Optimizing Transmission Rate in NOMA via Block Diagonalization Beamforming and Power Allocation

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***Abstract*—In this paper, we propose a new transmission scheme, relying on user clustering, transmit beamforming and power allocation, for non-orthogonal multiple access (NOMA) in a downlink cellular framework. Assuming that channel state information is available at the base station, the users are first partitioned into small clusters based on a weighted multi-objective optimization, so that each cluster may be served by a single beamforming vector.The complete set of beamforming vectors is jointly designed to remove inter-cluster interference by taking advantage of block diagonalization along with maximum ratio transmission. Subsequently, the NOMA power allocation factors for the users in each cluster are determined by solving an optimization problem, where the aim is to maximize the transmission sum rate subject to power and SINR constraints. The performance of the proposed NOMA scheme is validated by means of numerical analysis. Compared to the benchmark approaches, the results show significant improvements in terms of spectral efficiency and interference cancellation.**

***Index Terms*—non-orthogonal multiple access (NOMA), block diagonalization beamforming, maximum ratio transmission (MRT) beamforming, multi-objective optimization.**

I. INTRODUCTION

Multiple access radio technologies play a crucial role in improving system performance in cellular mobile networks. In the conventional orthogonal multiple access schemes, such as time division multiple access (TDMA), code division multiple access (CDMA), and orthogonal frequency division multiple access (OFDMA), time, frequency, or code resources are (respectively) allocated orthogonally to the different users. While these schemes can completely remove inter-user interference under ideal condition, this comes at the cost of limiting the number of supported users, and consequently, overall network capacity [1].

In the past few years, the so-called non-orthogonal multiple access (NOMA) schemes have received considerable attention as an enabling technology to meet the exacting demands of fifth generation (5G) wireless networks and beyond. In effect, by allowing multiple users to access overlapping time and frequency resources in the same spatial layer, NOMA has the potential to meet higher system throughput and solve the massive connectivity issue in future radio access technologies [2]. Basically, existing NOMA schemes can be categorized into code domain and power domain multiplexing. In code domain multiplexing, such as sparse code multiple access (SCMA) [3] and multi-user shared access (MUSA) [4], different users are assigned different codes that allow the control of interference. In power domain multiplexing, different power levels are allocated to different users according to their channel conditions [5]. Successive interference cancellation (SIC) is then employed at the receiver whereby the strongest signal is decoded first and subtracted from the received signal, after which the second strongest signal is extracted from the residual, and so on in a sequential manner. In this paper, the main focus is on power domain NOMA.

In [6], a multiuser beamforming approach is proposed for NOMA such that a single beamforming vector is shared within a cluster. Moreover, a clustering and power allocation algorithm is developed to reduce inter-cluster interference and improve the sum capacity. In [7], the system performance of uplink NOMA with an advanced SIC receiver is investigated along with enhanced scheduling based on proportional fairness (PF). The design of robust beamformers for downlink multiple-input-single-output (MISO) NOMA is addressed in [8], where channel uncertainty is handled through a worst-case model. A minimum total transmission power multicast beamforming approach with superposition coding (SC) is applied to NOMA in [9]. Authors in [10] evaluate the downlink performance of NOMA with different transceiver types and proposes a design where the user signals are jointly modulated at the transmitter but detected without interference cancellation at the receiver. Further applications of NOMA to advanced transmission scenarios within the traditional RAN framework are reported in [11]-[14].

In this paper, we investigate the joint application of user clustering, transmit beamforming and power allocation in a downlink NOMA setup, assuming that the channel state information is known at the base station (BS). A low-complexity *greedy* algorithm based on weighted multi-objective optimization is formulated for user clustering, which seeks to pair users whose respective channels exhibit high correlations yet large gain differences simultaneously. A block diagonalization beamforming approach along with maximum ratio transmission (MRT) is then employed to jointly design the set of beamforming vectors, i.e. one for each cluster, such that inter-cluster interference is eliminated. Finally, power-domain NOMA is applied to the transmitted signals within

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each cluster, whereby the BS transmits the superposition of corresponding user signals with different power levels. To this end, a power allocation scheme is proposed which maximizes the sum capacity under power and SINR constraints. While the resulting optimization problem is non-convex, it can be approximated as a difference of convex functions (DC) program, which is iteratively solved via the constrained concave convex procedure (CCCP) [15]. Through simulations, it is demonstrated that applying the proposed NOMA scheme with block diagonalization beamforming can greatly increase capacity and improve spectral efficiency in multi-user downlink transmissions.

