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ADAPTIVE SIGNAL-SUBSPACE PROCESSING BASED ON FIRST-ORDER PERTURBATION ANALYSIS

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ABSTRACT

This paper presents a new approach to adaptive signal-subspace processing of narrowband array data which is based on the application of first-order perturbation analysis. In the proposed approach, the correction term in the recursive estimate of the array covariance matrix at time k is viewed as a perturbation of the estimate at time $k-1$. Following this interpretation, the theory of perturbation of Hermitian matrices is applied in order to obtain a new recursion expressing the eigenstructure estimate of $R_x(k)$, the true array covariance matrix at time k, in terms of the eigenstructure estimate of $R_x(k-1)$. This algorithm can be realized by means of L linear combiners with non-linear weight-vector adaptation equations, where L is the signal-subspace dimensionality. These non-linear adaptation equations appear to be substitutes for the orthonormal weight constraints found in other algorithms. The results of preliminary simulations are discussed.

I. INTRODUCTION

Most of the modern, high-resolution array processing techniques are based on the application of so-called signal-subspace algorithms, which use the eigenstructure of the array output sample covariance matrix to obtain improved estimates of the directions of arrival of multiple plane waves [1]. While originally intended for narrowband applications, modified versions of these algorithms have also been used successfully in broadband situations [2].

These algorithms are based on the assumption of (statistical) stationarity of the observed data at the array output. In many applications, however, the sources under observation are moving and this assumption is not satisfied. The application of signal-subspace algorithms in time-varying environments therefore requires repeated eigenstructure computations of an updated sample covariance matrix, a task which is generally prohibitive.

Two different approaches have been suggested in the past for time-varying, or *adaptive*, signal-subspace processing. In the first approach [3], the determination of the signal-subspace is formulated as a constrained optimization problem which is realized via a stochastic gradient search based on a recursive estimation of the array covariance matrix. In the second approach [4], variations of Bunch's rank one eigenstructure updating algorithm are used to update the eigenstructure of the sample covariance matrix of the array at each iteration. The updated eigenstructure is then used to estimate the signal subspace.

In this paper, we present a new approach to narrowband adaptive signal-subspace processing which is based on the application of first-order perturbation analysis. In the proposed approach, we view the correction term in the recursive estimate of the sample covariance matrix at time k as a perturbation of the estimate at time $k-1$. By applying the theory of perturbation of Hermitian matrices, we obtain a recursion expressing the eigenstructure estimate of $R_x(k)$, the true array covariance matrix at time k, in terms of the eigenstructure estimate of $R_x(k-1)$. This algorithm can be realized by means of L linear combiners with non-linear weight vector adaptation equations, where L is the dimensionality of the signal-subspace. These non-linear adaptation equations appear to be substitutes for orthonormal constraints found in other algorithms. Preliminary simulation results for two stationary, closely spaced sources in noise show improved performance of the proposed algorithm when compared to Yang and Kaveh's instantaneous LMS-type signal-subspace algorithm [3].

This paper is organized in the following way. In Section II, we describe the observation model used for the sensor outputs and we formulate the problem of updating the eigenstructure estimate of the array output covariance matrix as a perturbation problem. In Section III, we use the methods of perturbation theory to solve this problem. Special attention is given to the difficulties posed by the degenerate nature of the noise-subspace. In Section IV, we present the simulation results and finally, in Section V, we comment on possible future work.

