A NEW BEAMFORMING ALGORITHM BASED ON SIGNAL SUBSPACE EIGENVECTORS

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ABSTRACT

A new beamforming algorithm, based on the eigendecomposion of the sample correlation matrix, has been introduced. The beamformer uses a weighted linear combination of the signal eigenvectors. Three versions of the beamformer have been proposed. It is shown that the proposed beamformer is a generalization of the delay-and-sum and the minimum variance beamformers. A linearly constrained minimum variance beamformer has also been derived. It is shown that the proposed approach induces robust beamformers.

1. INTRODUCTION

In various applications, one is concerned with extracting a desired signal immersed in noise and interference. Using an adaptive array, it is possible to avert the effect of interference and noise by an elaborate selection of array weights. Many algorithms maximize the array output signal to interference ratio (SINR) subject to knowing the direction of arrival (DOA) of the desired signal. In these cases, the weight vector is computed from the correlation matrix of interference and noise . We call this the signal-free correlation matrix (SFCM). However, if the desired signal DOA and the array geometry are known, one can use the correlation matrix of the received mixture of signal, noise, and interference, and attain the same result. Small errors in calibration and DOA estimation will cause signal cancellation [1, 2].

For most practical situations, noise and interference are mixed with signal, and the measurement of SFCM is not a simple task. To compute a signal-free correlation matrix, one can use the generalized sidelobe canceller (GSC) [3]. However, in this method, the calibration error or the desired signal DOA estimation error will cause a leakage of signal component which degrades the performance of method.

In many practical applications, the performance of detection depends on signal-to-interference ratio (SIR). For instance, in spread spectrum communications, penetration of a smart jammer into the system, may cause a destructive effect on the system performance [4]. In such cases, interference minimization, rather than noise plus interference minimization, proves useful.

As a result of a higher resolution, much interest has been given to beamforming based on eigendecomposition [4, 5], and adaptive eigensubspace algorithms [6, 7]. Usually, these methods are based on the eigendecomposition of SFCM.

Here, we introduce a beamforming method based on the eigendecomposition of the received signal covariance matrix. To apply this beamforming method, one should know the DOA estimate of the desired signal, the number of point jammers, and an estimate of the received noise power. The introduced method, which needs a relatively low computation, is able to produce exact nulls in the direction of jammers . It is also able to maximize the output SINR or SNR. Due to lack of space, throughout, we omit the proof for the theorems.

2. SIGNAL MODEL

We assume an L-element array with arbitrary geometry and p narrowband point sources. Let $\mathbf{x}(k)$ denote the complex data vector received by the array elements at the k'th sampling instant. Data vector $\mathbf{x}(k)$ can be expressed as a superposition of the received signals and noise as

$$\mathbf{x}(k) = \mathbf{A}(k)\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where $\mathbf{n}(k)$ is the noise vector which is assumed to be white, $\mathbf{s}(k)$ is the signal vector, and

$$\mathbf{A}(k) = [\mathbf{a}(\theta_1(k)), \cdots, \mathbf{a}(\theta_p(k))], \tag{2}$$

with $\mathbf{a}(\theta_i) = \mathbf{a}_i$ being the array steering vector at the direction θ_i. Using (1), and assuming σ^2 to be the noise power , the autocorrelation matrix of the received signal is obtained as

$$\mathbf{R}(k) = E\{\mathbf{x}(k)\mathbf{x}(k)^H\} = \mathbf{A}(k)\mathbf{\Gamma}\mathbf{A}(k)^H + \sigma^2\mathbf{I}, \tag{3}$$

where $\mathbf{\Gamma} = diag(\gamma_1, \cdots, \gamma_p)$ is the signal correlation matrix, $E\{.\}$ represents the expected value, and superscript H denotes Hermitian transposition. Diagonal form of $\mathbf{\Gamma}$ is a consequence of the fact that the received signals are assumed to be uncorrelated with each other.

