

Success-Probability-Based Power Allocation for Downlink PNC in Multi-way Relay Channels

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Abstract—In this paper, we propose a novel power allocation scheme for physical-layer network coding (PNC) in downlink multi-way relay channels (MWRC). The power allocation is formulated as a constrained optimization problem, where the aim is to maximize the success probability under a total power constraint when using Babai estimation for signal detection. Optimizing over this metric allows us to maximize the probability of successfully decoding a chain of network codes, which is of crucial importance in downlink multi-way PNC. Specifically, to meet the different requirements for transmission quality in applications, we consider different aggregate measures of success probability over the participating user terminals, i.e., the arithmetic mean, the geometric mean, and the maximin. For each measure, we formulate a constrained optimization and demonstrate the concavity of the objective, allowing us to obtain solution efficiently via iterative means. The performance of the proposed power allocation schemes for downlink PNC in MWRC is evaluated by means of computer simulations over Rayleigh fading channels. The results demonstrate the effectiveness of the proposed schemes in improving the success probability in the reception of a chain of network codes.

I. INTRODUCTION

Physical-layer network coding (PNC), firstly proposed in [1], is an attractive approach to increasing the network throughput by exploiting the broadcast nature of the wireless channel. Different from conventional network coding (NC) [2], [3] which usually requires 2 time slots for uplink and 1 time slot for downlink transmissions in a half-duplex two way relay channel (TWRC), PNC allows biparty users to send their signal simultaneously to the relay using only 1 time slot in the uplink. Upon receiving the superimposed signals, the relay encodes this information into an NC signal and broadcasts it in a subsequent time slot in the downlink. Each user then decodes the desired information from the other user by employing its self-information. Compared to the conventional NC scheme, PNC only requires 2 time slots in total which leads to a 33% throughput improvement in theory.

The use of PNC in multi-way relay channels (MWRC) [4], where multiple users share information through a single relay, is a natural extension of TWRC. In a multi-way PNC system between N users, the relay typically broadcasts a chain of $N-1$ network codes, or symbols, that are designed to be strongly correlated with each other. Due to the correlation, the decoding

performance of the complete set of messages at the user terminals highly depends on the probability of successfully detecting each individual network codes in such chains. Hence, it is of critical importance to devise mechanisms that can improve the probability of symbol detection for downlink PNC transmissions in MWRC.

Existing techniques for multi-user communications, e.g., precoding and power allocation schemes, are often devised based on power domain metrics, such as the signal-to-noise ratio (SNR), signal-to-interference-plus-noise ratio (SINR) and related quantities, including achievable information rates. For instance, the design of a precoding matrix for a multi-user system with an arbitrary number of antennas at the user terminals is addressed in [5] to mitigate the multi-user interference in the downlink channel. A block diagonalization approach is considered for the design of downlink multi-user precoders in [6], [7]. In particular, the precoding matrix is generated based on the QR decomposition of the relay-to-user channel matrix so as to improve the achievable sum rate of the system. In [8], as an alternative criterion to SINR and SNR, the authors present a so called signal-to-leakage-and-noise ratio (SLNR) precoding scheme that considers the leaked power from one user to other users in a multi-user multiple-input and-multiple-output (MIMO) system; the precoder design is thus based on maximizing the SLNR for all users.

Few works explicitly focus on improving the detection performance of a chain of correlated symbols, as needed for PNC in MWRC. In this regard, the success probability of Babai estimation introduced in [9], can provide a useful metric for determining and enhancing the integrity of a chain of network codes received at user terminals. Babai estimation is an efficient tool that can be applied to the solution of a variety of estimation problems in wireless communications. In particular, it provides a suboptimal solution with low complexity to integer least squares problems occurring in the estimation of certain linear models [10]. In this context, the success probability of Babai estimation characterizes the detection performance of a group of symbols within a successive detection process. However, these works mainly focus on theoretical performance analysis and do not utilize the success probability as a metric for practical system design.