II. PROBLEM FORMULATION

Consider an array of M sensors and let $x(k)$ denote the complex M-dimensional narrow-band output of the array at the kth sampling interval, or *snapshot*. Denote the covariance matrix of $x(k)$ by

$$R_x(k) = E[x(k)x^H(k)] \tag{1}$$

where the superscript H denotes complex conjugate transposition. Suppose that the observed wavefield consists of the superposition of L distinct sources on a background noise. More presicely, let

$$x(k) = A(k)s(k) + n(k) \tag{2}$$

where s(k) is a complex L-dimensional signal process, $A(k)$ is a $M \times L$ transmission matrix, and $n(k)$ is a complex M-dimensional background noise process. We shall assume that $s(k)$ and $n(k)$ are zero-mean, uncorrelated random processes with covariance matrices at time k given by

$$R_s(k) = E[s(k)s^H(k)] \tag{3}$$

$$R_n(k) = E[n(k)n^H(k)] = \sigma_n^2(k)\, I_M \tag{4}$$

where $\sigma_n^2(k)$ is the noise variance at time k and I_M is the $M \times M$ identity matrix. In this case, the array output covariance matrix satisfies

$$R_x(k) = A(k)\, R_s(k)\, A^H(k) + \sigma_n^2(k)\, I_M. \tag{5}$$

Suppose for the moment that the signal and noise are stationary random processes and that the transmission matrix is constant over time, so that the dependence on the time index k can be omitted temporarily. If the rank of A is equal to L, the number of sources, it is not difficult to prove the following: Let λ_i and q_i, $i = 1, \ldots, M$,

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denote the eigenvalues and corresponding eigenvectors of the covariance matrix R_x, with the eigenvalues arranged in descending order. Then,

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L > \lambda_{L+1} = \cdots = \lambda_M = \sigma_n^2 \quad (6)$$

$$A^H Q_n = 0 \quad (7)$$

where 0 denotes the $M \times M$ zero matrix and

$$Q_n = [q_{L+1}, q_{L+2}, \ldots, q_M]. \quad (8)$$

Hence, the number of sources L is given indirectly by the multiplicity of the smallest eigenvalue of R_x, while the column span of A is identical to that of the matrix

$$Q_s = [q_1, q_2, \ldots, q_L] \quad (9)$$

Accordingly, the span of Q_s and Q_n are referred to as the signal-subspace and the noise-subspace, respectively.

A problem of fundamental importance in array processing is the joint estimation of the number of sources L, the signal subspace Q_s and the noise variance σ_n^2. A variety of algorithms, such as MUSIC [1] for instance, use these parameters to generate high resolution estimates of the angles of arrival of multiple plane waves incoming on the array. Based on the above property of the covariance matrix R_x, such a joint estimate can be obtained quite naturally by first constructing an estimate of R_x, say $\hat{R}_x$, and then solving for the eigenvalues and eigenvectors of $\hat{R}_x$. In this context, a typical estimate of R_x is given by

$$\hat{R}_x = \frac{1}{K} \sum_{k=1}^{K} x(k) x^H(k). \quad (10)$$

The crucial part of this approach is actually the determination of the multiplicity of the smallest eigenvalue of R_x from the estimated eigenvalues. Various criteria, such as the Akaike information criterion (AIC) and the minimum description length (MDL) criterion, can be used for this task. In a stationary environment, a good estimator of R_x can be obtained by averaging the outer product $x(k)x^H(k)$ in (10) over a sufficiently large number of snapshots K. In this case, it can be shown that the joint estimates of L (using the MDL criterion), Q_s and σ_n^2 are asymptotically consistent (convergence to the exact values in the limit of large K) [5].

In many applications, however, $R(k)$ can only be considered stationary over a relatively small time interval which may not be sufficient to get statistically reliable estimates of the desired parameters. This may occur for instance when the bearing angles of the sources under observation vary with time. In this case, one is constrained to take into account the time-varying nature of the problem and to explicitely specify the dependence in k of the eigenvalues and eigenvectors of $R_x(k)$, namely: $\lambda_i(k)$ and $q_i(k)$. Moreover, the application of signal-subspace algorithms then requires repeated computation of the eigenstructure of a time-varying sample covariance matrix $\hat{R}_x(k)$, a task which is usually prohibitive.

