Cooperative Localization of Mobile Nodes in NLOS

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***Abstract*—In this paper, cooperative localization of mobile nodes in non-line of sight (NLOS) situation is considered using a constrained square root unscented Kalman filter (CSRUKF). The NLOS measurements are used as quadratic constraints, which form a convex feasible region inside which the positions of the mobile nodes are supposed to be. The CSRUKF consists of two main stages: square root unscented Kalman filter (SRUKF) and sigma point projection. In the former, a conventional SRUKF is used to estimate the state vector and the Cholesky factor of the error covariance matrix. In the latter, a new set of sigma points are generated, and the ones violating the constraints are projected onto the feasible region by solving a set of convex quadratically constrained quadratic programs (QCQP). Each QCQP can be solved independently and in parallel for each sigma point violating the constraint, thus the algorithm is suitable for distributed processing. The simulation results show that our algorithm can perform well in different NLOS scenarios.**

***Index Terms*—Constrained Kalman filter, convex optimization, cooperative localization, non-line of sight.**

I. Introduction

The global positioning system (GPS) is a conventional system for localization and navigation. However, GPS is not a suitable technology for indoor places and dense urban areas due to shadowing and multipath propagation. Furthermore, the high battery consumption of the GPS devices is a limiting factor. Therefore, a ground-based localization system which can be used in indoor places offers a suitable alternative for applications in healthcare, surveillance, military, and many more [1]. In particular, the use of a low power and low cost wireless sensor network (WSN), which consists of some fixed anchor nodes at known location along with the mobile sensor nodes being tracked, is of great interest for such applications.

Different types of measurements can be used for localization purposes, including: time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA) and received signal strength (RSS). Accurate TOA measurements can be obtained by employing ultra wide-band (UWB) technology due to its fine timing resolution [2]. Accordingly, TOA is one of the most popular types of measurements currently being used for localization. Although TOA measurements between two nodes require the latter to be time synchronized, the effects of clock drift and offset can be mitigated using a two-way ranging (TWR) protocol [1].

Localization with WSN can be divided into two categories, namely: non-cooperative and cooperative. In the non-cooperative category, each sensor only uses the signals obtained from its neighbouring anchors in order to be localized or tracked. In practice, the number of anchors is limited and may not be adequate to allow for accurate, non-ambiguous localization. Cooperation between mobile nodes, by making pairwise measurements and exchanging information, can result in better localization performance. Therefore, the use of a cooperative WSN has started to gain attention in recent years [3], [4].

In practice however, the direct view between many pairs of nodes (anchors or sensors) is blocked by objects in the environment, resulting in a non-line of sight (NLOS) situation. In NLOS, the received signal is obtained after reflection from a scatterer or penetration through the blocking objects. Hence, the travel time of the signal increases, resulting in positively biased range measurements [5]. Localization in NLOS has been studied extensively for non-cooperative networks, as summarized in [6]. In case the sensor motion can be modelled by a dynamic state equation, several techniques have been proposed in [7], [8], [9]. More recently, the authors proposed a constrained square root unscented Kalman filter (CSRUKF) which can perform well in different NLOS scenarios [10]. Cooperative localization in NLOS has been considered for static networks in [11], [12], [13]. However, to the best knowledge of the authors, there is no work in the literature which considers cooperative localization of mobile nodes with dynamic equations in NLOS scenarios, and where prior statistics about the NLOS biases are not assumed available.

In this paper, we extend the centralized CSRUKF proposed in [10] to the cooperative localization scenario. The main idea is based on projection of sigma points of an unscented Kalman filter onto the feasible region, as first considered in [14]. The projection operation is a convex quadratically constrained quadratic program (QCQP) which can be solved efficiently in polynomial time. We show how the optimization problem can be solved more efficiently by avoiding inverse operations and by reducing its size. Furthermore, the resulting QCQP problems are decoupled and their solutions can be obtained in a distributed fashion; thus making our algorithm even more interesting for practical applications. Through simulations, it is shown that the proposed CSRUKF can perform well even in severe NLOS situations.

