

A NEW ALGORITHM FOR JOINT DOA AND MULTIPATH DELAY ESTIMATION: SEPARABLE DIMENSION SUBSPACE METHOD

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ABSTRACT

With the growing capacity demand in wireless communication systems, space division multiplexing and space-time processing by means of antenna arrays are becoming ever more attractive as a technology to improve the system performance, especially for reduction of multipath effects. This paper presents a new low complexity and high accuracy algorithm to estimate the multipath delays and direction of arrivals (DOAs) simultaneously in wireless communication systems. By using separable dimension correlation processing, the temporal and spatial signal subspaces are formed and the joint two dimensional delay/DOA estimation problem is separated into two simpler one dimensional estimations.

1. INTRODUCTION

Three major performance and capacity limiting impairments in current mobile communication systems are: multipath fading, intersymbol interference (ISI) and co-channel interference (CCI). Especially, the ISI impairment resulting from delay spread constrains the maximum data rate. Current mobile communication systems, using temporal processing alone, cannot effectively address these impairments. By means of an antenna array, a combination of temporal and spatial processing can potentially yield good performance improvements over existing systems. Several joint delay and direction estimation algorithms for signals in multipath environments have thus been developed recently [1, 2, 3].

This paper proposes a new low complexity and high accuracy algorithm based on a separable dimension subspace method [7] to estimate the multipath delays and direction of arrivals (DOAs) simultaneously. With separable dimension processing, a joint spatial and temporal estimation problem is separated, i.e., the delays are first estimated by using a one-dimensional subspace method and then the DOAs are estimated for each estimated delay. In this way, the computational complexity of the proposed method is reduced while its performance for the joint delay/DOA estimation is

still satisfied as supported by computer simulations.

2. PROBLEM FORMULATION

Consider a base station receiving array composed of M antennas and assume that the single user signal of interest arrives at the base station via D paths, with the DOA of the i^{th} path denoted as θ_i ($i = 1, 2, \dots, D$). Then, the received complex baseband signal vector at the antenna array can be described as:

$$\mathbf{x}(t) = \sum_{i=1}^D \mathbf{a}(\theta_i) \beta_i r(t - \tau_i) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{a}(\theta_i)$ is an $M \times 1$ spatial steering vector for the i^{th} path, β_i is the complex fading factor of the i^{th} ray, $r(t)$ is a transmitted complex baseband signal, τ_i is the i^{th} path propagation delay and $\mathbf{n}(t)$ is a spatially and temporally white additive Gaussian noise with zero mean and equal covariance σ_n^2 .

In a linear time-invariant system, the transmitted signal $r(t)$ can be represented as a convolution of the data bits and a pulse shaping function $g(t)$, i.e. $r(t) = \sum_i s_i \cdot g(t - lT_s)$. Therefore, by passing the signal vector $\mathbf{x}(t)$ through a set of tapped-delay lines (TDL) of length Q and delay T_0 , as shown in Figure 1, and sampling the resulting outputs, a data matrix $\mathbf{X}[n]$ is formed as:

$$\begin{aligned} \mathbf{X}[n] &= [\mathbf{x}_1^T[n] \ \mathbf{x}_2^T[n] \ \dots \ \mathbf{x}_M^T[n]]^T \\ &= \sum_{i=1}^D \mathbf{a}(\theta_i) \beta_i \otimes [\mathbf{G}(\tau_i) \ \dots \ \mathbf{G}(\tau_i + LT_s)] \mathbf{s}[n] + \mathbf{N}[n] \end{aligned} \quad (2)$$

where symbol \otimes denotes the Kronecker product, $\mathbf{G}(\tau_i) = [g(t_0 - \tau_i), g(t_0 - T_0 - \tau_i), \dots, g(t_0 - (Q-1)T_0 - \tau_i)]^T$ is referred to as the temporal manifold, $g(\cdot)$ is the pulse shaping function, which models the total impulse response of the filters used in the system, t_0 is the sampling reference time of the n^{th} data, $\mathbf{s}[n] = [s(n)s(n-1) \ \dots \ s(n-L)]^T$ is a vector consisting of $L+1$ consecutive symbols, T_s is symbol duration and L is the length of the channel, which covers the range of delays $\{\tau_i\}_{i=1}^D$. We assume that

a training sequence is embedded in the transmitted signal; the training portion, represented here by $s[n]$, can be extracted by receiver and is assumed to be known. Typically, in TDMA systems, $L = L_g + \tau_{max}/T_s$, where $L_g T_s$ is the duration of the pulse shaping function $g(t)$ and τ_{max} is the maximum integer delay [4]. Likewise, in DS-CDMA systems, $L = 2N_c$, where $N_c = T_{cs}/T_c$, T_{cs} is the data symbol period and T_c is the chip duration [5].