Two different approaches have been suggested for time-varying, or adaptive, signal-subspace processing. In the first approach [3], the determination of the signal subspace at time k is formulated as a constrained optimization problem. The optimization is carried out over time via a stochastic gradient search based on a recursive estimation of the array covariance matrix $R_x(k)$. More precisely, denoting by $U_s(k)$ the estimate of $Q_s(k)$, the signal subspace at time k, we have

$$U_s'(k) = U_s(k-1) + \mu \nabla(k) \quad (11)$$

$$U_s(k) = Gram - Schmidt\ orthonormalization\ of\ U_s'(k) \quad (12)$$

where μ is a convergence factor and $\nabla(k)$ is an estimate of the gradient of an appropriate cost function. Different gradient estimators have been considered in [3], including

$$\nabla(k) = 2\, \hat{R}_x(k)\, U_s(k-1) \quad (13)$$

where $\hat{R}_x(k)$ is an estimate of $R_x(k)$ obtained recursively through

$$\hat{R}_x(k) = (1-\alpha)\, \hat{R}_x(k-1) + \alpha\, x(k)\, x^H(k). \quad (14)$$

In a second approach, proposed by Degroat and Robert [4], the eigenstructure of $\hat{R}_x(k)$ (14) is used to estimate the eigenstructure of the true covariance matrix $R_x(k)$. However, instead of recomputing the eigenstructure of $\hat{R}_x(k)$ at each iteration, the eigenstructure is updated recursively using a "stabilized" version of Bunch's rank one eigenstructure updating algorithm. It is important to note that in this approach, the exact eigenstructure of $\hat{R}_x(k)$ is sought at each time. In principle, the exact updating algorithm requires the solution of a non-linear equation. In practice, this equation is solved approximately through an iterative, finite steps algorithm.

We now present a new formulation of the eigenstructure updating problem which differs substantially from those described above. This formulation is based on the application of first-order perturbation analysis. To begin, we rewrite the updating equation (14) for $\hat{R}_x(k)$ as follows:

$$\hat{R}_x(k) = \hat{R}_x(k-1) + \varepsilon\, [x(k)x^H(k) - \hat{R}_x(k-1)] \quad (15)$$

where ε is a small positive parameter that admits the following interpretation: $1/\varepsilon$ can be viewed as the effective number of samples $x(k)$ used in the determination of $\hat{R}_x$ (this can be shown by solving the difference equation (15) for $\hat{R}_x(k)$). We note that the correction term $\varepsilon[x(k)x^H(k) - \hat{R}_x(k-1)]$ in (15) can be viewed as a perturbation of $\hat{R}_x(k)$. Because this pertubative term is linear in ε, it is referred to as a first-order perturbation.

Let $\gamma_i(k)$ and $u_i(k)$ denote the desired estimates of the true eigenvalues $\lambda_i(k)$ and corresponding eigenvectors $q_i(k)$ of $R_x(k)$. Rather than seeking estimates which are exact eigenvalues and eigenvectors of $\hat{R}_x(k)$ (15), we shall require that

$$\hat{R}_x(k)u_i(k) = \gamma_i(k)u_i(k) + O(\varepsilon^2) \quad (16)$$

$$u_i^H(k)u_j(k) = \delta_{ij} + O(\varepsilon^2) \quad (17)$$

for all k, where δ_{ij} is the Kronecker delta. That is, $\gamma_i(k)$ and $u_i(k)$ are eigenvalues and eigenvectors of $\hat{R}_x(k)$ up to error terms of order ε^2. The main justification for seeking estimates satisfying (16)-(17) is the possibility of applying the theory of perturbation of Hermitian matrices to construct recursive equations expressing the estimates at time k in terms of those at time $k-1$. Indeed, because the correction term $\varepsilon[x(k)x^H(k) - \hat{R}_x(k-1)]$ in (15) can be viewed as a first-order perturbation of $\hat{R}_x(k-1)$, the proper application of the theory should lead to perturbative series in ε expressing $\gamma_i(k)$ and $u_i(k)$ in terms of $\gamma_j(k-1)$ and $u_j(k-1)$, $j = 1, \ldots, M$. The recursive equations so obtained would enable us to *adapt* our estimates of the eigenstructure of $R_x(k)$ as the time index k is incremented, without having to completely recompute these estimates.

