Robust Transceiver Design for MISO Interference Channel with Energy Harvesting

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***Abstract*—In this paper, we consider the power splitting technique for multiple-input single-output (MISO) interference channel where the received signal is divided into two parts for information decoding and energy harvesting (EH) respectively. Specifically, assuming norm-bounded errors (NBE) in the channel state information (CSI), we study the robust joint beamforming and power splitting (JBPS) design problem, where the total transmission power is minimized subject to both signal-to-interference-plus-noise ratio (SINR) and EH constraints. We first propose an efficient approximation method based on semidefinite relaxation (SDR) for solving the highly non-convex JBPS problem, where the latter can be formulated as a semidefinite programming (SDP) problem. Then, a low complexity algorithm is proposed using EH relaxation and cutting-set philosophy, which partitions the original problem into an alternating sequence of optimization and worst-case analysis subproblems with guaranteed convergence. Finally, simulation results are presented to validate the robustness and efficiency of the proposed algorithms.** [1]

***Index Terms*—MISO interference channel, beamforming, power splitting, semidefinite programming, second-order cone programming, cutting-set method.**

I. INTRODUCTION

Recently, energy harvesting (EH) from the environment has attracted considerable interest since it is a promising solution to provide cost-effective and perpetual power supplies for wireless networks [1]. As a result, the unified study of simultaneous wireless information and power transfer (SWIPT) has drawn significant attention [1]–[5], [7], [8], as it opens new challenges and possibilities in the analysis and design of transmission schemes and protocols.

The fundamental concept of SWIPT was proposed in [1]–[3], which characterize the rate-energy (R-E) tradeoff in various system models. In [4], the authors proposed a practical receiver structure for the first time, and investigated the R-E region and optimal transmission scheme of a MIMO broadcasting channel. Moreover, two practical signal separation schemes were also considered in the form of : time switching (TS) and power splitting (PS).

Some recent studies have focused on multi-antenna SWIPT interference channel. The work in [5] considers a multiuser MISO SWIPT downlink system. The total transmission power at the base station is minimized subject to given signal-to-interference-plus-noise ratio (SINR) and harvested power constraints, and an optimal solution is proposed based on semidefinite relaxation (SDR) [6]. In [7] and [8], joint beamforming and power splitting (JBPS) design are studied for a K-user MISO interference channel with the same design criterion as that in [5]. Specially, the work [7] uses SDR to tackle the non-convex JBPS problem. In [8], this problem is reformulated as a second-order cone programming (SOCP) problem based on an alternative method named SOCP relaxation, and two sufficient conditions are given under which the relaxation is tight.

In most of the existing works, the channel state information (CSI) is assumed to be perfectly known. In practice however, CSI is prone to errors owing to various factors in practice, which may limit the system performance drastically. Assuming norm-bounded errors (NBE), this paper studies the robust JBPS design problem in a K-user MISO interference channel with multi-antenna transmit beamformers and single-antenna PS receivers. We first propose an efficient approximation method based on SDR for solving the highly non-convex JBPS problem, where the latter can be formulated as a semidefinite programming (SDP) problem. Then, a low complexity algorithm is proposed using EH relaxation and cutting-set philosophy. In this way, we can partition the original problem into an alternating sequence of optimization and worst-case analysis subproblems, which involve solving SOCP problems. The convergence of the cutting-set based algorithm is guaranteed and we also provide a method to recover a robust feasible solution to the original problem. The simulation results validate the near-optimal performance of the two designs.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Proposed System Model

We consider the K-user MISO interference channel where each transmitter is equipped with $N_k (k = 1, \ldots, K)$ antennas and each receiver is equipped with a single antenna. We assume that the kth transmitter sends its signal s_k to its intended receiver through beamforming vector $\mathbf{f}_k \in \mathbb{C}^{N_k \times 1}$, and that the transmitted signals s_k for different user k are statistically independent with zero mean and $E\{|s_k|^2\} = 1, \ \forall k$. Under these conditions, the received baseband signal at receiver k before power splitting is given by

$$r_k = \underbrace{\mathbf{h}_{kk}^H \mathbf{f}_k s_k}_{desired\ signal} + \underbrace{\sum\nolimits_{j=1, j\neq k}^{K} \mathbf{h}_{kj}^H \mathbf{f}_j s_j}_{interference} + n_k, \quad (1)$$

where $\mathbf{h}_{kj} \in \mathbb{C}^{N_k \times 1}$ denotes the conjugated complex channel vector between transmitter j and receiver k, and $n_k \in \mathbb{C}$ is

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the additive white Gaussian noise (AWGN) introduced by the receive antenna, which is assumed to have zero mean and variance σ_k^2.