Motivated by the work of [9], we propose a novel power allocation scheme for PNC in downlink MWRC. The power

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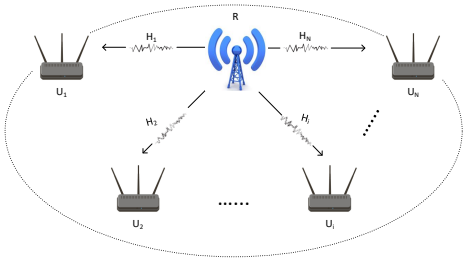


Fig. 1: Downlink model of PNC in MWRC. The relay broadcasts $N - 1$ network codes to the users.

allocation is formulated as a constrained optimization problem, where the aim is to maximize the success probability under a total power constraint when using Babai estimation for signal detection. Optimizing over this metric allows us to maximize the probability of successfully decoding a chain of network codes. Specifically, to meet diverse requirements for transmission quality in applications, we consider different aggregate measures of success probability over the participating user terminals, i.e., the arithmetic mean, the geometric mean, and the maximin. For each measure, we formulate constrained optimization problems and demonstrate the concavity of the objective, allowing us to obtain solution efficiently via iterative means. The performance of the proposed power allocation schemes for downlink PNC in MWRC is evaluated by means of computer simulations over Rayleigh fading channels. The results demonstrate the effectiveness of the proposed schemes in improving the success probability in the reception of a chain of network codes.

The paper is organized as follows: In Section II, we introduce the system model, including physical setup and transmission protocol of the downlink PNC scenario. In Section III, we introduce the backgrounds regarding the proposed methods. In Section IV, we formulate and solve the three optimization problems for power allocation based on different measures of success probability. In Section V, we provide simulation results to demonstrate the performance of the proposed power allocation schemes. Section VI concludes the paper.

II. SYSTEM MODEL

A. System Setup

The general system setup for download PNC in MWRC is illustrated in Fig.1. The relay, say R , is equipped with K antennas, while each user terminal, say U_i , $i = 1, \dots, N$, is equipped with M antennas. In this work, we focus on the so-called overdetermined problem where $M \geq K$. In practice, this corresponds to a situation where R and U_i are of similar scale, such as in collaborations among multiple base stations [11] or among mobile devices in vehicle-to-vehicle (V2V) communications [12]. The radio channel between R and U_i is assumed to be flat fading, and therefore represented by a matrix $\mathbf{H}_i \in \mathbb{C}^{M \times K}$. It is assumed that R has full knowledge of all the channels while each U_i has knowledge of its respective channel \mathbf{H}_i . U_i has a message to share with all other users,

which is denoted as $m_i \in \mathbb{F}_q$, where \mathbb{F}_q is a finite integer field of q elements. It is assumed that these messages are available at R , following uplink transmission as in [13].

In the downlink stage, R uses the messages in the set $\{m_1, m_2, \dots, m_N\}$ to generate a code vector $\mathbf{c} = [c_1, c_2, \dots, c_{N-1}]^T \in \mathbb{F}_q^{N-1}$, consisting of $N - 1$ codewords (or network codes) c_i to be broadcast to the N users over multiple time slots. To be specific, each entry c_i is generated through the application of a network coding function ϕ on a pair of user messages, i.e., (m_i, m_j) , $i \neq j$. In this work, the function $\phi : \mathbb{F}_q^2 \rightarrow \mathbb{F}_q$ is chosen as the modulo- q addition of its operands. That is, for any two integers $a, b \in \mathbb{F}_q$, we define:

$$\phi(a, b) = a \oplus b = (a + b) \bmod q \quad (1)$$

For MWRC, there exist several ways to generate \mathbf{c} [14]. Since our focus is on the downlink transmission, we select the sequential pairing strategy for simplicity, which is given as:

$$c_i = \phi(m_i, m_{i+1}), \quad i = 1, \dots, N - 1 \quad (2)$$

B. Downlink Transmission

The transmission scheme of downlink PNC in MWRC is described as follows. The relay R breaks up \mathbf{c} into $T = \lceil (N - 1)/K \rceil$ packets, where $\lceil \cdot \rceil$ denotes the ceiling function. Each packet can be expressed as a length- K vector¹ $\mathbf{c}_t \in \mathbb{F}_q^K$ where the index $t \in \{0, 1, \dots, T - 1\}$. The relay R then broadcasts the packet \mathbf{c}_t to all users over T consecutive time slots.

To this end, the relay maps each \mathbf{c}_t to a baseband signal vector $\mathbf{s}(t) \in \mathbb{C}^K$ for the transmission. The mapping is implemented element-wise through a bijective function $\varphi : \mathbb{F}_q \rightarrow \mathcal{C}$, where \mathcal{C} with cardinality q represents the transmitted signal constellation. Specifically, denoting by $s_i(t)$ and $c_{t,i}$ the i -th entries of $\mathbf{s}(t)$ and \mathbf{c}_t , respectively, we have:

$$s_i(t) = \varphi(c_{t,i}), \quad i = 1, 2, \dots, K. \quad (3)$$

The set \mathcal{C} depends on the particular modulation scheme employed for digital transmission. To simplify our discussion, we hereby set $q = 2$ and define $\mathcal{C} = \{-1, +1\}$.

