the $N \times T_e$ matrix $\mathcal{M} = \left[\mathcal{I}_{T_e}\ \mathbf{0}_{T_e \times (N-T_e)}\right]^T$. Using this modified weight vector, (14) is valid for the computation of the error signal. For updating the weights, (17) is modified to

$$\boldsymbol{\omega}^{k+1} = \boldsymbol{\omega}^k + \bar{\mu}\,\mathcal{G}\,\mathcal{M}\,\mathcal{M}^T\mathcal{G}^H\mathcal{S}^{k^H}\mathbf{E}^k \tag{18}$$

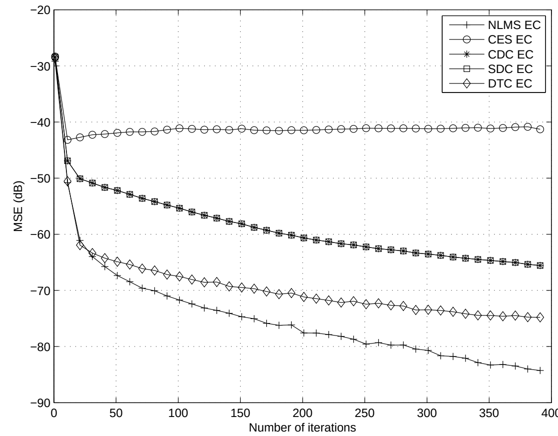

Fig. 1. Comparison of the convergence behaviour of the various echo cancellers, $T_e = N = 512$

where the gradient constraint $\mathcal{G}\ \mathcal{M}\ \mathcal{M}^T\mathcal{G}^H$ ensures that the last $(N - T_e)$ weights in the time domain are equal to zero. Applying the gradient constraint adds to the complexity of the algorithm, however, (18) can be used infrequently in combination with (15). As the simulation results in Section 5 shows, the infrequent update does not deteriorate the performance of the canceller significantly.

5. CONVERGENCE COMPARISONS

In this section, we use simulation experiments to evaluate the convergence behaviour of the proposed dual transform domain echo canceller and the existing echo cancellers. In the simulations, an ADSL system over the carrier serving area (CSA) loop $\#4$ is used. DMT modulation is employed where, tones 7-31 and 33-255 are allocated for the upstream and downstream, respectively; where each tone transmits a 4-QAM signal constellation. The downstream and upstream signal transmit with -40 dbm/Hz, and the external additive noise is white Gaussian noise at -140 dBm/Hz. The transmit block length in the upstream and downstream is 64 and 512, respectively and the corresponding cyclic prefix length is 5 and 40, respectively. The true echo channel transfer function contains 512 samples at 2.2 MHz, including the effect of the hybrid and the transmitter and receiver filters. In all the simulations, the step sizes are chosen in a way that all of the compared algorithms have a similar slope in the first part of the curve; having same initial rate of convergence.

In the first set of simulations, the length of the echo canceller T_e is assumed to be equal to the frame size. The echo cancellation methods examined are: normalized LMS algorithm, CES algorithm, CDC algorithm, SDC algorithm and dual trigonometric canceller (DTC) which is the implementation of the DTDC using the symmetric decomposition [5]. No dummy data is sent on the unused tones as required in [1]. As represented in Fig.1, the CES canceller has the worst performance because of the lack of excitation on the unused tones. The SDC and CDC cancellers perform similarly, as discussed before, due to their similar update formula. The DTC method has the closest performance to the NLMS method with much reduced complexity.