The organization of the paper is as follows: The system model and problem formulation are presented in Section II. The proposed algorithm is described in Section III. Its performance is compared to an existing technique through simulations in Section IV. Finally, Section V concludes the paper.

Notation: The identity matrix of size N is denoted by $\boldsymbol{I}_N$.

For $i \leq j$, $\boldsymbol{q}(i:j)$ denotes a vector of size $j-i+1$ obtained by extracting the i-th to j-th entries of vector $\boldsymbol{q}$, inclusively. $(\boldsymbol{U})_j$ denotes the j-th column of matrix $\boldsymbol{U}$ and $\boldsymbol{U}(i:j)$ is a sub-matrix which includes rows i to j of matrix $\boldsymbol{U}$.

II. Problem Statement

We consider a 2-dimensional (2D) WSN (the extension to 3D is straightforward) comprised of N sensor nodes with unknown locations $\boldsymbol{x}_k^i \in \mathbb{R}^2$ and unknown speeds $\dot{\boldsymbol{x}}_k^i \in \mathbb{R}^2$ for $i = \{1, \ldots, N\}$ at discrete time instant k, and of M anchors with known and fixed locations $\boldsymbol{a}^i$ for $i = \{N+1, \ldots, N+M\}$. The sensors move according to a random acceleration model as

$$\boldsymbol{x}_{k+1}^i = \boldsymbol{x}_k^i + \dot{\boldsymbol{x}}_k^i \delta t + \boldsymbol{w}_k^i \frac{\delta t^2}{2}, \quad i = 1, \ldots, N \tag{1}$$

where δt is the time step duration and $\boldsymbol{w}_k^i \in \mathbb{R}^2$ is a zero-mean white Gaussian random process.

Pairwise range measurements between neighbouring nodes are obtained either by one way ranging and prior network synchronization, or through TWR, modelled as

$$r_k^{ij} = \begin{cases} \|\boldsymbol{x}_k^i - \boldsymbol{a}^j\| + n_k^{ij}, & (i,j) \in \mathcal{L}_a \\ \|\boldsymbol{x}_k^i - \boldsymbol{x}_k^j\| + n_k^{ij}, & (i,j) \in \mathcal{L}_s \\ \|\boldsymbol{x}_k^i - \boldsymbol{a}^j\| + b_k^{ij} + n_k^{ij}, & (i,j) \in \mathcal{N}_a \\ \|\boldsymbol{x}_k^i - \boldsymbol{x}_k^j\| + b_k^{ij} + n_k^{ij}, & (i,j) \in \mathcal{N}_s \end{cases} \tag{2}$$

where

$\mathcal{L}_a : \{(i,j) : \text{LOS link between } i\text{-th sensor and } j\text{-th anchor}\}$
$\mathcal{L}_s : \{(i,j) : \text{LOS link between } i\text{-th and } j\text{-th sensors}\}$
$\mathcal{N}_a : \{(i,j) : \text{NLOS link between } i\text{-th sensor and } j\text{-th anchor}\}$
$\mathcal{N}_s : \{(i,j) : \text{NLOS link between } i\text{-th and } j\text{-th sensors}\}$

In (2), n_k^{ij} is a zero-mean Gaussian noise with known variance σ_n^2, and b_k^{ij} is the NLOS bias which is a positive random variable and has been modelled as exponential, uniform, shifted Gaussian and other distributions in different works [5], [15]. Prior knowledge about the distribution of NLOS may not be easy to obtain beforehand for practical online applications; hence we assume that we do not have any information about the statistics of b_k^{ij} including its mean and variance. We only assume that the NLOS links are identified accurately for every time instant as done in may works [16], [17].