The rest of the paper is organized as follows: Section 2 introduces the system model. In Section 3, the proposed NOMA based approach using user clustering, block diagonalization beamforming and optimal power allocation is described. The simulation results are presented in Section 4, followed by conclusion in Section 5.

II. SYSTEM MODEL

In this work, we consider a NOMA-based downlink cellular transmission system with one BS as illustrated in Fig. 1. The transmitter is equipped with N antennas and serves $2K$ single-antenna users. The latter are partitioned into K non-overlapping clusters on the basis of their channel conditions (see Section III.A) so that each cluster consists of two users [6],[9]: a strong user (i.e., with higher channel gain) and a weak user. The two users in a given cluster jointly share a common beamforming vector. Hence, the BS utilizes the NOMA superposition of signals and beamforming simultaneously.

Let $\mathbf{h}_{k,i} \in \mathbb{C}^{N\times 1}$ denote the channel vector from BS to the ith user in the kth cluster, where $i \in \{1,2\}$, and $k \in \{1,...,K\}$. The channel vectors are assumed to be known at the BS and without loss of generality, we assume that the first user is the strongest, i.e. $\|\mathbf{h}_{k,1}\| > \|\mathbf{h}_{k,2}\|$. The received signal at the ith user in the kth cluster is given by

$$y_{k,i} = \mathbf{h}_{k,i}^{H}\mathbf{x}_k + \sum_{m=1,m\neq k}^{K} \mathbf{h}_{k,i}^{H}\mathbf{x}_m + n_{k,i} \tag{1}$$

where $\mathbf{x}_k \in \mathbb{C}^{N\times 1}$ is the transmitted signal by the BS towards the kth cluster, and $n_{k,i} \sim \mathcal{CN}(0, \sigma_{k,i}^2)$ is an additive white Gaussian noise term.

It is assumed that the BS applies power domain NOMA within each cluster. That is, it transmits the superposition of the individual data symbols with different power levels to all users in a cluster simultaneously with the same radio resources, such as time slot and frequency channel. Hence, we have

$$\mathbf{x}_k = \mathbf{w}_k(\sqrt{p_{k,1}}s_{k,1} + \sqrt{p_{k,2}}s_{k,2}) \tag{2}$$

where $\mathbf{w}_k \in \mathbb{C}^{N\times 1}$ is the beamforming vector of the kth cluster, while $p_{k,i} > 0$ and $s_{k,i} \in \mathbb{C}$ are the transmission power and data symbol of the the ith user within this cluster, respectively. We assume that the data symbols of the different users are independent and, without loss of generality, that they have unit variance, i.e. $E[|s_{k,i}|^2] = 1$.

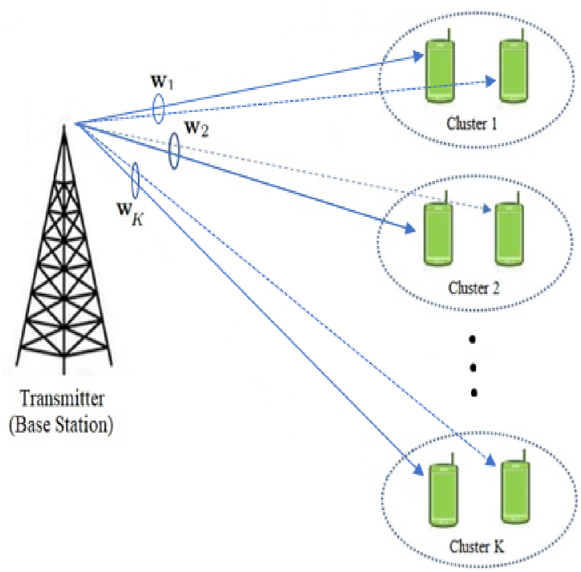

Fig. 1. Downlink NOMA cellular system model.