For the positive-definite correlation matrix $\mathbf{R}$ one can find a set of eigenvalues $(\lambda_i+\sigma^2)$'s and orthonormal eigenvectors $\mathbf{q}_i$'s such that:

$$\mathbf{R}\mathbf{q}_i = (\lambda_i + \sigma^2)\mathbf{q}_i \qquad \text{for} \qquad 1 \leq i \leq L.$$

We assume that λ_i's are in decreasing order, i.e. $(\lambda_1 + \sigma^2) \geq \cdots \geq (\lambda_L + \sigma^2)$. It can be shown that $\lambda_i = 0$ for $i > p$.

Eigenvectors $\mathbf{Q} = [\mathbf{q}_1 \cdots \mathbf{q}_L]$ can be divided into two matrices as $\mathbf{Q} = [\mathbf{Q}_s | \mathbf{Q}_n]$ where the columns of $\mathbf{Q}_s$ and $\mathbf{Q}_n$, respectively, span the orthogonal signal and noise subspaces. We can prove the following theorem.

Theorem 1: Defining $\mathbf{\Lambda}_s = \text{diag}(\lambda_1 + \sigma^2 \cdots \lambda_p + \sigma^2)$, the following equalities are valid

$$\mathbf{A}^H\mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}\mathbf{Q}_s^H\mathbf{A} = \mathbf{\Gamma}^{-1} \tag{4}$$

$$\mathbf{Q}_s^H\mathbf{A}\mathbf{\Gamma}\mathbf{A}^H\mathbf{Q}_s = (\mathbf{\Lambda}_s - \sigma^2\mathbf{I}). \tag{5}$$

3. REDUCED–RANK BEAMFORMER

To extract the n'th signal source (impinging on the array from direction θ_n), we propose the following beamforming weight vector

$$\mathbf{w}_{n,\epsilon} = \sum_{i=1}^{p} \frac{\mathbf{q}_i\mathbf{q}_i^H}{\epsilon\lambda_i + (1-\epsilon)\sigma^2}\mathbf{a}_n \qquad \text{for} \qquad 0 \leq \epsilon \leq 1 . \tag{6}$$

This beamforming method, which needs the knowledge of the desired signal DOA and the number of point signal sources, has certain properties for various values of ϵ. We study this beamforming method for three different values of $\epsilon = 1,\ 0.5,\ 0$, (noted by SC–1 ,SC–2 and SC–3, respectively).

3.1. Special Case–1 (SC–1)

For this case, we compute the weight vector $\mathbf{w}_n$ as

$$\mathbf{w}_n = \sum_{i=1}^{p} \frac{\mathbf{q}_i\mathbf{q}_i^H}{\lambda_i}\mathbf{a}_n = \mathbf{Q}_s(\mathbf{\Lambda}_s - \sigma^2\mathbf{I})^{-1}\mathbf{Q}_s^H\mathbf{a}_n. \tag{7}$$

Theorem 2: The pattern for the *SC-1* beamformer has null (exactly zero) in the direction of interferers, i.e.

$$\mathbf{w}_n^H\mathbf{a}_i = 0 \qquad \text{for} \qquad i = 1, \cdots, p \quad \text{and} \quad i \neq n \tag{8}$$

Theorem 3: The output SINR of the SC–1 beamformer is restricted to λ_1/σ^2 and λ_p/σ^2, i.e.

$$\frac{\lambda_p}{\sigma^2} \leq \left(\frac{S}{I+N}\right)_o \leq \frac{\lambda_1}{\sigma^2} \tag{9}$$

and for the case of only one signal source ($p = 1$) the output SNR is equal to λ_1/σ^2.