In the next section, a first-order perturbation analysis is carried out in order to obtain the desired recursions.

III. FIRST-ORDER PERTURBATION ANALYSIS

Among all possible solutions $\lambda_i(k)$ and $u_i(k)$ to (16)-(17), we shall be seeking those satisfying recursive equations of the type

$$\gamma_i(k) = \gamma_i^{(0)}(k) + \gamma_i^{(1)}(k)\varepsilon, \quad \gamma_i^{(0)}(k) = \gamma_i(k-1) \quad (18)$$

$$u_i(k) = u_i^{(0)}(k) + u_i^{(1)}(k)\varepsilon, \quad u_i^{(0)}(k) = u_i(k-1). \quad (19)$$

Since $\hat{R}_x(k)$ (15) is a convergent power series in ε around $\hat{R}_x(k-1)$, the theory of perturbation of Hermitian matrices [6] asserts that the eigenvalues and eigenvectors of $\hat{R}_x(k)$ (if properly selected) can be expanded in power series of the same type, convergent for ε small, around the eigenvalues and eigenvectors of $\hat{R}_x(k-1)$. This theory

also provides methods for determining these power series. Although $\gamma_i(k-1)$ and $u_i(k-1)$ are not exact eigenvalues and eigenvectors of the unperturbed matrix $\hat{R}_x(k-1)$, but rather approximations of order ε^2, it is still possible to apply the theory in order to determine the correction terms $\gamma_i^{(1)}(k)$ and $u_i^{(1)}(k)$ in the finite expansions (18)-(19). The general approach is described below.

To simplify the notations, we define

$$F = \hat{R}_x(k-1) \tag{20}$$

$$G = x(k)x^H(k) - \hat{R}_x(k-1), \tag{21}$$

so that

$$\hat{R}_x(k) = F + \varepsilon G, \tag{22}$$

and we temporarily write $\gamma_i^{(j)}(k)$ and $u_i^{(j)}(k)$ as $\gamma_i^{(j)}$ and $u_i^{(j)}$, respectively. Substituting (18), (19) and (22) in (16), and multiplying term by term, we get

$$Fu_i^{(0)} + (Fu_i^{(1)} + Gu_i^{(0)})\varepsilon = \gamma_i^{(0)}u_i^{(0)} + (\gamma_i^{(1)}u_i^{(0)} + \gamma_i^{(0)}u_i^{(1)})\varepsilon + O(\varepsilon^2). \tag{23}$$

But according to (16), $Fu_i^{(0)} = \gamma_i^{(0)}u_i^{(0)} + O(\varepsilon^2)$. Substituting in (23) and requiring that the resulting equation be satisfied for all ε in the neighborhood of 0, we obtain

$$Fu_i^{(1)} + Gu_i^{(0)} = \gamma_i^{(1)}u_i^{(0)} + \gamma_i^{(0)}u_i^{(1)}. \tag{24}$$

Proceeding in a similar manner with (17), we obtain

$$(u_i^{(1)}, u_j^{(0)}) + (u_i^{(0)}, u_j^{(1)}) = 0 \tag{25}$$

where $(u, v) = u^H v$ denotes the scalar product between M-components column vectors. The desired correction terms $\gamma_i^{(1)}$ and $u_i^{(1)}$ can be found as the solutions to (24) and (25). As such, we note that $\gamma_i^{(1)}$ and $u_i^{(1)}$ depend only on the new data vector $x(k)$ and the old estimates $\hat{R}_x(k-1)$, $\gamma_i(k-1)$ and $u_i(k-1)$.