The signal $\mathbf{x}(t)$ sent from R is written as: $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$ where the diagonal matrix $\mathbf{A} = \text{diag}(\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_K}) \in \mathbb{R}_+^{K \times K}$ is the power allocation matrix, with P_i being the power allocated to the i^{th} antenna. The signal received at U_i is thus given by:

$$\mathbf{y}_i(t) = \mathbf{H}_i \mathbf{x}(t) + \mathbf{n}_i(t) = \mathbf{H}_i \mathbf{A} \mathbf{s}(t) + \mathbf{n}_i(t) \quad (4)$$

where $\mathbf{n}_i(t) \in \mathbb{C}^{M \times 1}$ is the additive noise at U_i , which is modeled as a complex circular Gaussian vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}$.

By estimating $\mathbf{s}(t)$ for $t = 0, 1, \dots, T - 1$ and inverting the mapping (3), each user can determine all the codewords c_i and restore the code vector \mathbf{c} on its side. User U_i then utilizes its own message m_i and \mathbf{c} to decode the messages m_j of U_j for all $j \neq i$. This concludes the general process of the

¹To simplify the analysis, we ensure every packet to be length- K by appending a vector $\bar{\mathbf{c}} \in \mathbb{F}_q^l$, consisting of $l = KT - (N - 1)$ pseudo codewords, to \mathbf{c} . We assume that $\bar{\mathbf{c}}$ is known to all terminals prior to the communication, so that it can be correctly removed by any receiving terminal.

downlink broadcast phase. From now on, we will focus on the transmission of a single packet in a specific time slot, hence the time index t will be dropped for convenience.

III. SUCCESS PROBABILITY OF BABAI ESTIMATION

In this section, we review the underlying principles of Babai estimation and summarize key results for its success probability in terms of the signal power. A conventional power allocation method serving as a benchmark is also discussed.

A. Babai Estimation

After dropping time index t , the model in (4) becomes:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x} + \mathbf{n}_i = \mathbf{H}_i \mathbf{A} \mathbf{s} + \mathbf{n}_i \quad (5)$$

for $i = 1, \dots, N$. The maximum likelihood estimate of the transmitted signal \mathbf{s} is the solution of the integer least squares (ILS) problem:

$$\min_{\mathbf{s}_i \in \{-1, +1\}^K} \|\mathbf{y}_i - \mathbf{H}_i \mathbf{A} \mathbf{s}\| \quad (6)$$

Since the ILS problem is NP-Hard, an optimal solution to the detection problem generally requires high time complexity. To reduce the computation load for user terminals with limited capability, such as mobile phones, we adopt a suboptimal solution approach, called the Babai estimation [9], which allows each U_i to estimate \mathbf{s} by successively canceling interference.

Specifically, the so-called Babai point \mathbf{s}^B for overdetermined problem (6) where $M \geq K$ and $s_i^B \in \{-1, +1\}$ is defined below:

$$\begin{aligned} s_K^B &= \lfloor \Re(b_K) \rfloor_C, \quad b_K = \frac{\tilde{y}_K}{\sqrt{P_K} r_{KK}^{(i)}} \\ s_j^B &= \lfloor \Re(b_j) \rfloor_C, \quad b_j = \frac{\tilde{y}_j - \sum_{k=j+1}^K \sqrt{P_k} r_{jk}^{(i)} s_k^B}{\sqrt{P_j} r_{jj}^{(i)}} \end{aligned} \quad (7)$$

for $j = K-1, \dots, 1$, where $\Re(\cdot)$ denotes the real part and the operator $\lfloor \cdot \rfloor$ rounds to the nearest value in $C = \{-1, +1\}$, $\mathbf{R}_i = [r_{jk}^{(i)}]_{K \times K}$ is the upper triangular matrix from the QR factorization of \mathbf{H}_i , i.e.:

$$\mathbf{H}_i = [\mathbf{Q}_1, \mathbf{Q}_2] \begin{bmatrix} \mathbf{R}_i \\ \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{y}} = \mathbf{Q}_1^H \mathbf{y}_i. \quad (8)$$

Without loss of generality, according to [9], the diagonal entries of \mathbf{R}_i can always be set to non-negative values through simple matrix transformation of QR decomposition in (8).