The NLOS biases are large random variables and for UWB applications it can be assumed that $b_k^{ij} + n_k^{ij} \geq 0$, which is equivalent to stating that for all the NLOS measurements

$$\|\boldsymbol{x}_k^i - \boldsymbol{a}^j\| \leq r_k^{ij}, \quad (i,j) \in \mathcal{N}_a \tag{3}$$
$$\|\boldsymbol{x}_k^i - \boldsymbol{x}_k^j\| \leq r_k^{ij}, \quad (i,j) \in \mathcal{N}_s \tag{4}$$

In order to increase the robustness against large noise samples we use the following constraints instead

$$\|\boldsymbol{x}_k^i - \boldsymbol{a}^j\| \leq r_k^{ij} + \epsilon\sigma_n, \quad (i,j) \in \mathcal{N}_a \tag{5}$$
$$\|\boldsymbol{x}_k^i - \boldsymbol{x}_k^j\| \leq r_k^{ij} + \epsilon\sigma_n, \quad (i,j) \in \mathcal{N}_s \tag{6}$$

where $\epsilon \geq 0$ is a tuning parameter which increases the chance that the inequalities hold true. Note that the new inequalities in (5)-(6) might also hold true for LOS measurements; therefore, if a link is LOS but wrongly identified as being NLOS, the inequalities in (5)-(6) have a higher chance to be satisfied compared to the ones in (3)-(4).

In some works, the NLOS biases have been modelled by the random walk model, and therefore, they are included in the state vector and estimated together with other unknowns [7], [9]. However, random walk approximately models the relationship between b_k^{ij} and b_{k+1}^{ij}, and selecting a suitable variance for the increment of the random walk is not easy for dynamic environments. We therefore, avoid including them in the state vector and estimating them, however, we remove the NLOS measurements from the observation vector and instead use these measurements to impose the constraints in (5) and (6) on the positions of sensors. The measurement vector $\boldsymbol{z}_k$ is obtained by stacking together all the LOS measurements r_k^{ij} for $(i,j) \in \mathcal{L}_a \cup \mathcal{L}_s$. The state of all the sensors can also be expressed in a vector form as $\boldsymbol{s}_k = [\boldsymbol{x}_k^1, \boldsymbol{x}_k^2, \ldots, \boldsymbol{x}_k^N, \dot{\boldsymbol{x}}_k^1, \dot{\boldsymbol{x}}_k^2, \ldots, \dot{\boldsymbol{x}}_k^N]^T \in \mathbb{R}^{4N}$. We can finally formulate the constrained state space model as

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{s}_k) + \boldsymbol{n}_k \tag{7}$$
$$\boldsymbol{s}_k = \boldsymbol{F}\boldsymbol{s}_{k-1} + \boldsymbol{G}\boldsymbol{w}_k \tag{8}$$
$$\text{s.t.} \quad \|\boldsymbol{x}_k^i - \boldsymbol{a}^j\| \leq r_k^{ij} + \epsilon\sigma_n, \quad (i,j) \in \mathcal{N}_a \tag{9}$$
$$\|\boldsymbol{x}_k^i - \boldsymbol{x}_k^j\| \leq r_k^{ij} + \epsilon\sigma_n, \quad (i,j) \in \mathcal{N}_s \tag{10}$$

where $\boldsymbol{h}(\boldsymbol{s}_k)$ is the vector of true ranges, $\boldsymbol{n}_k$ is the vector of measurement errors with zero-mean and covariance matrix $\mathbf{R} = \sigma_n^2 \mathbf{I}$, $\boldsymbol{w}_k = [(\boldsymbol{w}_k^1)^T, (\boldsymbol{w}_k^2)^T, \ldots, (\boldsymbol{w}_k^N)^T]^T$ is a zero-mean Gaussian vector with covariance matrix $\boldsymbol{Q}$ and

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{I}_{2N} & \delta t \boldsymbol{I}_{2N} \\ \boldsymbol{0}_{2N} & \boldsymbol{I}_{2N} \end{bmatrix} \in \mathbb{R}^{4N \times 4N}, \quad \boldsymbol{G} = \begin{bmatrix} \frac{\delta t^2}{2} \boldsymbol{I}_{2N} \\ \delta t \boldsymbol{I}_{2N} \end{bmatrix} \in \mathbb{R}^{4N \times 2N} \tag{11}$$

In the following, we show how to estimate $\boldsymbol{s}_k$ based on the history of the range measurements and the constraints.