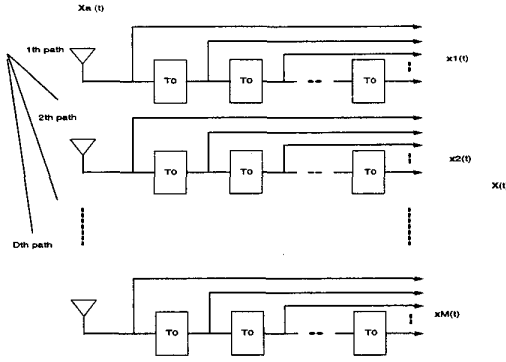


Fig. 1. Array front-end with TDLs

The equation (2) can be rewritten as:

$$\begin{aligned} \mathbf{X}[n] &= \sum_{i=1}^D \mathbf{a}(\theta_i) \beta_i \otimes \begin{bmatrix} \tilde{g}(t_0 - \tau_i) \\ \vdots \\ \tilde{g}(t_0 - (Q-1)T_0 - \tau_i) \end{bmatrix} + \mathbf{N}[n] \\ &= [\mathbf{a}(\theta_1) \cdots \mathbf{a}(\theta_D)] \diamond [\tilde{\mathbf{G}}(\tau_1) \cdots \tilde{\mathbf{G}}(\tau_D)] \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_D \end{pmatrix} + \mathbf{N}[n] \\ &= \mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\tau}) \mathbf{B} + \mathbf{N}[n] \end{aligned}$$

where \diamond denotes the Khatri-Rao product (see [6]), which represents column-Kronecker product;

$$\tilde{\mathbf{G}}(\tau_i) = [\mathbf{G}(\tau_i) \cdots \mathbf{G}(\tau_i + LT_s)] \mathbf{s}[n]$$

is a modified temporal manifold, which is the convolution between the training sequence and delayed shaping function;

$$\mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\tau}) = [\mathbf{a}(\theta_1) \cdots \mathbf{a}(\theta_D)] \diamond [\tilde{\mathbf{G}}(\tau_1) \cdots \tilde{\mathbf{G}}(\tau_D)]$$

is a spatio-temporal manifold and $\mathbf{B} = [\beta_1 \beta_2 \cdots \beta_D]^T$.

Thus, based on the available samples $\{\mathbf{X}[n]\}_{n=1}^N$, the problem of interest here is to estimate the DOAs θ_i and the multipath delays τ_i ($i = 1, 2, \dots, D$) simultaneously with subspace methods.

3. SUBSPACE PARTITION

Assume that the number of paths D , the maximum path delay τ_{max} , the array response $\mathbf{a}(\cdot)$ and the pulse shaping function $g(\cdot)$ are known. Also assume that the complex path fading factor $\{\beta_i\}_{i=1}^D$ remain constant during a data symbol period.

Define

$$\mathbf{r}'_h = \frac{1}{M} \sum_{l=1}^M E[x_l(nT_s - h \cdot T_0) \mathbf{x}_l^H(n)], \quad h = 0, 1, \dots, D-1 \quad (4)$$

$$\boldsymbol{\eta}'_l = \frac{1}{Q} \sum_{h=0}^{Q-1} E[x_l(nT_s - h \cdot T_0) \mathbf{Y}^H(nT_s - h \cdot T_0)] \quad l = 1, 2, \dots, D \quad (5)$$

where

$$\mathbf{Y}(t) = [x_1(t) \cdots x_M(t)]^T$$

and $E(\cdot)$ denotes mathematical expectation. We refer to $\{\mathbf{r}'_h\}_{h=0}^{D-1}$ and $\{\boldsymbol{\eta}'_l\}_{l=1}^D$ as the set of temporal vectors and spatial vectors, respectively. It can be shown that the linear space spanned by these sets of vectors are equal to the range space of $\tilde{\mathbf{G}}(\boldsymbol{\tau})$ and $\mathbf{a}(\boldsymbol{\theta})$, respectively. That is, let $\mathbf{r} = [\mathbf{r}'_1, \mathbf{r}'_2, \dots, \mathbf{r}'_D]$ and $\boldsymbol{\eta} = [\boldsymbol{\eta}'_1, \boldsymbol{\eta}'_2, \dots, \boldsymbol{\eta}'_D]$, then we have