If all the eigenvalue estimates $\gamma_i(k-1)$ where distinct, the solution of (24)-(25) would be relatively straightforward. However, because of the particular structure of the array covariance matrix $R_x(k)$ (5) whose $M-L$ smallest eigenvalues are degenerate, any reasonable estimation procedure should eventually result in $\gamma_{L+1}(k), \ldots, \gamma_M(k)$ clustering together as k increases. In turn, this would make the recursion obtained under the assumption of distinct $\gamma_i(k-1)$ unstable. One way out of this dilemma is to further impose the following condition on the $M-L$ smallest eigenvalue estimates:

$$\gamma_{L+1}(k) = \cdots = \gamma_M(k). \tag{26}$$

That is, we construct the recursive eigenvalue estimates in such a way that the smallest one has the proper multiplicity. Of course, this requires that the signal-subspace dimensionality L be known in advance.

A similar problem occurs if some of the eigenvalues $\lambda_i(k)$ of $R_x(k)$ are degenerate for $1 \le i \le L$. In principle, this problem can be handled in the same way as above. However, to simplify the presentation, we shall assume that

$$\gamma_1(k) > \cdots > \gamma_L(k). \tag{27}$$

We refer to (27) as the *non-degenerate signal-subspace* assumption. Further comments will be made in the conclusion concerning the question of adaptive determination of eigenvalue multiplicities.

The solution of (24)-(25) under the assumption (26)-(27) is carried out in [7]. One important caracteristic of the resulting correction terms is that they depend only on the new data vector $x(k)$ and the previous estimates $\gamma_i(k-1)$ and $u_i(k-1)$, and not on the estimate $\hat{R}_x(k-1)$ of the array covariance matrix, as suggested by (24)-(25). Once the correction terms are available, the formulation of a com- plete eigenstructure updating algorithm based on the recursive equations (18)-(19) is straightforward. This algorithm is stated below.

Data needed at time k:

$x(k)$

$u_i(k-1), \quad i = 1, \ldots, L$

$\gamma_i(k-1), \quad i = 1, \ldots, L+1$

Computation of correction terms:

$$y_i(k) = (u_i(k-1), x(k)), \quad i = 1, \ldots, L \tag{28}$$

$$\gamma_i^{(1)}(k) = |y_i(k)|^2 - \gamma_i(k-1), \quad i = 1, \ldots, L \tag{29}$$

$$\gamma_{L+1}^{(1)}(k) = \frac{1}{(M-L)} \{ (x(k), x(k)) - \sum_{i=1}^{L} |y_i(k)|^2 \} - \gamma_{L+1}(k-1) \tag{30}$$

$$v_i(k) = y_i(k)u_i(k-1), \quad i = 1, \ldots, L \tag{31}$$

$$b_{ji}(k) = \begin{cases} 1, & j = i \\ \dfrac{\gamma_j(k-1) - \gamma_{L+1}(k-1)}{\gamma_j(k-1) - \gamma_i(k-1)}, & j \neq i \end{cases} \tag{32}$$

$$u_i^{(1)}(k) = \frac{y_i^*(k)}{\gamma_i(k-1) - \gamma_{L+1}(k-1)} \{ x(k) - \sum_{j=1}^{L} b_{ji}(k)v_j(k) \} \tag{33}$$

Updating eigenstructure:

$$\gamma_i(k) = \gamma_i(k-1) + \varepsilon\gamma_i^{(1)}(k), \quad i = 1, \ldots, L+1 \tag{34}$$

$$u_i(k) = u_i(k-1) + \varepsilon u_i^{(1)}(k), \quad i = 1, \ldots, L \tag{35}$$

The dimensionality of the signal subspace, L, and the initial values $\gamma_i(0)$, $i = 1, \ldots, L+1$ and $u_i(0)$, $i = 1, \ldots, L$ are needed to start the recursion. They can be obtained for example by performing an eigendecomposition of an initial estimate $\hat{R}_x(0)$ of the array covariance matrix.