Upon substitution of (2) into (1), we can express the received signal at the ith user in the kth cluster as a sum of the desired signal, the interference from the other user in that cluster (intra-cluster interference), the inter-cluster interference and the noise. The effect of inter-cell interference is neglected as it is out of scope of this paper and can be considered in the future works. In power domain NOMA, the weaker signal in a given cluster (i.e., $s_{k,2}$) will be apportioned a larger amount of the available transmit power for that cluster (i.e., $p_{k,2} > p_{k,1}$), enabling the mitigation of intra-cluster interference. Specficially, SIC will be implemented by the strong user to decode the signal $s_{k,2}$ of the weak user, subtract it from the combined signal, and then extract the desired signal $s_{k,1}$. In our proposed approach, the inter-cluster interference will be removed by the joint design of the transmit beamforming vectors $\{\mathbf{w}_k\}_{k=1}^{K}$ used for the different clusters. The detailed derivations are presented in the next Section.

III. PROPOSED ALGORITHM

In this section, we first present a *greedy* algorithm for user clustering. Then, a block diagonalization beamforming approach is developed to remove inter-cluster interference. Lastly, the NOMA power allocation factors are determined by sum rate maximization.

A. User clustering

User clustering can play an important role in improving the performance of transmit beamforming and power allocation within the above NOMA framework. In order to cluster users in a system, an exhaustive search can be implemented to achieve maximum capacity. However, this type of approach entails high computational complexity, which grows exponentially with the number of users. Alternatively, under the assumption of known CSI, we can employ metrics derived from the channel vectors of the users to sort and group them into clusters.

High correlation between the channel vectors of the users in a cluster can provide a better beamforming performance. In effect, if users in a cluster have highly correlated channels, they can share a common beamforming vector; in this way, more degrees of freedom will be left for the cancellation of inter-cluster interference (as explained in Section III.B). Besides, it has been shown in [16] that a large channel gain difference between the two users in a cluster can improve the performance of power domain NOMA by increasing the achievable sum rate. As a result, an effective clustering algorithm should seek to pair users with channel vectors exhibiting large correlation and gain difference simultaneously, which represents a multi-objective problem.

In this paper, we approach this problem by implementing a widely-used method for multi-objective optimization, i.e. the so-called weighted sum method. Specifically, this method transforms the multiple objectives into a single objective function by aggregating the former into a convex combination. The proposed user clustering approach is summarized in the following algorithm, where $\mathbf{h}_i$ for $i = 1, \ldots, 2K$ are the known channel vectors of the $2K$ users prior to clustering.

- **Step 0:** Sort users on the basis of their channel gains in descending order, i.e.: $\|\mathbf{h}_1\| \geq \|\mathbf{h}_2\| \geq \cdots \geq \|\mathbf{h}_{2K}\|$. We refer to the first K users with index $i \in \mathcal{K} = \{1, \ldots, K\}$, as the strong users, and to the remaining users with index $j \in \bar{\mathcal{K}} = \{K+1, \ldots, 2K\}$ as the weak users.
- **Step 1:** Calculate the channel correlations and channel gain differences between the strong and weak users

  $$\mathrm{C}_{i,j} = \frac{|\mathbf{h}_i^H \mathbf{h}_j|}{\|\mathbf{h}_i\| \|\mathbf{h}'_j\|} \qquad \forall i \in \mathcal{K}, j \in \bar{\mathcal{K}}$$

  $$\mathrm{D}_{i,j} = |\|\mathbf{h}_i\| - \|\mathbf{h}_j\|| \quad \forall i \in \mathcal{K}, j \in \bar{\mathcal{K}}$$