Let $\rho_k \in [0,1]$ denote the power splitting (PS) ratio for receiver k, which means that a portion ρ_k of the signal power is used for signal detection and the remaining portion $1-\rho_k$ of the power is diverted to an energy harvester.[2] Thus, the available signal for information decoding (ID) at receiver k can be expressed as

$$r_k^{\text{ID}} = \sqrt{\rho_k}(\mathbf{h}_{kk}^H \mathbf{f}_k s_k + \sum\nolimits_{j=1, j\neq k}^{K} \mathbf{h}_{kj}^H \mathbf{f}_j s_j + n_k) + v_k, \quad (2)$$

where v_k is the additional AWGN circuit noise due to phase offsets and non-linearities during baseband conversion [4] with zero mean and variance ω_k^2. Then, the SINR at the kth receiver is given by

$$\Gamma_k = \frac{\rho_k |\mathbf{h}_{kk}^H \mathbf{f}_k|^2}{\rho_k (\sum_{j=1, j\neq k}^{K} |\mathbf{h}_{kj}^H \mathbf{f}_j|^2 + \sigma_k^2) + \omega_k^2}. \quad (3)$$

Besides, the total harvested power that can be stored by receiver k is equal to

$$P_k^{\text{EH}} = \xi_k (1-\rho_k)(\sum\nolimits_{j=1}^{K} |\mathbf{h}_{kj}^H \mathbf{f}_j|^2 + \sigma_k^2), \quad (4)$$

where $\xi_k \in (0,1]$ denotes the energy conversion efficiency of the kth EH unit.

B. Channel Error Model

In practice, perfect CSI at the transmitters is unavailable due to many factors such as channel estimation error, quantization error, and feedback error/delay. Let $\widehat{\mathbf{h}}_{kj} \in \mathbb{C}^{N_k \times 1}$ denote the estimated channel vector between transmitter j and receiver k, then the actual CSI can be expressed as $\mathbf{h}_{kj} = \widehat{\mathbf{h}}_{kj} + \mathbf{e}_{kj}$, where $\mathbf{e}_{kj}$ denotes the CSI error vector.

To model the CSI errors, the well-known NBE model [9] is adopted, where we assume that the channel error vector $\mathbf{e}_{kj}$ is bounded in its Euclidean norm, that is, $\|\mathbf{e}_{kj}\| \le \eta_{kj},\ \forall j,k$, where η_{kj} is a known positive constant. It should be emphasized that the actual error $\mathbf{e}_{kj}$ is assumed to be unknown while the corresponding upper bound η_{kj} can be obtained using the preliminary knowledge of the type of imperfection and/or coarse knowledge of the channel type and its main characteristics [10].

C. Optimization Problem

In this work, we focus on the JBPS design [7]. In order to minimize the total transmission power subject to both SINR and EH constraints for all possible CSI errors, we consider the following robust JBPS design problem

$$\begin{aligned} \min_{\{\mathbf{f}_k, \rho_k\}} \quad & \sum\nolimits_{k=1}^{K} \|\mathbf{f}_k\|^2 \\ \text{s.t.} \quad & \Gamma_k \ge \gamma_k,\ P_k^{\text{EH}} \ge \psi_k, \\ & 0 \le \rho_k \le 1,\ \forall \|\mathbf{e}_{kj}\|^2 \le \eta_{kj}^2,\ \forall j,k, \end{aligned} \quad (5)$$

where γ_k and ψ_k are the corresponding SINR and EH targets for receiver k. Solving the robust design problem (5) is more challenging than the non-robust problem in [7], [8] because there is an infinite number of constraints (due to the NBE model) and moreover each of the SINR (EH) constraint is not convex. Both these properties make problem (5) very difficult to address. It can be shown that the feasibility of problem (5) is independent of the energy harvesting constraints similar to Lemma 3.1 & Lemma 3.2 in [5]. The feasibility problem has not been well studied in the literature, which remains an interesting avenue for research.

[2]The structure of a MISO system with PS-based energy harvesting receiver can be found in Fig. 2 of [7].

III. Proposed SDR-based Robust Design

In this section, the celebrated SDR technique is applied to convert the semi-infinite constraints in (5) into linear matrix inequalities. Then, by applying S-Procedure [11], the semi-infinite programming problem is transformed into an equivalent problem with finite convex constraints. Lastly, we provide a heuristic rank-one-solution recovery method.