III. Proposed Centralized Algorithm

The proposed CSRUKF consists of two main stages: unconstrained nonlinear Kalman filter (SRUKF [18]), and projection of the sigma points violating the constraints.

A. Unconstrained SRUKF

If there are no constraints on the state vector, then a nonlinear Kalman filter can be used for the *a posteriori* estimation of state and the corresponding error covariance matrix, i.e., $\boldsymbol{s}_{k|k}$ and $\boldsymbol{\Sigma}_{k|k}$, respectively. We use a SRUKF for this aim, as proposed in [18], and find $\boldsymbol{s}_{k|k}$ and the Cholesky factor of the error covariance matrix denoted by $\boldsymbol{U}_{k|k}$, as described in Algorithm 1, where $L = 4N$. A detailed explanation about selection of parameters η_α and ϵ and their relation with the weights $w^{(l)}$ can be found in [10].

Algorithm 1 SRUKF

Initialize $\boldsymbol{s}_{0|0}$ and set $\boldsymbol{\Sigma}_{0|0}$ to a large symmetric positive-definite (SPD) diagonal matrix.

Set η_α and ϵ

for $k = 1, \ldots, K$ **do**

Prediction

$$\boldsymbol{s}_{k|k-1} = \boldsymbol{F}\boldsymbol{s}_{k-1|k-1} \tag{12}$$

$$\boldsymbol{U}_{k|k-1} = \text{qr}\left\{ \begin{bmatrix} \boldsymbol{U}_{k-1|k-1}\boldsymbol{F}^T \\ \boldsymbol{Q}^{1/2}\boldsymbol{G} \end{bmatrix} \right\} \tag{13}$$

Correction

$$s^{(l)}_{k|k-1} = \begin{cases} \boldsymbol{s}_{k|k-1} \\ \boldsymbol{s}_{k|k-1} + \sqrt{\eta_\alpha}(\boldsymbol{U}_{k|k-1})_l, l = 1, \ldots, L \\ \boldsymbol{s}_{k|k-1} - \sqrt{\eta_\alpha}(\boldsymbol{U}_{k|k-1})_l, l = L+1, \ldots, 2L \end{cases} \tag{14}$$

$$\boldsymbol{z}^{(l)}_{k|k-1} = \boldsymbol{h}(\boldsymbol{s}^{(l)}_{k|k-1}), \quad l = 0, \ldots, 2L \tag{15}$$

$$\hat{\boldsymbol{z}}_{k|k-1} = \sum_{l=0}^{2L} w^{(l)}\boldsymbol{z}^{(l)}_{k|k-1} \tag{16}$$

$$\boldsymbol{\Sigma}^{\boldsymbol{s},\boldsymbol{z}}_{k|k-1} = \sum_{l=0}^{2L} w^{(l)}(\boldsymbol{s}^{(l)}_{k|k-1} - \boldsymbol{s}_{k|k-1})(\boldsymbol{z}^{(l)}_{k|k-1} - \hat{\boldsymbol{z}}_{k|k-1})^T \tag{17}$$

$$\mathbf{e}^{(l)}_z = \sqrt{w^{(l)}}(\boldsymbol{z}^{(l)}_{k|k-1} - \hat{\boldsymbol{z}}_{k|k-1}), \quad l = 0, \ldots, 2L$$

$$\boldsymbol{U}_{\boldsymbol{z}_k} = \text{qr}\left\{ \left[\mathbf{e}^{(0)}_z, \mathbf{e}^{(1)}_z, \ldots, \mathbf{e}^{(2L)}_z, \boldsymbol{R}^{\frac{1}{2}}\right]^T \right\} \tag{18}$$