$$\mathcal{R}(\mathbf{r}) = \mathcal{R}(\tilde{\mathbf{G}}(\boldsymbol{\tau})) \quad \mathcal{R}(\boldsymbol{\eta}) = \mathcal{R}(\mathbf{a}(\boldsymbol{\theta})) \quad (6)$$

where $\mathcal{R}(\cdot)$ denotes the range space of its matrix argument, $\tilde{\mathbf{G}}(\boldsymbol{\tau}) = [\tilde{\mathbf{G}}(\tau_1) \cdots \tilde{\mathbf{G}}(\tau_D)]$ and $\mathbf{a}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1) \cdots \mathbf{a}(\theta_D)]$. (3)

4. SEPARABLE DIMENSIONAL ALGORITHM

Equation (6) suggests that a subspace method may be used independently to estimate DOA and delay parameters. Specifically, we may use correlation processing in spatial and temporal dimension respectively to get the estimates of $\{\mathbf{r}'_h\}_{h=0}^{D-1}$ and $\{\boldsymbol{\eta}'_l\}_{l=1}^D$; These estimates are then used separately to generate null subspace projections. Finally, the path delay and DOAs can be estimated with subspace methods by two one-dimensional searches. This leads to the following algorithm:

Step 1. Formation of temporal and spatial projection matrices

(1) Estimation of temporal vectors:

$$\hat{\mathbf{r}}'_h = \frac{1}{M} \sum_{l=1}^M \frac{1}{N} \sum_{n=1}^N x_l(nT_s - h \cdot T_0) \mathbf{x}_l^H(n) \quad (7)$$

$$\hat{\mathbf{r}}_h = (\hat{\mathbf{r}}'_h - \hat{\sigma}^2 \mathbf{e}_h)^H, \quad h = 0, 1, \dots, D-1 \quad (8)$$

where $\mathbf{e}_h = \underbrace{[0 \cdots 0]_{1 \times h} 1 0 \cdots 0}$ and $\hat{\sigma}^2$ is an estimate of the noise variance.

(2) Gram-Schmidt (GS) orthogonalization and formation of temporal projection matrix \mathbf{P}_τ : From the vectors $\{\hat{\mathbf{r}}_h\}_{h=0}^{D-1}$, we can get D orthogonal vectors, $\{\mathbf{q}_k\}_{k=1}^D$ via GS orthogonalization. Let $\mathbf{Q}_\tau = [\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_D]$, then compute the temporal projection matrix $\mathbf{P}_\tau = \mathbf{I} - \mathbf{Q}_\tau \mathbf{Q}_\tau^H$, which spans the null space of $\{\tilde{\mathbf{G}}(\tau_k)\}_{k=1}^D$.

(3) Estimation of spatial vectors:

$$\hat{\boldsymbol{\eta}}'_l = \frac{1}{Q} \sum_{h=0}^{Q-1} \frac{1}{N} \sum_{n=1}^N x_l(nT_s - h \cdot T_0) \mathbf{Y}^H(nT_s - h \cdot T_0) \quad (9)$$

$$\hat{\boldsymbol{\eta}}_l = (\hat{\boldsymbol{\eta}}'_l - \hat{\sigma}^2 \mathbf{e}_l)^H, \quad l = 1, 2, \dots, D \quad (10)$$

(4) Gram-Schmidt (GS) orthogonalization and formation of spatial projection matrix \mathbf{P}_θ : Via Gram-Schmidt orthogonalization of $\{\hat{\boldsymbol{\eta}}_l\}_{l=1}^D$, the D orthogonal vectors, $\{\zeta_l\}_{l=1}^D$ and spatial orthogonal projection matrix $\mathbf{P}_\theta = \mathbf{I} - \mathbf{Q}_\theta \mathbf{Q}_\theta^H$ are obtained, where $\mathbf{Q}_\theta = [\zeta_1 \zeta_2 \cdots \zeta_D]$.

Step 2. Multipath delays and direction of arrivals estimation

The path delays $\{\tau_k\}_{k=1}^D$ are estimated as the D largest peaks of the function $P(\tau) = (\tilde{\mathbf{G}}^H(\tau) \mathbf{P}_\tau \tilde{\mathbf{G}}(\tau))^{-1}$, searching over the delay sector of interest, measured by symbol period T . Likewise, the DOAs $\{\theta_k\}_{k=1}^D$ are estimated by searching over the direction sector of interest to get the D largest peaks of the function $P(\theta) = (\mathbf{a}^H(\theta) \mathbf{P}_\theta \mathbf{a}(\theta))^{-1}$.