A. SDP with Rank Relaxation

According to [6], we introduce a new optimization variable $\mathbf{F}_k \triangleq \mathbf{f}_k \mathbf{f}_k^H$. Moreover, it is not difficult to see that the SINR and EH constraints in (5) can be reformulated respectively as the following two inequalities

$$\begin{aligned} &\min_{\forall \|\mathbf{e}_{kk}\|^2 \le \eta_{kk}^2} \tfrac{1}{\gamma_k} \mathbf{h}_{kk}^H \mathbf{F}_k \mathbf{h}_{kk} \\ &- \sum\nolimits_{j\neq k}^{K} \max_{\forall \mathbf{e}_{kj}^H \mathbf{e}_{kj} \le \eta_{kj}^2} \mathbf{h}_{kj}^H \mathbf{F}_j \mathbf{h}_{kj} \ge \sigma_k^2 + \tfrac{\omega_k^2}{\rho_k}, \end{aligned} \quad (6)$$

$$\sum\nolimits_{j=1}^{K} \min_{\forall \|\mathbf{e}_{kj}\|^2 \le \eta_{kj}^2} \mathbf{h}_{kj}^H \mathbf{F}_j \mathbf{h}_{kj} \ge \frac{\psi_k}{\xi_k (1-\rho_k)} - \sigma_k^2, \quad (7)$$

Then, we can rewrite problem (5) as follows

$$\begin{aligned} \min_{\{\mathbf{F}_k, \rho_k\}} \quad & \sum\nolimits_{k=1}^{K} \text{Tr}(\mathbf{F}_k) \\ \text{s.t.} \quad & \tfrac{1}{\gamma_k} \mathbf{h}_{kk}^H \mathbf{F}_k \mathbf{h}_{kk} - \sum\nolimits_{j\neq k}^{K} \mathbf{h}_{kj}^H \mathbf{F}_j \mathbf{h}_{kj} \ge \sigma_k^2 + \tfrac{\omega_k^2}{\rho_k}, \\ & \sum\nolimits_{j=1}^{K} \mathbf{h}_{kj}^H \mathbf{F}_j \mathbf{h}_{kj} \ge \tfrac{\psi_k}{\xi_k (1-\rho_k)} - \sigma_k^2, \\ & 0 \le \rho_k \le 1,\ \mathbf{F}_k \succeq \mathbf{0},\ \text{rank}(\mathbf{F}_k) = 1, \\ & \forall \mathbf{e}_{kj}^H \mathbf{e}_{kj} \le \eta_{kj}^2,\ \forall j,k. \end{aligned} \quad (8)$$

This problem can be relaxed by dropping the non-convex rank-one constraint $\text{rank}(\mathbf{F}_k) = 1$, leading to a convex SDP problem. By applying the S-Procedure, the infinitely many constraints can be reformulated into finite convex constraints.

We first observe that each term in (6) and (7) involves independent CSI errors. Hence, we introduce two auxiliary variables

$$p_{kj} = \max_{\forall \mathbf{e}_{kj}^H \mathbf{e}_{kj} \le \eta_{kj}^2} \mathbf{h}_{kj}^H \mathbf{F}_j \mathbf{h}_{kj},\ \forall k,\ j \neq k, \quad (9)$$

$$q_{kj} = \min_{\forall \mathbf{e}_{kj}^H \mathbf{e}_{kj} \le \eta_{kj}^2} \mathbf{h}_{kj}^H \mathbf{F}_j \mathbf{h}_{kj},\ \forall k,\ j \neq k, \quad (10)$$

where p_{kj} is the maximum cochannel power from transmitter j to receiver k and q_{kj} denotes the minimum power available for EH from transmitter j to receiver k. According to the S-Procedure and with the help of these two variables, the SINR constraints in problem (8) can be rewritten as (11) and (12), shown at the top of the next page, where $\alpha_k = 1/\rho_k$, and $\lambda_{kj},\ \forall k,j$ are slack variables. Similarly, we can recast the EH constraints as (13) and (14) with $\beta_k = 1/(1-\rho_k)$, and

$$\mathbf{U}_k(\mathbf{F}_k, \{p_{kj}\}_{j\neq k}, \lambda_{kk}, \alpha_k) \triangleq \begin{bmatrix} \frac{1}{\gamma_k}\mathbf{F}_k + \lambda_{kk}\mathbf{I} & \frac{1}{\gamma_k}\mathbf{F}_k\widehat{\mathbf{h}}_{kk} \\ \frac{1}{\gamma_k}\widehat{\mathbf{h}}_{kk}^H\mathbf{F}_k & \frac{1}{\gamma_k}\widehat{\mathbf{h}}_{kk}^H\mathbf{F}_k\widehat{\mathbf{h}}_{kk} - \sum_{j=1,j\neq k}^K p_{kj} - \sigma_k^2 - \omega_k^2\alpha_k - \lambda_{kk}\eta_{kk}^2 \end{bmatrix} \succeq \mathbf{0} \quad (11)$$