  Then set $k = 1$.
- **Step 2:** Select a user pair based on following weighted sum method

  $$(i^*, j^*) = \arg \max_{i \in \mathcal{K}, j \in \bar{\mathcal{K}}} \ \beta \mathrm{C}_{i,j} + (1-\beta) \mathrm{D}_{i,j} \tag{3}$$

  where $\beta \in [0, 1]$ is a weighting factor. Set $\mathbf{h}_{k,1} = \mathbf{h}_{i^*}$ and $\mathbf{h}_{k,2} = \mathbf{h}_{j^*}$.
- **Step 3:** Remove users i^* and j^* from the unassigned users, i.e. update $\mathcal{K}$ and $\bar{\mathcal{K}}$. Set $k = k + 1$.
- **Step 4:** Repeat Step 2-3 until all users are assigned.

B. Block diagonalization beamforming

Considering that CSI is available at the BS, block diagonalization beamforming can be adopted in a single-cell multi-user MIMO system to remove the inter-cluster interference and enhance the quality of service (QoS) for intra-cluster users [17]. Specifically, by projecting the transmitted signal onto the null-space of the interfering channels, the proposed transmit beamforming can eliminate the inter-cluster interference, as further detailed below.

In order to eliminate inter-cluster interference, the transmit beamforming vectors of the kth cluster, $\mathbf{w}_k$, should be generated such that

$$\mathbf{h}_{m,i}^H \mathbf{w}_k = \mathbf{0} \quad \forall\ m \neq k,\ i = 1, 2 \tag{4}$$

In other words, the beamforming vector of each cluster should be in the null-space of interfering channels from other clusters. Let us define $\mathbf{H}_k = [\mathbf{h}_{k,1} \ \ \mathbf{h}_{k,2}] \in \mathbb{C}^{N \times 2}$ and

$$\mathbf{H}_{-k} = [\mathbf{H}_1 \ldots \mathbf{H}_{k-1} \ \mathbf{H}_{k+1} \ldots \mathbf{H}_K] \in \mathbb{C}^{N \times 2(K-1)} \tag{5}$$

for $k \in \mathcal{K}$. Hence, we seek $\mathbf{w}_k$ orthogonal to the column span of $\mathbf{H}_{-k}$, i.e., $\mathbf{H}_{-k}^H \mathbf{w}_k = \mathbf{0}$. Here, it is assumed that the number of anteannas N is larger than $2(K-1)$; also, without loss in generality, we set $\|\mathbf{w}_k\| = 1$.

The singular value decomposition (SVD) can be employed to calculate the beamforming vectors. Applying the SVD to $\mathbf{H}_{-k}$ yields

$$\mathbf{H}_{-k} = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H \tag{6}$$

where $\mathbf{U}_k \in \mathbb{C}^{N \times N}$ and $\mathbf{V}_k \in \mathbb{C}^{2(K-1) \times 2(K-1)}$ are unitary matrices and $\mathbf{\Sigma}_k \in \mathbb{R}^{N \times 2(K-1)}$ is the rectangular diagonal matrix of singular values. Let r denote the rank of matrix $\mathbf{H}_{-k}$, which corresponds to the number of non-zero diagonal entries in $\mathbf{\Sigma}_k$. The null-space of the interfering channel matrix $\mathbf{H}_{-k}$ is spanned by the left singular vectors (i.e. columns of matrix $\mathbf{U}_k$) associated to the zero singular values of $\mathbf{H}_{-k}$. We represent these basis vectors by the following matrix

$$\mathbf{B}_k = [\mathbf{u}_{r+1,k} \ \mathbf{u}_{r+2,k} \ \ldots \mathbf{u}_{N,k}] \tag{7}$$

where $\mathbf{u}_{i,k}$ denotes the ith column of $\mathbf{U}_k$.