3.2. Special Case–2 (SC–2)

For this case, we compute the weight vector $\mathbf{w}_n$ as

$$\mathbf{w}_n = \sum_{i=1}^{p} \frac{\mathbf{q}_i\mathbf{q}_i^H}{\lambda_i + \sigma^2}\mathbf{a}_n = \mathbf{Q}_s\mathbf{\Lambda}_s^{-1}\mathbf{Q}_s^H\mathbf{a}_n. \tag{10}$$

It can be shown that here, $\mathbf{w}_n = \mathbf{R}^{-1}\mathbf{a}_n$, which is the MV solution for the array weight vector — the MV beamformer maximizes the array output signal to interference and noise ratio (SINR) [5].

A shortcoming of the MV beamformer is its sensitivity to signal DOA uncertainty and array calibration error — this causes signal cancellation [8]. Define the output SINR sensitivity with respect to the steering vector error ($\Delta\mathbf{a} = \tilde{\mathbf{a}} - \mathbf{a}$) as

$$\mathcal{S}^{\mathbf{w}}_{\text{SINR}_o,\mathbf{a}} = \frac{|\Delta\text{SINR}_o|}{||\Delta\mathbf{a}||^2}\ , \tag{11}$$

where

$$\Delta\text{SINR}_o = \text{SINR}_o|_{\tilde{\mathbf{a}}=\mathbf{a}+\Delta\mathbf{a}} - \text{SINR}_o|_{\tilde{\mathbf{a}}=\mathbf{a}}\ . \tag{12}$$

It can be shown that the SC–2 beamformer is less sensitive to the steering vector error (due to DOA uncertainty or uncalibrated array) when compared to the MV method — the sensitivity of MV beamformer increases rapidly with input SNR.

3.3. Special Case–3 (SC–3)

For this reduced-rank beamformer, the weight vector, $\mathbf{w}_n$, is

$$\mathbf{w}_n = \sum_{i=1}^{p} \frac{\mathbf{q}_i\mathbf{q}_i^H}{\sigma^2}\mathbf{a}_n = \frac{1}{\sigma^2}\mathbf{Q}_s\mathbf{Q}_s^H\mathbf{a}_n. \tag{13}$$

Using $\mathbf{Q}_s\mathbf{Q}_s^H = \mathbf{I} - \mathbf{Q}_n\mathbf{Q}_n^H$ and noting that the signal steering vector is orthogonal to the noise subspace, (13) can be written as

$$\mathbf{w}_n = \frac{1}{\sigma^2}\mathbf{a}_n. \tag{14}$$

If the true $\mathbf{a}_n$ is known, the weight vector (14) produces the well-known delay-and-sum beamformer.

Definition: For an array with $\mathbf{w}$ as a weight vector, we define the sensitivity of an array output SNR with respect to the array steering vector error ($\Delta\mathbf{a} = \tilde{\mathbf{a}} - \mathbf{a}$) as

$$\mathcal{S}^{\mathbf{w}}_{SNR_o,\mathbf{a}} = \frac{|\Delta SNR_o|}{||\Delta\mathbf{a}||^2} \tag{15}$$

where

$$\Delta SNR_o = SNR_o|_{\tilde{\mathbf{a}}=\mathbf{a}+\Delta\mathbf{a}} - SNR_o|_{\tilde{\mathbf{a}}=\mathbf{a}} \,. \tag{16}$$

It can be proved that the output SNR for the SC–3 beamformer is less sensitive to the array steering vector error than the delay-and-sum beamformer.

For SC–3, the array output SNR is

$$\frac{S_o}{N_o} = L\left(\frac{S}{N}\right)_i \,. \tag{17}$$

We have proved that the maximum output SNR for an array with L elements is L times the input SNR. Thus, SC–3 maximizes the output SNR.