Step 3. Delay and DOA pairing

With estimated delays $\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_D$, if we select the estimated DOAs $\hat{\theta}_i$ ($i = 1, 2, \dots, D$) to minimize the cost function \mathcal{L} , then the delays $\hat{\tau}_i$ and the DOAs $\hat{\theta}_i$ can be paired. The cost function \mathcal{L} is:

$$\mathcal{L} = \mathbf{D}^H(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\tau}}) \mathbf{E}_n \mathbf{E}_n^H \mathbf{D}(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\tau}}) \quad (11)$$

where $\mathbf{D}(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\tau}}) = \mathbf{a}(\hat{\boldsymbol{\theta}}) \diamond \tilde{\mathbf{G}}(\hat{\boldsymbol{\tau}})$ is joint spatial-temporal vector and \mathbf{E}_n is a matrix whose columns are the eigenvectors corresponding to the smallest eigenvalues of covariance matrix $\mathbf{R}_x = E[\mathbf{X}[n] \mathbf{X}^H[n]]$.

5. ESTIMATION OF UNKNOWN NOISE COVARIANCE

In the case of unknown noise covariance, the performance of separable dimension subspace estimation method will be degraded. In equation (7) and equation (8), we can see that the vector \mathbf{r}_h will be noise free when the TDL delay

T_0 is greater than the correlation time of noise. Therefore, to improve the estimation performance in the case of unknown noise covariance, we can change the length of tapped-delay-lines (TDL) from Q to $2Q$ ($Q \geq D$) to estimate unknown noise covariance σ^2 and then remove it from temporal/spatial vectors.

Let the first Q TDL outputs of the l^{th} sensor be represented by the vector $\mathbf{x}_l = [x_l(nT_s), x_l(nT_s - T_0), \dots, x_l(nT_s - (Q-1)T_0)]^T$ and the later Q TDL outputs by the vector $\bar{\mathbf{x}}_l = [x_l(nT_s - QT_0), x_l(nT_s - (Q+1)T_0), \dots, x_l(nT_s - (2Q-1)T_0)]^T$. Then, the sample covariance matrix for cross \mathbf{x}_l and $\bar{\mathbf{x}}_l$ is

$$\hat{\mathbf{R}}_l = \frac{1}{N} \sum_{n=1}^N [\mathbf{x}_l(n) \bar{\mathbf{x}}_l(n)^H] \quad (12)$$

We can estimate the temporal vector \mathbf{r}_h

$$\hat{\mathbf{r}}_h = \frac{1}{M} \sum_{l=1}^M \frac{1}{N} \sum_{n=1}^N x_l(nT_s - (h-1)T_0) \bar{\mathbf{x}}_l^H(n) \quad (13)$$

With the method described in Section 4 and the estimated vector $\hat{\mathbf{r}}_h$, we can estimate the multipath delay $\hat{\tau}_i$, $i = 1, 2, \dots, D$ with equation $P(\tau) = (\tilde{\mathbf{G}}^H(\tau) \mathbf{P}_\tau \tilde{\mathbf{G}}(\tau))^{-1}$.

To estimate the signal covariance matrix \mathbf{R}_s , we can reconstruct the modified temporal manifold $\tilde{\mathbf{G}}(\hat{\tau}_i)$ and $\tilde{\mathbf{G}}'(\hat{\tau}_i)$ with the estimated delay, $\hat{\tau}_i$, herein $\tilde{\mathbf{G}}'(\hat{\tau}_i) = [\hat{g}(t_0 - QT_0 - \hat{\tau}_i), \dots, \hat{g}(t_0 - 2QT_0 - \hat{\tau}_i)]^T$, and have

$$\hat{\mathbf{R}}_s = [\tilde{\mathbf{G}}^H(\hat{\boldsymbol{\tau}}) \tilde{\mathbf{G}}(\hat{\boldsymbol{\tau}})]^{-1} \tilde{\mathbf{G}}^H(\hat{\boldsymbol{\tau}}) \hat{\mathbf{R}}_l \tilde{\mathbf{G}}(\hat{\boldsymbol{\tau}}) [\tilde{\mathbf{G}}'^H(\hat{\boldsymbol{\tau}}) \tilde{\mathbf{G}}'(\hat{\boldsymbol{\tau}})]^{-1} \quad (14)$$