Any beamforming vector chosen from this null-space is capable of eliminating the interference from other clusters. However, an arbitrary selection of beamforming vector from this null-space can have detrimental effects on users within the cluster. Consequently, we use the projection of the MRT [18] beamformer onto this null-space, as the desired beamforming vector. Specifically, let $\mathbf{P}_k = \mathbf{B}_k \mathbf{B}_k^\dagger$ denote the projection operator for $\mathbf{B}_k$, where $\mathbf{B}_k^\dagger = (\mathbf{B}_k^H \mathbf{B}_k)^{-1} \mathbf{B}_k^H$ is the left-pseudo inverse of $\mathbf{B}_k$. We can express the projected MRT beamforming vector as follows

$$\mathbf{w}_k = \frac{\mathbf{P}_k \mathbf{h}_{k,1}}{\|\mathbf{P}_k \mathbf{h}_{k,1}\|} \tag{8}$$

In our proposed approach, we use $\mathbf{h}_{k,1}$ as the MRT beamforming vector, as opposed to $\mathbf{h}_{k,2}$. We have been able to observe that MRT based beamforming using the channel vector of the strong user in (8) leads to better sum rate for the cluster.

C. Power allocation for NOMA

According to (4), the received signal at the strong user in the kth cluster can be rewritten as follows

$$y_{k,1} = \mathbf{h}_{k,1}^H \mathbf{w}_k \sqrt{p_{k,1}} s_{k,1} + \mathbf{h}_{k,1}^H \mathbf{w}_k \sqrt{p_{k,2}} s_{k,2} + n_{k,1} \tag{9}$$

In order to implement SIC, the strong user should be able to detect and cancel the signal of the weak user. Hence, the SINR

Algorithm 1: The CCCP-based iterative algorithm.

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Step 0: Initialize $\mathbf{p}_k^{(0)} \in \mathbb{R}^2$, choose $\varepsilon \geq 0, K_{max}$, and set $t = 0$.
Step 1: If $|q(\mathbf{p}_k^{(t+1)}) - q(\mathbf{p}_k^{(t)})| \leq \varepsilon$ or $t > T_{max}$: STOP.
Step 2: Convex approximation of $q(\mathbf{p}_k)$ at $\mathbf{p}_k^{(t)}$:
    $q^{(t)}(\mathbf{p}_k) = f(\mathbf{p}_k) - g(\mathbf{p}_k^{(t)}) - \nabla g(\mathbf{p}_k^{(t)})^T(\mathbf{p}_k - \mathbf{p}_k^{(t)})$
Step 3: Solve:
    $\mathbf{p}_k^{(t+1)} = \arg\min_{p_{k,i} \in \mathcal{P}} \; q^{(t)}(\mathbf{p}_k)$
Step 4: Put $t \leftarrow t+1$ and go to (Step 1).
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seen by the strong user before and after the cancellation are given respectively by[1]

$$\mathrm{SINR}'_{k,1} = \frac{|\mathbf{h}_{k,1}^H \mathbf{w}_k|^2 p_{k,2}}{|\mathbf{h}_{k,1}^H \mathbf{w}_k|^2 p_{k,1} + \sigma_{k,1}^2} \tag{10}$$

$$\mathrm{SINR}_{k,1} = \frac{|\mathbf{h}_{k,1}^H \mathbf{w}_k|^2 p_{k,1}}{\sigma_{k,1}^2} \tag{11}$$

For successful performance of SIC, the condition $\mathrm{SINR}'_{k,1} \geq \mathrm{SINR}_{k,1}$ should be satisfied. The weak user does not perform SIC and decode its signal directly. Consequently, the received SINR is

$$\mathrm{SINR}_{k,2} = \frac{|\mathbf{h}_{k,2}^H \mathbf{w}_k|^2 p_{k,2}}{|\mathbf{h}_{k,2}^H \mathbf{w}_k|^2 p_{k,1} + \sigma_{k,2}^2} \tag{12}$$

To guarantee the fairness between the useres, our aim is to determine the power factors of the NOMA-based signals in a cluster such that the transmission sum rate is maximized subject to appropriate power and QoS constraints. This problem can be formulated as below

$$\max_{p_{k,i} > 0} \quad \mathrm{R}_{k,1} + \mathrm{R}_{k,2} \tag{13a}$$

$$\text{s.t.} \quad p_{k,1} + p_{k,2} \leq P_{max} \tag{13b}$$

$$\mathrm{SINR}_{k,1}^{(2)} \geq \mathrm{SINR}_{k,1} \tag{13c}$$

where P_{max} is the maximum transmission power per cluster and $R_{k,i} = \log_2(1 + \mathrm{SINR}_{k,i})$ is the transmission rate.