3.4. Improved LCMV method

As mentioned earlier, the SC–2 beamformer has the properties of MV method with a smaller sensitivity. In the MV method, the weight vector is the solution of the following minimization

$$\min_{\mathbf{w}}\{\mathbf{w}^H\mathbf{R}\mathbf{w}\} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}(\theta_i) = g \tag{18}$$

where g is a constant. By the method of Lagrange multipliers, the solution to this minimization is

$$\mathbf{w} = g\frac{\mathbf{R}^{-1}\mathbf{a}}{\mathbf{a}^H\mathbf{R}^{-1}\mathbf{a}} \,. \tag{19}$$

The single linear constraint in (18) can be generalized to a multiple linear constraint. For instance, to produce a beampattern with a unit gain in the direction of sources, θ_1 and θ_2, the desired constraint may be expressed as

$$\begin{bmatrix} \mathbf{a}^H(\theta_1) \\ \mathbf{a}^H(\theta_2) \end{bmatrix} \mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} . \tag{20}$$

If there are $m < L$ linear constraints on $\mathbf{w}$, it is possible to write them in the matrix form $\mathbf{C}^H\mathbf{w} = \mathbf{f}$, where the $L \times m$ matrix $\mathbf{C}$ and the m-dimensional vector $\mathbf{f}$ are the constraint matrix and the response vector, respectively. It is assumed that the constraints are linearly independent — the constraint matrix has rank m. The solution of (18) is then

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{C}[\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C}]^{-1}\mathbf{f} \tag{21}$$

which is called the linear constraint minimum variance (LCMV) weight vector. Similar to (6), we define the following weight vector which satisfies the constraint $\mathbf{C}^H\mathbf{w} = \mathbf{f}$,

$$\mathbf{w} = \mathbf{H}\mathbf{C}[\mathbf{C}^H\mathbf{H}\mathbf{C}]^{-1}\mathbf{f} \,, \tag{22}$$

where $\mathbf{H}$ is defined as

$$\mathbf{H} \stackrel{def}{=} \sum_{i=1}^{p} \frac{\mathbf{q}_i\mathbf{q}_i^H}{\epsilon\lambda_i + (1-\epsilon)\sigma^2}\Bigg|_{\epsilon=0.5} \,. \tag{23}$$

We call this technique, the improved LCMV (ILCMV) algorithm. Replacing Karhunen-Loeve expansion in (21), it is straightforward to prove that when the columns of $\mathbf{C}$ are a subset of the columns of $\mathbf{A}$, the weight vector (21) is the same as $\mathbf{w}$ in (22). However, the simulation results show an improvement for ILCMV when compared to LCMV.

4. SIMULATION RESULTS

In the following examples, we use a uniform circular array (UCA) with L omnidirectional antenna elements. The interelement spacing is assumed to be $\lambda/2$ where λ is the received signal wavelength. Three stationary point signal sources with the same power are used in simulations.

The effect of ϵ in (6) on the produced pattern for $\epsilon = 0, 0.2, 0.4, 0.6, 0.8$ is shown in Fig. 1, for the desired source at 180°, and interfering sources at 125°, and 280° ($L = 8$). The figure shows that as ϵ increases, the two relative nulls of the beampattern move towards the jammers and become deeper.

In the second example, we choose a random DOA for signal and jammers. Fig. 2 shows the average output SIR, SINR, and SNR as a function of ϵ for the proposed beamformer choosing a random DOA for signals. The input SNR is assumed to be 3dB and $L = 8$ is considered. The curves show that the output SIR decreases rapidly with decreasing ϵ, however, the changes in SNR and SINR are not substantial.

In Fig. 3, the produced beampatterns with LCMV and ILCMV methods are compared (for $L = 15$). Here, the signal source DOAs are at 80° and 240°, and an interferer is located at 160°. The results clearly show the robustness of the proposed method against DOA uncertainty and array calibration error.

REFERENCES

[1] B. Friedlander, "A Signal Subspace Method for Adaptive Interference Cancellation," *IEEE Trans. Acoust., Speech and Sig. Proc.*, Vol. ASSP-36, No. 12, pp. 1853–1845, Dec. 1988.

[2] A. P. Applebaum and D. J. Chapman, "Adaptive Array with Main Beam Constraints," *IEEE Trans. Ant. and Prop.*, Vol. AP-24, No. 5, pp. 650–662, Sep 1976.