For the case $T_e = N$, the computational complexity of the discussed schemes is given in Table 1. The complexity is expressed as

Table 1. Complexity Comparison

Application	Echo Emulation	Adaptive Update	Total
NLMS	N^2	N^2	$2N^2$
CES	$2N + \frac{(N-\upsilon-1)^2}{2}$	$N\log_2 N - N + 4$	$N\log_2 N + N + \frac{(N-\upsilon-1)^2}{2} + 4$
CDC	$2N\log_2 N + N + 6$	$2N + \frac{1}{\psi}(1.5N\log_2 N - 2.5N + 6)$	$2N\log_2 N + 3N + 6 + \frac{1}{\psi}(1.5N\log_2 N - 2.5N + 6)$
SDC	$3N\log_2 N + 3N + 8$	$2N$	$3N\log_2 N + 5N + 8$
DTC	$2N\log_2 N + 2N + 2$	$\frac{N}{2}\log_2 N + 4N + \frac{1}{\phi}(N\log_2 N + 4N)$	$2.5N\log_2 N + 6N + 2 + \frac{1}{\phi}(N\log_2 N + 4N)$

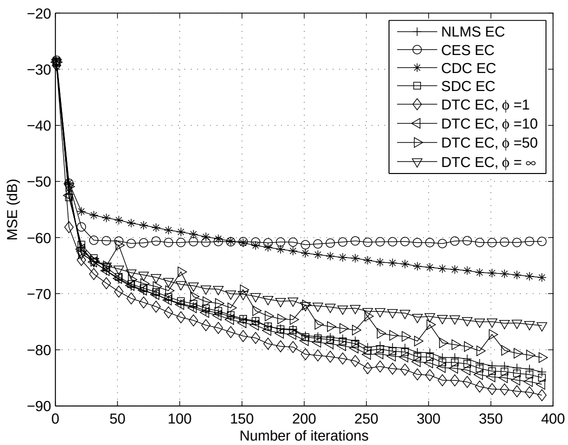

Fig. 2. Comparison of the convergence behaviour of various echo cancellers with the DTC algorithm with different interval of gradient constraint update, $T_e = 300$

the number of real multiplications at symbol rate, and for the DFT complexity, the split-radix FFT algorithm is assumed to be used. For the CDC and DTC algorithms part of the weight update is done infrequently, which is indicated by ψ and ϕ, respectively. As it can be seen, the NLMS method requires the largest number of computations and the CES algorithm requires the least, however, the latter is sensitive to the lack of excitation on the unused tones. For large frame sizes, the SDC method is more efficient to implement than the CDC scheme. The proposed DTC algorithm has approximately the same computational cost as CDC and SDC methods, while it does not suffer from the lack of excitation on the tap-input vector used in the update step.

In the second set of simulations, the length of the echo canceller $T_e = 300$, which is shorter than the frame size. For the SDC method, we have expanded the algorithm, since in [3] it is only discussed for $T_e = N$. In the expanded version, the last $N - T_e$ weights in the time domain are forced to zero. For the DTC algorithm, different intervals for applying the gradient constraint to the transformed weight vector areexamined, where for $\phi = 1$, the constraint in (18) is applied at each iteration; for $\phi = 10, 50$, it is applied at every 10^{th} and 50^{th} iteration, respectively and for $\phi = \infty$, no constraint is applied and (15) is used to update the weights. As seen in Fig. 2, the DTC method with no gradient constraint performs better than the CDC and CES algorithms. It is interesting to note the improved performance of the expanded SDC algorithm which is similar to the performance of NLMS and DTC with $\phi = 10$. This improvement is due to the fact that the NLMS, SDC and DTC algorithms calculate the estimated echo in the time domain. Consequently, the zeroing of the last $N - T_e$ weights of the weight vector reduces the error floor in these cancellers directly. On the other hand, in the CES and CDC algorithms, where a part of the echo emulation is done in the frequency domain, higher error floor is observed. Since, in these methods, the weights in the frequency domain do not correspond to the transformation of the time domain weight vector with last $N-T_e$ weights forced to zero.

6. CONCLUSION

In this paper, we have studied the dual transform domain canceller for DMT-based echo cancellers, presented in [5], in detail. We derive the proof for the update formula used in the dual domain and extend this algorithm for the case where the length of the echo canceller is smaller than the frame size. A comprehensive comparative performance evaluation is performed between the proposed method and the existing methods for echo cancellation, using convergence curves and computational complexity calculations. The given results show that this algorithm is not affected by the lack of excitation on the unused tones and has a faster convergence with a low error floor. Additionally, these benefits are gained with no extra computational cost compare to the CDC and SDC algorithms.

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