$$\boldsymbol{T}_k = \boldsymbol{\Sigma}^{\boldsymbol{s},\boldsymbol{z}}_{k|k-1}\boldsymbol{U}^{-1}_{\boldsymbol{z}_k} \tag{19}$$

$$\boldsymbol{s}_{k|k} = \boldsymbol{s}_{k|k-1} + \boldsymbol{T}_k\boldsymbol{U}^{-T}_{\boldsymbol{z}_k}(\boldsymbol{z}_k - \hat{\boldsymbol{z}}_{k|k-1}) \tag{20}$$

$$\boldsymbol{U}_{k|k} = \text{cholupdate}\{\boldsymbol{U}_{k|k-1}, \boldsymbol{T}_k, -1\} \tag{21}$$

end for

B. Projection of Sigma Points

Assume that the *a posteriori* state estimate and the Cholesky factor of the *a posteriori* error covariance matrix have been obtained using a SRUKF without taking the constraints into account. To impose the constraints on the estimated state and error covariance matrix, similar to [14], a new set of sigma points are generated according to

$$\boldsymbol{s}^{(l)}_{k|k} = \begin{cases} \boldsymbol{s}_{k|k}, & l = 0, \\ \boldsymbol{s}_{k|k} + \sqrt{\eta_\alpha}(\boldsymbol{U}^T_{k|k})_l, & l = 1, \ldots, L, \\ \boldsymbol{s}_{k|k} - \sqrt{\eta_\alpha}(\boldsymbol{U}^T_{k|k})_{l-L}, & l = L+1, \ldots, 2L. \end{cases} \tag{22}$$

The generated sigma points form an uncertainty ellipsoid with $\boldsymbol{s}_{k|k}$ at its centre as illustrated in Fig. 1 for the simple case of $L = 2$, i.e., one sensor with only position coordinates as the state vector. After the generation of sigma points $\boldsymbol{s}^{(l)}_{k|k}$, those which violate the constraints are projected onto the convex

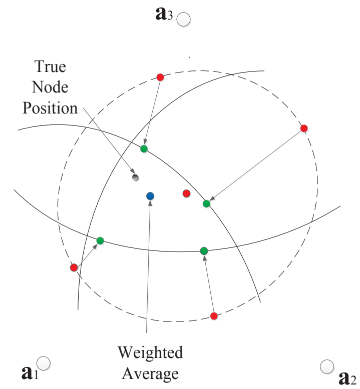

Fig. 1: The sigma points before and after projection are illustrated with red and green circles, respectively for a single node. The weighted average falls inside the feasible region.

feasible region through

$$\mathcal{P}(\boldsymbol{s}^{(l)}_{k|k}) = \arg\min_{\boldsymbol{q}} \left\{(\boldsymbol{q} - \boldsymbol{s}^{(l)}_{k|k})^T\boldsymbol{W}_k(\boldsymbol{q} - \boldsymbol{s}^{(l)}_{k|k})\right\}, \tag{23}$$

$$\text{s.t. } \left\|\boldsymbol{q}(2i-1:2i) - \boldsymbol{a}^j\right\| \le r^{ij}_k + \epsilon\sigma_n, \quad (i,j) \in \mathcal{N}_a$$
$$\left\|\boldsymbol{q}(2i-1:2i) - \boldsymbol{q}(2j-1:2j)\right\| \le r^{ij}_k + \epsilon\sigma_n, \ (i,j) \in \mathcal{N}_s$$

where $\boldsymbol{W}_k$ is a SPD weighting matrix [19], [20]. One reasonable choice is $\boldsymbol{W}_k = \boldsymbol{\Sigma}^{-1}_{k|k}$, which gives the smallest estimation error covariance matrix when a linear KF is applied to a system with linear dynamic equations and with zero-mean Gaussian observation and excitation noises [20].

The optimization problem in (23) is a QCQP, which is convex since $\boldsymbol{W}_k$ is SPD and the constraints are convex [21, p.153]. As the constraints are only on the first $2N$ elements of the state vector, i.e., the position coordinates, it is possible to reduce the size of the QCQP problem.