Equation (28) can be realized by means of L linear combiners, with the complex weight vector of the ith combiner given by $u_i(k-1)$. The output $y_i(k)$ of the ith combiner is used to evaluate the correction term $\gamma_i^{(1)}(k)$ (29). To evaluate the correction term $\gamma_{L+1}(k)$ (30), we must further evaluate $(x(k), x(k))$, the energy of $x(k)$. In this respect, we note that the term between brackets in (30) can be interpreted as an estimate of the energy of $x(k)$ along the noise subspace. The quantity $v_i(k)$ (31) is the projection of $x(k)$ along $u_i(k-1)$ (assuming $u_i(k-1)$ has unit lenght, which is true to the first order in ε). The coefficients $b_{ji}(k)$ (32) are computed from the previous eigenvalues $\gamma_i(k-1)$. The correction term $u_i^{(1)}(k)$ is obtained by substracting a linear combination of the $v_j(k)$ (using the $b_{ji}(k)$ as coefficients) from $x(k)$, and then scaling the result. In (34)-(35), the correction terms are used to form the estimates at time k.

We note that the above recursion is non-linear, that is: the updated eigenvalues $\gamma_i(k)$ and corresponding eigenvectors $u_i(k)$ at time k depend non-linearly on the eigenvalues $\gamma_i(k-1)$ and $u_i(k-1)$ at time $k-1$. In particular, equation (33), which can be interpreted as a weight-vector adaptation equation for the ith linear combiner, is non-linear in $u_i(k-1)$. This non-linearity appears to be a substitute for the orthonormal weight constraints found in other algorithms (see (12)).

IV. SIMULATIONS

Computer simulations were used to compare the new adaptive signal-subspace algorithm presented in this paper to the instantaneous LMS-type algorithm proposed in [3]. The purpose of this preliminary study was to investigate the convergence properties of the two algorithms when operating on stationary data, and to compare

the relative accuracy of the resulting eigenstructure estimates. The simulations are described below.

The scenario considered is identical to that of [3]. A uniform linear array of $M=8$ sensors is used to monitor $L=2$ uncorrelated plane waves of common frequency incoming from directions $\theta_1=9^o$ and $\theta_2=12^o$. The intersensor spacing is equal to half of the wavelength. The plane waves are monitored in the presence of uncorrelated background noise as described in Section II. The signal-to-noise ratios of the two plane wave signals at the sensor outputs are given by $\mathrm{SNR}_1=\mathrm{SNR}_2=20\mathrm{dB}$.

The initial values needed to implement the first-order perturbative algorithm were obtained as follows. A sequence of 10 independent array output vectors $x(k)$, with the proper statistical characteristics, was used to form an initial estimate $\hat{R}_x$ of the true array covariance matrix as in (10). The 2 largest eigenvalues of $\hat{R}_x$, arranged in descending order, were used as the initial values $\gamma_1(0)$ and $\gamma_2(0)$, respectively. The corresponding eigenvectors were used as the initial estimates $u_1(0)$ and $u_2(0)$, respectively. The initial value $\gamma_3(0)$ was obtained by averaging the remaining 6 smallest eigenvalues of $\hat{R}_x$. The same vectors $u_1(0)$ and $u_2(0)$ were used to initialize the instantaneous LMS-type algorithm.

Once initialized, both algorithms were applied to the same sequence of independent array output vectors $x(k)$, $k=1,\ldots,400$. The performance of each algorithm was evaluated in terms of the following measure:

$$J(k)=\frac{1}{\sqrt{L}}\|U_s(k)U_s^H(k)-Q_sQ_s^H\| \tag{36}$$

where $\|.\|$ denotes the Euclidean norm of a matrix. The quantity $J(k)$, also referred to as a learning curve, measures the normalized error between the projector on the true signal subspace, i.e. $Q_sQ_s^H$, and an estimate of this projector at time k given by $U_s(k)U_s^H(k)$. An average learning curve was finally obtained by performing 40 independent experiments as above and averaging $J(k)$ (36) over these experiments (we note that the same initial values were used for all of the 40 independent experiments).