$$\mathbf{V}_{kj}(\mathbf{F}_j, p_{kj}, \lambda_{kj}) \triangleq \begin{bmatrix} -\mathbf{F}_j + \lambda_{kj}\mathbf{I} & -\mathbf{F}_j\widehat{\mathbf{h}}_{kj} \\ -\widehat{\mathbf{h}}_{kj}^H\mathbf{F}_j & p_{kj} - \widehat{\mathbf{h}}_{kj}^H\mathbf{F}_j\widehat{\mathbf{h}}_{kj} - \lambda_{kj}\eta_{kj}^2 \end{bmatrix} \succeq \mathbf{0}, j \neq k \quad (12)$$

$$\mathbf{X}_k(\mathbf{F}_k, \{q_{kj}\}_{j\neq k}, \mu_{kk}, \beta_k) \triangleq \begin{bmatrix} \mathbf{F}_k + \mu_{kk}\mathbf{I} & \mathbf{F}_k\widehat{\mathbf{h}}_{kk} \\ \widehat{\mathbf{h}}_{kk}^H\mathbf{F}_k & \widehat{\mathbf{h}}_{kk}^H\mathbf{F}_k\widehat{\mathbf{h}}_{kk} + \sum_{j=1,j\neq k}^K q_{kj} - \frac{\psi_k\beta_k}{\xi_k} + \sigma_k^2 - \mu_{kk}\eta_{kk}^2 \end{bmatrix} \succeq \mathbf{0} \quad (13)$$

$$\mathbf{Y}_{kj}(\mathbf{F}_j, q_{kj}, \mu_{kj}) \triangleq \begin{bmatrix} \mathbf{F}_j + \mu_{kj}\mathbf{I} & \mathbf{F}_j\widehat{\mathbf{h}}_{kj} \\ \widehat{\mathbf{h}}_{kj}^H\mathbf{F}_j & \widehat{\mathbf{h}}_{kj}^H\mathbf{F}_j\widehat{\mathbf{h}}_{kj} - q_{kj} - \mu_{kj}\eta_{kj}^2 \end{bmatrix} \succeq \mathbf{0}, j \neq k \quad (14)$$

slack variables $\mu_{kj},\ \forall k, j$. Then, problem (8) can be expressed as

$$\begin{aligned} \min_{\{\mathbf{F}_k,\, \alpha_k,\, \beta_k,\, \lambda_{kj},\, \mu_{kj},\, p_{kj},\, q_{kj}\}} \quad & \sum_{k=1}^K \text{Tr}(\mathbf{F}_k) \\ \text{s.t.} \quad & (11), (12), (13) \text{ and } (14), \\ & \alpha_k \geq 1,\ \beta_k \geq 1, \\ & \text{invp}(\alpha_k) + \text{invp}(\beta_k) \leq 1, \\ & \mathbf{F}_k \succeq \mathbf{0},\ \lambda_{kj} \geq 0,\ \mu_{kj} \geq 0,\ \forall j, k, \end{aligned} \quad (15)$$

where $\text{invp}(x)$ denotes the inverse of positive portion, i.e. $1/\max\{x, 0\}$. The set of constraints involving $\text{invp}(\cdot)$ must be satisfied with equality at optimality; otherwise the objective value can be further decreased by decreasing α_k's. The above problem is a convex SDP problem which can be solved by a standard solver [12].

B. Proposed Rank-one-solution Recovery Method

Let $\mathbf{F}_k^*$ denote the optimal solution obtained by solving the relaxed problem (15), which provides a lower bound for the original problem (8). If $\mathbf{F}_k^*$ happens to be of rank one, then the principal eigenvector $\mathbf{f}_k^*$ of $\mathbf{F}_k^*$, such that $\mathbf{F}_k^* = \mathbf{f}_k^*\mathbf{f}_k^{*H}$, $\|\mathbf{f}_k^*\| = \sqrt{f_k}$, will be an optimal solution to problem (5).[3] In this work, we provide a good heuristic solution inspired by [7] when higher-rank solutions are returned by solving problem (15).

Before we proceed to introduce the rank-one recovery method, we calculate the worst-case channels for given beamforming vectors and PS ratios first. Assuming that $\{\mathbf{f}_k^*\}$ (the principle eigenvector of $\mathbf{F}_k^*$) and $\{\rho_k^*\}$ have been determined in the previous subsection, then the worst-case CSI errors which minimize the SINR of receiver k, are the solution to the following problems

$$\min_{\{\mathbf{e}_{kk}\}} |(\widehat{\mathbf{h}}_{kk}^H + \mathbf{e}_{kk}^H)\mathbf{f}_k^*| \quad \text{s.t.} \quad \|\mathbf{e}_{kk}\|^2 \leq \eta_{kk}^2, \quad (16)$$

$$\max_{\{\mathbf{e}_{kj}\}} |(\widehat{\mathbf{h}}_{kj}^H + \mathbf{e}_{kj}^H)\mathbf{f}_j^*| \quad \text{s.t.} \quad \|\mathbf{e}_{kj}\|^2 \leq \eta_{kj}^2,\ j \neq k. \quad (17)$$

The above constrained optimization problems have closed-from solutions which can be obtained by Cauchy-Schwarz inequality or Lagrange multiplier method. We omit the detailed derivation due to limited space, and let $\overline{\mathbf{e}}_{kj}$ denote the optimum solution to problem (16) and (17).