Recalling that $\boldsymbol{\Sigma}_{k|k} = \boldsymbol{U}^T_{k|k}\boldsymbol{U}_{k|k}$, the objective function in (23) can be expressed as $(\boldsymbol{q} - \boldsymbol{s}^{(l)}_{k|k})^T\boldsymbol{U}^{-1}_{k|k}\boldsymbol{U}^{-T}_{k|k}(\boldsymbol{q} - \boldsymbol{s}^{(l)}_{k|k})$. To avoid the inverse operation, we define

$$\boldsymbol{u} = \boldsymbol{U}^{-T}_{k|k}(\boldsymbol{s}^{(l)}_{k|k} - \boldsymbol{q}). \tag{24}$$

Then it follows that

$$\boldsymbol{q} = \boldsymbol{s}^{(l)}_{k|k} - \boldsymbol{U}^T_{k|k}\boldsymbol{u}. \tag{25}$$

We partition the lower triangular matrix $\boldsymbol{U}^T_{k|k}$ as follows:

$$\boldsymbol{U}^T_{k|k} = \begin{bmatrix} \boldsymbol{L}_{11} & \boldsymbol{0} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix}, \tag{26}$$

where $\boldsymbol{L}_{11} \in \mathbb{R}^{2N\times2N}$ and $\boldsymbol{L}_{22} \in \mathbb{R}^{2N\times2N}$ are lower triangular. Then from (25) we have

$$\boldsymbol{q}(1:2N) = \boldsymbol{s}^{(l)}_{k|k}(1:2N) - \boldsymbol{L}_{11}\boldsymbol{u}(1:2N). \tag{27}$$

Using (25) and (27), and noting that the optimal $\boldsymbol{u}(2N+1:4N) = \boldsymbol{0}$ (since the constraints only depend on $\boldsymbol{u}(1:2N)$) we can reformulate the QCQP problem (23) as (28). This convex

$$\min_{\boldsymbol{u}(1:2N)} \left\{\boldsymbol{u}^T(1:2N)\boldsymbol{u}(1:2N)\right\} \tag{28}$$
$$\text{s.t. } \left\|\boldsymbol{s}_{k|k}^{(l)}(2i-1:2i) - \boldsymbol{L}_{11}(2i-1:2i)\boldsymbol{u}(1:2N) - \boldsymbol{a}^j\right\| \le r_k^{ij} + \epsilon\sigma_n,\ (i,j)\in\mathcal{N}_a$$
$$\left\|\boldsymbol{s}_{k|k}^{(l)}(2i-1:2i) - \boldsymbol{L}_{11}(2i-1:2i)\boldsymbol{u}(1:2N) - \boldsymbol{s}_{k|k}^{(l)}(2j-1:2j) + \boldsymbol{L}_{11}(2j-1:2j)\boldsymbol{u}(1:2N)\right\| \le r_k^{ij} + \epsilon\sigma_n,\ (i,j)\in\mathcal{N}_s$$

QCQP problem can now be solved efficiently using iterative techniques [21]. After finding the optimal $\boldsymbol{u}(1:2N)$, we can compute the optimal $\boldsymbol{q}$ using (25) and the fact that the optimal $\boldsymbol{u}(2N+1:4N) = \boldsymbol{0}$ as follows:

$$\mathcal{P}(\boldsymbol{s}_{k|k}^{(l)}) \triangleq \boldsymbol{q} = \boldsymbol{s}_{k|k}^{(l)} - \begin{bmatrix}\boldsymbol{L}_{11}\\ \boldsymbol{L}_{21}\end{bmatrix}\boldsymbol{u}(1:2N). \tag{29}$$