Fig. 1 shows the average learning curves of the two algorithms for $\varepsilon=.02$ and $\mu=.00025$, where ε is the perturbation parameter introduced in this paper and μ is the convergence factor used in [3]. With this choice of parameters, both algorithms yield similar steady state normalized error. In this case, however, the new algorithm converges about twice as fast as the LMS-type algorithm. Fig. 2 shows the average learning curves for $\varepsilon=.02$ and $\mu=.001$. In this case, the convergence rate of the two algorithms is similar. However, the steady state normalized error of the new algorithm is about half that of the LMS-type algorithm.

V. CONCLUSION

We have presented a new algorithm for adaptive signal-subspace processing which is based on the application of first-order perturbation analysis. Preliminary simulation results indicate that this algorithm can achieve substantially better performance than Yang and Kaveh's instantaneous LMS-type algorithm [3].

A final comment is in order concerning the non-degenerate signal-subspace assumption made in Section III. This assumption requires that the number of sources L be known, and that the L largest eigenvalues of the true array covariance matrix remain non-degenerate at all time. In practice, the number of sources L must be estimated from the data. Moreover, the multiplicity of the L largest eigenvalues may change as the sources move in time. As a remedy to these potential problems, we note that it is possible to develop a first-order perturbative version of the AIC criterion for the adaptive determination of L. Moreover, it is possible to extend the approach proposed in this paper to the case of degenerate signal-subsapce eigenvalues. These questions will be addressed in the future.

VI. REFERENCES

[1] S. U. Pillai, "Array Signal Processing," New York: Springer-Verlag, 1989.

[2] H. Wang and M. Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wide-band sources," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, pp.823-831, Aug. 1985.

[3] J.-F. Yang and M. Kaveh, "Adaptive eigensubspace algorithms for direction or frequency estimation and tracking," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 241-251, Feb. 1988.

[4] R. D. DeGroat and R. A. Roberts, "Efficient, numerically stabilized rank-one eigenstructure updating," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 301-316, Feb. 1990.

[5] M. Wax and T. Kailath, "Determining the number of signals by information theoretic criteria," in *Proc. ASSP Spectrum Estimation Workshop II*, Tampa, FL, 1983, pp.192-196.

[6] F. Rellich, "Perturbation theory of eigenvalue problems," New York: Gordon and Breach, 1969.

[7] B. Champagne, "Adaptive signal-subspace processing based on first-order perturbation analysis," In preparation.

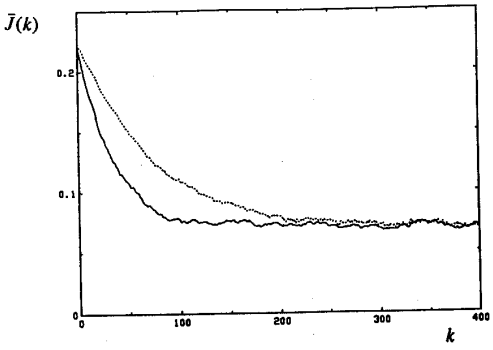

Fig. 1. Avereage learning curves of the first-order perturbative signal-subspace algorithm (continuous curve) and the instantaneous LMS-type algorithm (dashed curve): $\varepsilon=.02$, $\mu=.00025$.

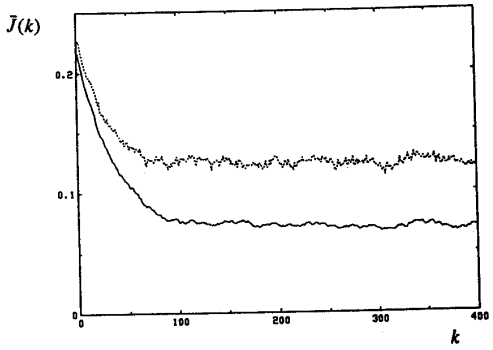

Fig. 2. Avereage learning curves of the first-order perturbative signal-subspace algorithm (continuous curve) and the instantaneous LMS-type algorithm (dashed curve): $\varepsilon=.02$, $\mu=.001$