Similarly, the worst-case CSI errors which minimize the harvested energy of receiver k are the solution to the following problem

$$\min_{\{\mathbf{e}_{kj}\}} |(\widehat{\mathbf{h}}_{kj}^H + \mathbf{e}_{kj}^H)\mathbf{f}_j^*| \quad \text{s.t.} \quad \|\mathbf{e}_{kj}\|^2 \leq \eta_{kj}^2,\ \forall j, k. \quad (18)$$

[3] f_k is the largest eigenvalue of $\mathbf{F}_k^*$.

TABLE I
ALGORITHM-1 : PROPOSED SDP-BASED ROBUST DESIGN

1. Solve problem (15) to obtain the optimal $\{\mathbf{F}_k^*\}$ and $\{\rho_k^*\}$.
2. For each k, find the principal component $\{\mathbf{f}_k^*\}$ of $\mathbf{F}_k^*$. If $\text{rank}(\mathbf{F}_k^*) = 1,\ \forall k$, then $\{\mathbf{f}_k^*\}$ and $\{\rho_k^*\}$ are the optimum solution and exit the algorithm, otherwise go to Step 3.
3. Solve problem (19) with $\{\mathbf{f}_k^*\}$ to obtain $\{\varphi_k\}$ and $\{\rho_k\}$.
4. Return the beamforming vectors $\{\sqrt{\varphi_k}\mathbf{f}_k^*\}$ and $\{\rho_k\}$.

Let $\widetilde{\mathbf{e}}_{kj}$ denote the optimal solution of (18), which can be calculated following the same idea as that for problem (16) and (17). We note that $\widetilde{\mathbf{e}}_{kk} = \overline{\mathbf{e}}_{kk}$ and in the case $j \neq k$, $\widetilde{\mathbf{e}}_{kj}$ and $\overline{\mathbf{e}}_{kj}$ can not be simultaneously attained for the same channel realizations. However, we employ both $\widetilde{\mathbf{e}}_{kj}$ and $\overline{\mathbf{e}}_{kj}$ to guarantee the robustness of the joint design.

With the worst-case analysis described above, we recover the rank-one solution by scaling up $\mathbf{f}_k^*$ by $\sqrt{\varphi_k}$ and then jointly optimize $\{\varphi_k\}$ and $\{\rho_k\}$ to satisfy both the worst-case SINR and EH constraints. Specifically, we can formulate the optimization as the following SOCP problem with given $\{\mathbf{f}_k^*\}$

$$\begin{aligned} \min_{\{\rho_k, \varphi_k\}} \quad & \sum_{k=1}^K \varphi_k \|\mathbf{f}_k^*\|^2 \\ \text{s.t.} \quad & \|[2\omega_k, x_k - \rho_k]\| \leq x_k - \rho_k, \\ & \|[2\sqrt{\tfrac{\psi_k}{\xi_k}}, y_k + \rho_k - 1]\| \leq y_k - \rho_k + 1, \\ & 0 \leq \rho_k \leq 1,\ \ \varphi_k > 0,\ \ x_k \geq 0,\ y_k \geq 0,\ \forall k, \end{aligned} \quad (19)$$

where x_k and y_k are defined as $x_k = u_{kk}\varphi_k/\gamma_k - \sum_{j=1,j\neq k}^K \varphi_j v_{kj} - \sigma_k^2$, $y_k = \sum_{j=1}^K \varphi_j u_{kj} + \sigma_k^2$ and $u_{kj} = |(\widehat{\mathbf{h}}_{kj}^H + \widetilde{\mathbf{e}}_{kj}^H)\mathbf{f}_j^*|^2,\ \forall j, k$, $v_{kj} = |(\widehat{\mathbf{h}}_{kj}^H + \overline{\mathbf{e}}_{kj}^H)\mathbf{f}_j^*|^2,\ \forall k,\ j \neq k$. Problem (19) can be efficiently solved by off-the-shelf algorithms. The proposed SDR-based robust design for problem (5) is summarized in TABLE I.

IV. PROPOSED ROBUST DESIGN BASED ON CUTTING-SET METHOD

It is well known that solving an SDP problem is generally more computationally expensive than solving a SOCP problem. In this section, we propose an iterative algorithm based on SOCP relaxation and cutting-set method [13] to solve problem (5), where the optimization problem in each iteration is formulated as a SOCP problem. The cutting-set method is an effective technique to solve worst-case convex optimization problems with parameter uncertainty [9]. The uncertain parameters are assumed to belong to some given uncertainty sets. The proposed algorithm involves solving an alternating sequence of optimization and worst-case analysis subproblems. In the optimization subproblem, we solve problem (5) with fixed channel vectors. In the worst-case analysis subproblem, we compute the worst-case channel vectors with fixed beamforming vectors and PS ratios. Also, a robust solution recovery method is provided to ensure the robustness of the algorithm.