[3] A. Farina, "Antenna-Based Signal Processing Techniques for Radar Systems." Artech House, 1992.

[4] A. M. Haimovich and Y. Bar-Ness, "An Eigenanalysis Interference Canceller," *IEEE Trans. on Sig. Proc.*, Vol. SP-39, No. 1, pp. 76–84, Jan. 1991.

[5] P. A. Zulch, et al, "Comparison of Reduced-rank Signal Processing Techniques," 32nd Asilmor Conference Signal System and Computer, pp. 421–425, 1998.

[6] B. Champagne, "Adaptive Eigendecomposition of Data Covariance Matrices Based on First-Order Perturbation." *IEEE Trans. on Sig. Proc.*, Vol. SP-42, No. 10, pp. 2758–2770, Oct. 1994.

[7] J. F. Yang and M. Kaveh, "Adaptive Eigensubspace Algorithms for Direction or Frequency Estimation and Tracking," *IEEE Trans. Acoust., Speech and Sig. Proc.*, Vol. ASSP-36, No. 2, pp. 241–251, Feb. 1988.

[8] H. Cox, R. M. Zeskind, and M. M. Owen, "Robust Adaptive Beamforming," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-35, pp. 1365–1378, Oct. 1987.

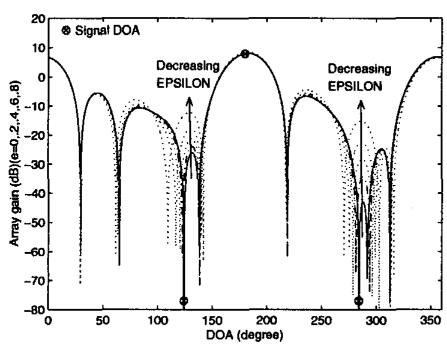

Figure 1: The effect of ϵ on the produced pattern for $\epsilon = 0, 0.2, 0.4, 0.6, 0.8$. The desired source is located at 180°, and interfering sources are at 125°, and 280° ($L = 8$).

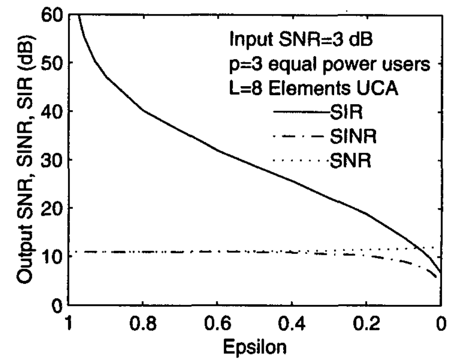

Figure 2: The average output SIR, SINR, and SNR as a function of ϵ for an 8-element UCA.

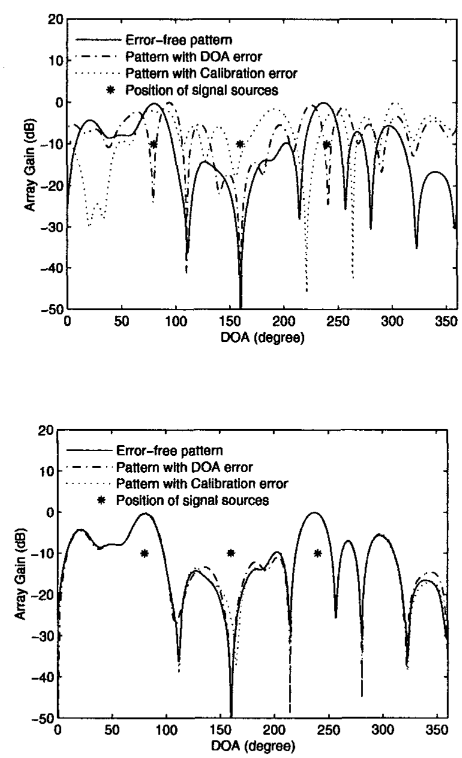

Figure 3: Beampattern for the LCMV (top) and the ILCMV (bottom) algorithms.