Since problem (13) is in general non-convex, we reformulate it into a more tractable form. By defining $\alpha_{k,i} = |\mathbf{h}_{k,i}^H \mathbf{w}_k|^2$ and, $\mathbf{p}_k = [p_{k,1} \; p_{k,2}]$, problem (13) can be stated compactly as

$$\min_{\mathbf{p}_k} \quad q(\mathbf{p}_k) := f(\mathbf{p}_k) - g(\mathbf{p}_k) \tag{14a}$$

$$\text{s.t.} \quad p_{k,1} + p_{k,2} \leq P_{max} \tag{14b}$$

$$\mathrm{SINR}'_{k,1} \geq \mathrm{SINR}_{k,1} \tag{14c}$$

where

$$\begin{aligned} f(\mathbf{p}_k) = -\log_2(1 + \frac{\alpha_{k,1} p_{k,1}}{\sigma_{k,1}^2}) - \log_2(\alpha_{k,2} p_{k,2} \\ + \alpha_{k,2} p_{k,1} + \sigma_{k,2}^2) \end{aligned} \tag{15}$$

$$g(\mathbf{p}_k) = -\log_2(\alpha_{k,2} p_{k,1} + \sigma_{k,2}^2) \tag{16}$$

We note that $f(\mathbf{p}_k)$ and $g(\mathbf{p}_k)$ are both convex functions of the power factors $p_{k,i}$, which means that the new objective $q(\mathbf{p}_k)$ represents a difference of two convex functions. Thus, it can be efficiently solved using the iterative CCCP.

By linearizing the non-convex part of the objective, we obtain convex sub-problems. Specifically, the first-order Taylor expansion of $\mathbf{g}(\mathbf{p}_k)$ around the current point in the kth iteration is expressed as

$$g(\mathbf{p}_k) = g(\mathbf{p}_k^{(t)}) + \nabla g(\mathbf{p}_k^{(t)})^T(\mathbf{p}_k - \mathbf{p}_k^{(t)}) \tag{17}$$

where $\nabla g(\mathbf{p}_k^{(t)})$ denotes the gradient of $g(\mathbf{p}_k)$ with respect to vector $\mathbf{p}_k$, evaluated at $\mathbf{p}_k^{(t)}$. Consequently,a sub-optimal solution to (14) can be efficiently found by iteratively solving a sequence of convex sub-problems.

The CCCP-based iterative algorithm proposed for solving problem (13) is summarized in Algorithm 1. For simplicity in notation, we define the feasible set as $\mathcal{P} = \{\mathbf{p}_k | p_{k,i} > 0, \text{(13a)}, \text{(13b)}\}$.

Repeated application of the CCCP iteration will eventually lead to a stationary solution of problem (13) [15]. The limit point of the iterates generated by the proposed CCCP algorithm also satisfies the KKT conditions of the DC program (14), which thus guarantees convergence to a local optimal solution of problem (13).

IV. NUMERICAL RESULTS

In this section, numerical experiments are carried out to illustrate the performance of the proposed NOMA scheme. For this purpose, we consider a single cell with one BS, which is equipped with N antennas for downlink transmission. In our simulations, we model the channel coefficients between each transmit antenna of the BS and the receive antenna of each user as the product of a path loss and complex Rayleigh fading coefficient, with zero-mean and unit variance. The (linear) path loss is given by $L = \gamma(d_0/d)^\eta$, where d is the distance between the transmit and receive antennas, $d_0 = 100$m is the reference distance, $\gamma = 0.05$ (-13dB) is a constant, and $\eta = 2$ is the path loss exponent. The cell radius is 200m over which the users are distributed uniformly. Throughout the experiments, it is assumed that the noise variance is the same for all users, which is normalized to $\sigma_i^2 = 31$ dBm.