A. The Optimization and Worst-case Analysis Subproblems

The first subproblem involves the computation of beamforming vectors and receive PS ratios for a given set $\mathcal{H}$ of channel vectors $\{\mathbf{h}_{kj}^{(i)}\}_{i=1}^{E}$, $\forall j,k$, where $\mathbf{h}_{kj}^{(i)}$ is the ith worst-case channel vectors, and E is the size of set $\mathcal{H}$. It is worth noting that $\{\mathbf{h}_{kj}^{(1)}\}$ is the estimated imperfect CSI, i.e. $\{\widehat{\mathbf{h}}_{kj}\}$, and E may be increased through each iteration. Inspired by [8], the original problem can be relaxed as the following problem by replacing the EH constraints with the sum of SINR and EH constraints

$$\begin{aligned} \min_{\{\mathbf{f}_k,\rho_k\}} \quad & \textstyle\sum_{k=1}^{K}\|\mathbf{f}_k\|^2 \\ \text{s.t.} \quad & \tfrac{1}{\gamma_k}|\mathbf{h}_{kk}^{(i)H}\mathbf{f}_k|^2-\textstyle\sum_{j\neq k}^{K}|\mathbf{h}_{kj}^{(i)H}\mathbf{f}_j|^2\geq\sigma_k^2+\tfrac{\omega_k^2}{\rho_k}, \\ & (1+\tfrac{1}{\gamma_k})|\mathbf{h}_{kk}^{(i)H}\mathbf{f}_k|^2\geq\tfrac{\psi_k}{\xi_k(1-\rho_k)}+\tfrac{\omega_k^2}{\rho_k}, \\ & 0\leq\rho_k\leq 1,\ \ i=1,\ldots,E,\ \forall j,k. \end{aligned} \tag{20}$$

We note that problem (20) is equivalent to the problem with perfect CSI in the case $E=1$; the worst-case channel vectors only add more constraints and will not affect the essence of problem (20). Hence, (20) can be rewritten as the following SOCP problem[4]

$$\begin{aligned} \min_{\{\mathbf{f}_k,a_k,b_k,c_k,d_k\}} \quad & t \\ \text{s.t.} \quad & \|[\mathbf{f}_1^T,\ldots,\mathbf{f}_K^T]\|\leq t, \\ & \|[\mathbf{I}_k^{(i)T},\sigma_k,c_k]\|\leq\tfrac{1}{\sqrt{\gamma_k}}\Re(\mathbf{h}_{kk}^{(i)H}\mathbf{f}_k),\ \forall i, \\ & \|[c_k,d_k]\|\leq\sqrt{1+\tfrac{1}{\gamma_k}}\Re(\mathbf{h}_{kk}^{(i)H}\mathbf{f}_k),\forall i, \\ & \|[2(\tfrac{\psi_k}{\xi_k})^{\frac{1}{4}},d_k-b_k]\|\leq d_k+b_k, \\ & \|[2\sqrt{\omega_k},c_k-a_k]\|\leq c_k+a_k, \\ & \|[a_k,b_k]\|\leq 1,\ a_k\geq 0,\ b_k\geq 0,\ \forall k, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \mathbf{I}_k^{(i)} = [&\mathbf{h}_{k1}^{(i)H}\mathbf{f}_1,\ldots,\mathbf{h}_{k(k-1)}^{(i)H}\mathbf{f}_{k-1}, \\ &\mathbf{h}_{k(k+1)}^{(i)H}\mathbf{f}_{k+1},\ldots,\mathbf{h}_{kK}^{(i)H}\mathbf{f}_K]^T \end{aligned} \tag{22}$$

denotes the interference received by receiver k under the ith worst-case channel vectors, $a_k^2=\rho_k$, $b_k^2=1-\rho_k$, $c_k^2\geq\frac{\omega_k^2}{\rho_k}$, and $d_k^2\geq\frac{\psi_k}{\xi_k(1-\rho_k)}$ are slack variables.