After finding the projected sigma points through (29), the state and the Cholesky factor of the error covariance matrix may be estimated through weighted averaging and QR factorization, respectively as

$$\boldsymbol{s}_{k|k}^{\mathcal{P}} = \sum_{l=0}^{2L} w^{(l)}\mathcal{P}(\boldsymbol{s}_{k|k}^{(l)}), \tag{30}$$

$$\boldsymbol{U}_{k|k}^{\mathcal{P}} = \text{qr}\left\{[\mathbf{e}_{\mathcal{P}}^{(0)}, \mathbf{e}_{\mathcal{P}}^{(1)}, \ldots, \mathbf{e}_{\mathcal{P}}^{(2L)}]^T\right\}, \tag{31}$$

where

$$\mathbf{e}_{\mathcal{P}}^{(l)} = \sqrt{w^{(l)}}(\mathcal{P}(\boldsymbol{s}_{k|k}^{(l)}) - \boldsymbol{s}_{k|k}^{\mathcal{P}})), \quad l = 0, \ldots, 2L.$$

Finally, in the next iteration of the unconstrained SRUKF, the constrained *a posteriori* state estimate $\boldsymbol{s}_{k|k}^{\mathcal{P}}$ and the Cholesky factor of the corresponding error covariance matrix $\boldsymbol{U}_{k|k}^{\mathcal{P}}$ replace $\boldsymbol{s}_{k|k}$ and $\boldsymbol{U}_{k|k}$, respectively as

$$\boldsymbol{s}_{k|k} = \boldsymbol{s}_{k|k}^{\mathcal{P}}, \tag{32}$$

$$\boldsymbol{U}_{k|k} = \boldsymbol{U}_{k|k}^{\mathcal{P}}. \tag{33}$$

C. Centralized Algorithm Summary

The proposed two stage CSRUKF is summarized in Algorithm 2.

Algorithm 2 CSRUKF

Initialize $\boldsymbol{s}_{0|0}$ and set $\boldsymbol{\Sigma}_{0|0}$ to a large SPD diagonal matrix
Set η_α and ϵ
for $k = 1, \ldots, K$ **do**
    Estimate $\boldsymbol{s}_{k|k}$ and $\boldsymbol{U}_{k|k}$ using a conventional SRUKF as described in Algorithm 1 [18].
    Generate the sigma points using (22).
    For every sigma point whose first two elements fall outside the feasible region solve (28) and find the projected point (29).
    Estimate $\boldsymbol{s}_{k|k}^{\mathcal{P}}$ using (30) and $\boldsymbol{U}_{k|k}^{\mathcal{P}}$ using (31).
    Update the *a posteriori* estimates, i.e., (32) and (33).
end for

IV. Simulation Results

To evaluate the performance of the proposed technique, we consider a 2D area with $M = 4$ anchors and $N = 5$ mobile sensors. The sensors are initially place uniformly on the plane, and move independently accordingly to the considered motion model in (1) with $\delta t = 0.2$ for 100 time steps. The anchors are located at positions $\boldsymbol{a}^6 = [0,0]$, $\boldsymbol{a}^7 = [0,10]$, $\boldsymbol{a}^8 = [10,0]$, and $\boldsymbol{a}^9 = [10,10]$. We assume that if the distance between the nodes is less than 10m then they are regarded as neighbours and they obtain pairwise range measurements. The true range between neighbouring nodes is disturbed with a Gaussian noise with zero-mean and standard deviation $\sigma_n = 0.1$m in order to model the range measurement. Ranging with centimetres accuracy is in accordance with IEEE 802.15.4a in indoor environments with LOS connection [22]. For the NLOS links, a positive exponential random variable with mean and standard deviation of 5m is also added to the obtained ranges. The tuning parameters in CSRUKF are set as $\epsilon = 3$ and $\eta_\alpha = 0.8$. The convex QCQP problems are solved using Sedumi solver [23] and Yalmip optimization package [24].