Fig. 2 compares the average achievable rate per user versus transmit power among the methods in [6], [9] and the proposed NOMA beamforming. In this regard, the number of antennas is $N = 20$ and the number of users is $2K = 10$. With the method in [6], despite the fact that the rate of the strong user is high, the weak user experiences extremely low rate, which does not change even with higher transmit power. While the performance of the weak user in the proposed approach is close to that of [9], the strong user has higher rate compared to [9]. Consequently, better total sum rate is obtained as further illustrated next.

Fig. 3 illustrates the impact of the number of users on the total sum rate. In this regard, the maximum transmit power

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[1] For conciseness, we refer to (11) as an SINR, even though formally under perfect cancellation it is more like SNR.

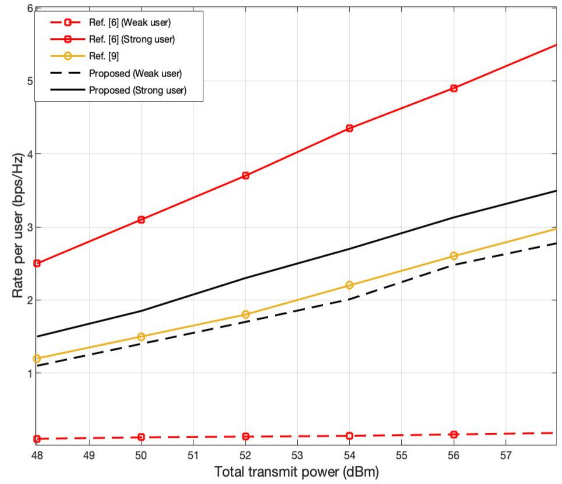

Fig. 2. Rate per user versus transmission power per cluster.

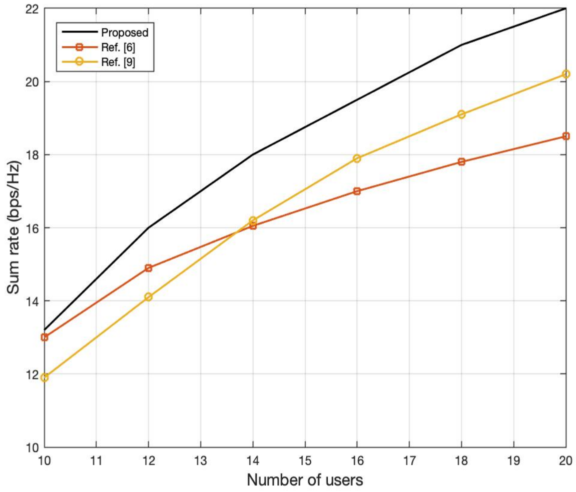

Fig. 3. Sum rate versus number of users.

per cluster is set to $P_{max} = 48$ dBm. It is noteworthy that the algorithm proposed in [6] is only able to remove inter-cluster interference for strong users while weak users suffer from inter-cluster interference. Consequently, an increase in the number users degrades the sum rate upward trend. Also, in contrast to [9], our proposed approach considers the channel gain difference between the two users in a cluster in addition to channel correlation. We thereby observe that the proposed method achieves better rate performance when compared to benchmark approaches.

V. CONCLUSION

In this paper, we studied the application of the power-domain NOMA along with downlink beamforming in a cellular system. Assuming that CSI is available, a greedy algorithm based on channel correlations and the channel gain differences was applied to partition users in a pairwise manner. The set of beamforming vectors was designed for the users in a cluster by using block diagonalization scheme to eliminate inter-cluster interference. The power factors of NOMA-based signals were determined by sum rate maximization subject to power and SINR constraints. Through simulations, it was shown that applying the proposed NOMA beamforming scheme can effectively eliminate inter-cluster interference and improve spectral efficiency.

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