B. Iterative Algorithm for the JBPS Design

We start the iterative algorithm with a set $\mathcal{H}$ of channel vectors, which initially contains only the imperfect CSI, i.e., $\mathcal{H}=\{\widehat{\mathbf{h}}_{kj}\}$. The worst-case SINR and relaxed EH are computed as $\Gamma_k^*=\Gamma_k(\mathbf{f}_k^*,\rho_k^*,\overline{\mathbf{e}}_{kk},\widetilde{\mathbf{e}}_{kj})$ and $P_k^{\text{EH}*}=P_k^{\text{EH}}(\mathbf{f}_k^*,\rho_k^*,\overline{\mathbf{e}}_{kk},\widetilde{\mathbf{e}}_{kj})$, where $\{\mathbf{f}_k^*,\rho_k^*\}$ are the beamforming vectors and PS ratios computed from subproblem (20), and $\{\overline{\mathbf{e}}_{kk},\widetilde{\mathbf{e}}_{kj}\}$ are the worst-case channel vectors obtained from subproblems (16) and (17). During the worst-case analysis subproblem in each iteration, the set $\mathcal{H}$ may be expanded if Γ_k^* or $P_k^{\text{EH}*}$ violate the corresponding constraints, and the iteration will be terminated if both Γ_k^* and $P_k^{\text{EH}*}$ satisfy the constraints. During the optimization subproblem, the total transmission power is minimized for the increased number of worst-case channels, thereby improving robustness. It is worth noting that the cutting-set method is shown to converge to the robust optimal point [13] of the SOCP relaxed problem due to exact worst-case analysis.

[4]Note that, we can restrict $\mathbf{h}_{kk}^{(i)H}\mathbf{f}_k$ to be positive when $E=1$, which incurs no loss of optimality (i.e. the so-called phase rotation technique). However, this condition will not hold true in the case $E>1$, thus the absolute value operator is replaced by the real-part operator $\Re(\cdot)$ resulting in a SOCP that can be solved by interior-point method. This makes the optimal solution of (21) suboptimal to (20). Nevertheless, the robust solution recovery method will ensure the robustness of the algorithm.

TABLE II
ALGORITHM-2 : PROPOSED ROBUST BASED ON CUTTING-SET METHOD

1. **for** the nth iteration
   - Update the set $\mathcal{H}$ of worst-case channel vectors.
   - Solve problem (21) to obtain $\{\mathbf{f}_k^*\}$ and $\{\rho_k^*\}$.
   - Compute worst-case channel vectors according to (16) and (17), add $\{\widehat{\mathbf{h}}_{kk}+\overline{\mathbf{e}}_{kk}\}_{k=1}^{K}$ and $\{\widehat{\mathbf{h}}_{kj}+\widehat{\mathbf{e}}_{kj}\}_{k=1,j\neq k}^{K}$ to set $\mathcal{H}$ or stop the iteration according to certain thresholds.
2. **end**
3. Repeat steps 1-2 until the algorithm converges and return $\{\widetilde{\mathbf{f}}_k^*\}$.
4. Employ steps 3-4 in *Algorithm-1* with $\{\widetilde{\mathbf{f}}_k^*\}$ to obtain a robust feasible solution.

While the solution $\{\widetilde{\mathbf{f}}_k^*\}$ and $\{\widetilde{\rho}_k^*\}$ obtained by the cutting set method may not be robust to all possible channel realizations in the uncertainty region due to SOCP relaxation, we can employ the method exposed in Section III-B to recover a robust solution. The proposed robust design based on the cutting-set method is summarized in TABLE II.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed robust JBPS algorithms numerically. We assume there are $K=3$ transmit-receive pairs and all transmitters are equipped with $N_k=N,\ k\in\{1,2,3\}$ antennas unless otherwise specified. We also assume that each transmit-receive pair has the same set of parameters in all our simulations, i.e., $\gamma_k=\gamma$, $\psi_k=\psi$, $\xi_k=\xi=1$, $\sigma_k^2=\sigma^2=-30$ dBm, $\omega_k^2=\omega^2=-20$ dBm and $\eta_{kj}=\eta,\ \forall j,k$ for simplicity. Moreover, the pre-assumed channel vectors $\{\widehat{\mathbf{h}}_{kj}\}$ are randomly generated from independent and identically distributed Rayleigh fading with average power 1. All convex problems are solved by CVX [12] on a desktop Intel (i3-2100) CPU running at 3.1GHz and 4GB RAM.

1) Feasibility Rate: We first present the feasibility rates of the two robust JBPS designs. In the simulation, feasibility is claimed for a design $\mathbf{f}_k^*$ and ρ_k^* if Γ_k^* and $P_k^{\text{EH}*}$ are greater than the targets or CVX reports an infeasible/fail status. The feasibility of the non-robust design [7] are tested with 100 channel error vectors satisfying the NBE model for each channel realization. Fig. 1 presents the simulation results obtained by testing over 1000 channel realizations. One can observe that the feasibility rate of *Algorithm-2* increases with an increase in the number of iterations. Moreover, *Algorithm-1* and *Algorithm-2* (4 or 8 iterations) exhibit similar feasibility rate with the bound provided by solving problem (15). The non-robust method fails to satisfy both the SINR and EH constraints almost all the time under NBE model.