With estimated signal covariance matrix $\hat{\mathbf{R}}_s$ and $\tilde{\mathbf{G}}(\hat{\boldsymbol{\tau}})$, the noise covariance matrix can be estimated by

$$\sigma^2 \mathbf{I} = \mathbf{R}'_l - \tilde{\mathbf{G}}(\hat{\boldsymbol{\tau}}) \hat{\mathbf{R}}_s \tilde{\mathbf{G}}^H(\hat{\boldsymbol{\tau}}) \quad (15)$$

where $\mathbf{R}'_l = E[\mathbf{x}_l(n) \mathbf{x}_l(n)^H]$.

6. COMPUTER SIMULATIONS

We assume that a transmitted signal with $D = 3$ paths arrives at a linear array of $M = 6$ sensors with half-wavelength spacing. The multipath delays are $[0, 0.5, 1.2] T$, $T = 1$, and the direction of arrivals are $[15^\circ, 40^\circ, 70^\circ]$. The path fading is $[1, 0.85, 0.8]$. Additive white Gaussian noise is added, the corresponding SNR is 10 dB. 100 samples are accumulated. The pulse shape function is a raised cosine with 0.35 excess bandwidth, the TDL length is $Q = 6$ and delay $T_0 = 0.5$.

Fig. 2 and Fig 3 show the estimation results of multipath delays and DOAs with 30 trials by using the proposed separable dimension subspace method. Simulations and performance comparison with other proposed algorithms such as JADE [2] are presented in Fig. 4 and Fig. 5.

7. CONCLUSIONS

A new algorithm based on separable dimension subspace method is proposed for joint estimation of DOAs and multipath delays in the wireless communication systems. In this paper, spatial and temporal separable dimension correlation processing are used to replace EVD (Eigen Value Decomposition) or SVD (Singular Value Decomposition). Therefore, compared to other joint DOA and delay estimation methods, the computational complexity of the proposed method is relatively small. The presented algorithm has been tested by computer simulation studies and has been found to perform satisfactorily.

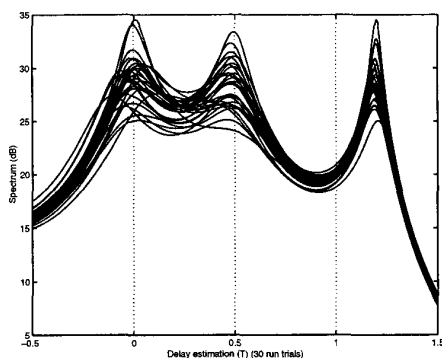


Fig. 2. Delay estimation with 30 trials

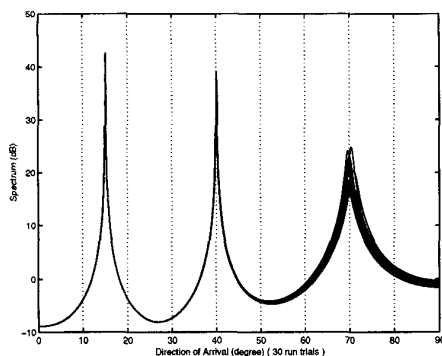


Fig. 3. DOA estimation with 30 trials

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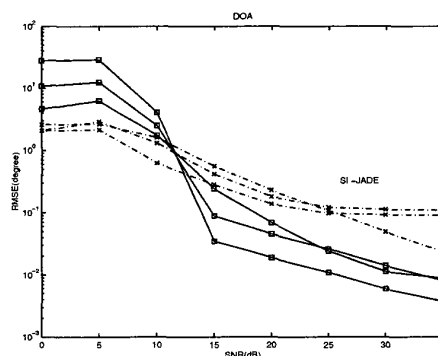


Fig. 4. DOA estimation performance

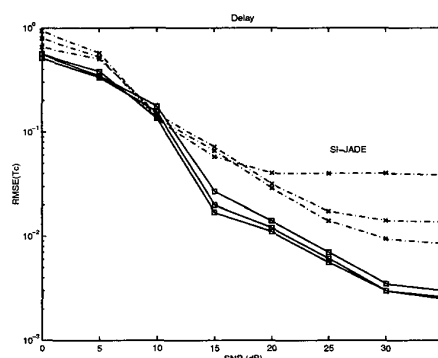


Fig. 5. Delay estimation performance

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