To test the algorithm, we consider three different NLOS scenarios where the probability of a link being NLOS, denoted as $P_{\mathcal{N}}$ varies from a low to a high value, as follows:

- Small ratio of NLOS to LOS links: $P_{\mathcal{N}} = 0.05$
- Moderate ratio of NLOS to LOS links: $P_{\mathcal{N}} = 0.5$
- Large ratio of NLOS to LOS links: $P_{\mathcal{N}} = 0.95$

For comparison purposes, we consider an unconstrained SRUKF with rejection of NLOS links, denoted by "SRUKF Outlier Rejection". If there are enough LOS measurements available for each node, then outlier rejection is the right strategy, but in the absence of enough LOS nodes the performance of this method might be severely degraded. As a performance metric, the cumulative distribution function (CDF) of the network positioning error, defined as

$$\text{CDF}(e_k) = \Pr\left\{\frac{1}{N}\sum_{i=1}^{N}\|\boldsymbol{s}_{k|k}(2i-1:2i) - \boldsymbol{s}_k(2i-1:2i)\| \le e_k\right\}$$

is evaluated empirically for different values of $P_{\mathcal{N}}$.

As observed in Fig. 2, for low ratio of NLOS to LOS links, the SRUKF with outlier rejection performs almost the same as our proposed CSRUKF. This confirms that using a few NLOS links as constraints might not improve the localization performance. However, for $P_{\mathcal{N}} = 0.5$, the performance of Kalman filtering with outlier rejection is degraded noticeably (it sometimes even diverges) while that of the proposed CSRUKF is less than 1m with 90% chance. For large ratios of NLOS to LOS links, the outlier rejection technique diverges because it essentially uses the prediction step of the SRUKF

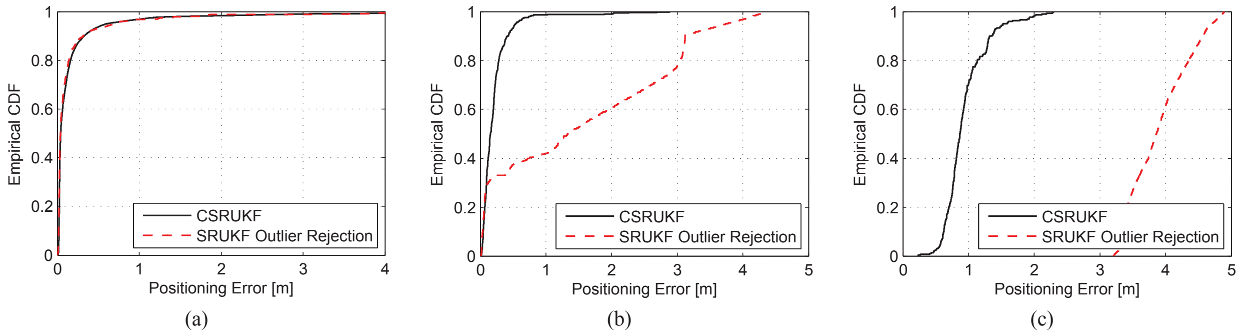

Fig. 2: CDFs of the network positioning error in different scenarios: (a) $P_{\mathcal{N}} = 0.05$; (b) $P_{\mathcal{N}} = 0.5$; (c) $P_{\mathcal{N}} = 0.95$.

(the measurement vector $\boldsymbol{z}_k$ is empty most of the times), while the proposed technique has a decent performance.

V. Conclusion and Future Work

A two-stage CSRUKF was considered in this work for cooperative localization in NLOS scenarios. In the first step, only the LOS measurements are considered in the observation vector and an unconstrained SRUKF is applied to the considered state space model. Then, a set of sigma points are generated and the ones violating the constraints are projected onto the convex feasible region. The constraints are obtained by using the NLOS measurements as quadratic constraints. The projection operation becomes a convex QCQP, and by reducing its size it is solved efficiently using convex optimization packages. The simulation results showed that the proposed CSRUKF could perform robustly under severe NLOS conditions. Solving the QCQPs might have a relatively high computational cost for a large WSN, however, due to their independence from each other they could be solved in parallel over multiple processors. Future work will focus on implementing a distributed CSRUKF.

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