2) Transmission Power: We illustrate the performance of the two robust designs in terms of average transmission power over 1000 problem instances. Fig. 2 shows performance comparison among the two robust designs, and the transmission power are averaged over problem instances where the robust designs are all feasible. It is observed that, as a price paid for guaranteed worst-case performance, the robust designs require higher average transmission power than the non-robust design. However, *Algorithm-1* and *Algorithm-2* (4 or 8 iterations) are very power-efficient since their performance are very close to the performance bound provided by solving problem (15).

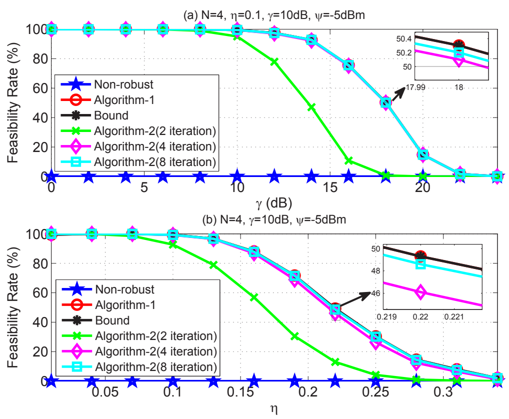

Fig. 1. (a). Feasibility rate (%) versus various γ, (b). Feasibility rate (%) versus various η.

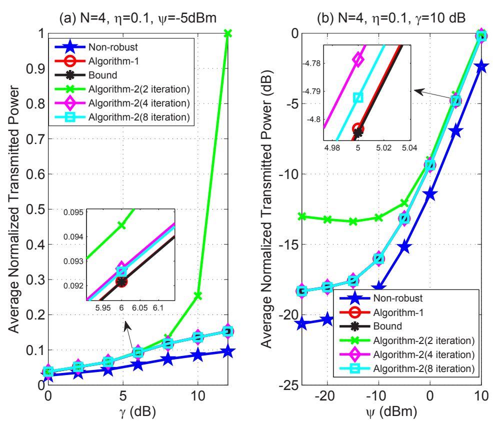

Fig. 2. (a). Transmission power versus SINR target γ, (b). Transmission power versus EH threshold ψ

3) Time Complexity: We then compare the performance of the robust and non-robust designs in terms of average execution time[5] over 100 channel realizations. Fig. 3 presents the average execution time for a particular parameter combination and different values of K. It is observed that the time consumed by all three algorithms increases with K. However, *Algorithm-2* consumes much less time than *Algorithm-1*. Considering its performance gain over the non-robust design, this property makes *Algorithm-2* very promising and suitable for systems with large antenna arrays.

[5]The complexity order of *Algorithm-1* is : $\mathcal{O}(\sqrt{2K^2(N+1)+KN}\cdot n\cdot[2K^2(N+1)^3+KN^3+2nK^2(N+1)^2+nKN^2+n^2]+\mathcal{O}(\sqrt{4K}\cdot 2K\cdot[2K\times 3^2+4K^2])$, $n=\mathcal{O}(KN^2+4K^2)$. The complexity order of *Algorithm-2* is $\mathcal{O}(I\sqrt{6K+4EK+2}\cdot n\cdot[(KN+1)^2+EK(K+2)^2+(3K+EK)\times 3^2+n^2])$, $n=\mathcal{O}(KN+4K)$, I is the iteration number. We suppress the $\ln(1/\varepsilon)$ term for an ε-optimal solution and omit the detailed derivation due to space limitation.

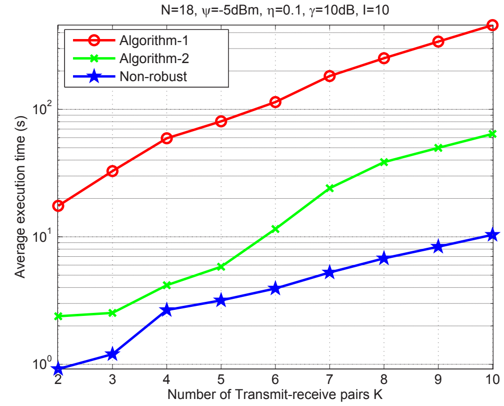

Fig. 3. Time complexity comparison versus K with fixed N.

VI. Conclusion

In this paper, we considered robust JBPS design for MISO interference channel under NBE model. Two robust designs were proposed to handle the highly non-convex problem. In the SDR-based design, we showed that the original problem can be relaxed as an SDP, which provided a lower bound for the robust JBPS problem. In the cutting-set based design, we demonstrated that robust solutions can also be obtained by solving an alternating sequence of optimization and worst-case analysis subproblems. The simulation results validated the near-optimal performance of the two designs.

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