B. Success Probability of Babai Estimator

In the context of Babai estimation, the probability of successfully detecting a series of successive signals provides a convenient metric for characterizing the integrity of the detected chain of signals. Specifically, the so-called *success probability* of \mathbf{s}^B at U_i is defined as [9]:

$$\begin{aligned} \rho_i &= \Pr(\mathbf{s}^B = \mathbf{s}) = \Pr(s_1^B = s_1 | s_2^B = s_2, \dots, s_K^B = s_K) \\ &\quad \dots \Pr(s_{K-1}^B = s_{K-1} | s_K^B = s_K) \Pr(s_K^B = s_K) \end{aligned} \quad (9)$$

From the definition of \mathbf{s}^B , we can conclude that when $s_j = -1$, $s_j^B = s_j$ if and only if (iff) $\Re(b_j) \in (-\infty, 0]$, and when $s_j = 1$, $s_j^B = s_j$ iff $\Re(b_j) \in (0, +\infty)$. Thus, based on [9],

the probability of $s_j^B = s_j$, given previous signals have been correctly detected, is given as:

$$\begin{aligned} &\Pr(s_j^B = s_j | s_{j+1}^B = s_{j+1}, \dots, s_K^B = s_K) \\ &= \Pr(s_j = -1) \Pr(\Re(b_j) \in (-\infty, 0] | s_{j+1}^B = s_{j+1}, \dots, s_K^B = s_K) \\ &\quad + \Pr(s_j = 1) \Pr(\Re(b_j) \in (0, +\infty) | s_{j+1}^B = s_{j+1}, \dots, s_K^B = s_K) \\ &= \frac{1}{2} \frac{1}{\sqrt{\pi} \sigma} \left[\int_{-\infty}^0 e^{-\frac{(t-(-1))^2}{\sigma^2 / (P_j (r_{jj}^{(i)})^2)}} dt + \int_0^{+\infty} e^{-\frac{(t-(+1))^2}{\sigma^2 / (P_j (r_{jj}^{(i)})^2)}} dt \right] \\ &= \frac{1}{2} \left(1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma) \right) \end{aligned} \quad (10)$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Then we have:

$$\rho_i = \prod_{j=1}^K \frac{1}{2} \left(1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma) \right). \quad (11)$$

C. Conventional Adaptive Power Allocation

As power allocation is generally considered essential for multi-user systems, conventional strategies mainly focus on boosting the spectral efficiency. A typical example of such a strategy, as being discussed in [15], is to maximize the channel capacity. Specifically, the capacity of the channel between R and U_i is expressed as: $C_i = W \sum_{j=1}^K \log_2 \left(1 + \frac{\lambda_{i,j} P_j}{\sigma^2} \right)$, where W is the channel bandwidth which can be regarded as a constant value and $\lambda_{i,j}$ for $j = 1, \dots, K$ are the eigenvalues of $\mathbf{H}_i \mathbf{H}_i^H$. The optimal power allocation is obtained by maximizing the average channel capacity of all R - U channels, i.e.,

$$\max_{P_1, \dots, P_K} \frac{1}{N} \sum_{i=1}^N \left[W \sum_{j=1}^K \log_2 \left(1 + \frac{\lambda_{i,j} P_j}{\sigma^2} \right) \right] \text{ s.t. : } \sum_{j=1}^K P_j = P_T, P_j \geq 0. \quad (12)$$

where P_T is the total transmit power at the relay. However, this metric is not practical for a PNC system whose performance highly depends on the success probability of the chain of network codes, i.e., the signal vector \mathbf{s} in (5), being correctly received by user terminals. Hence, it is beneficial to find an alternative solution for the PNC system in MWRC.

IV. PROPOSED METHODS

The success probability of Babai estimation, i.e. the probability of an entire code chain being correctly received at U_i , is a critical performance measure for the network codes. Hence, allocating power at relay R to improve this probability is a meaningful way of enhancing the reliability of downlink PNC transmission. In this section, the power allocation is formulated as constrained optimization problems, where the aim is to maximize the success probability under power constraints. Specifically, we consider three different aggregate measures of success probability over the participating user terminals, namely, the arithmetic mean, the geometric mean, and the maximin.

A. Average Success Probability (Arithmetic Mean)

We first focus on optimizing the average success probability at the user terminals, which benefits the reception reliability

of the multi-way system over an ensemble of channel realizations. In our approach, the average success probability is estimated as the arithmetic mean of the success probability over all users terminals. Considering (11) for $i = 1, \dots, N$, the average success probability is given as:

$$\rho_{ave} = \frac{1}{N} \sum_{i=1}^N \rho_i = \frac{1}{N} \sum_{i=1}^N \left[\prod_{j=1}^K \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right]. \quad (13)$$

An optimization problem can be formulated to maximize (13) subject to a total power constraint P_T , i.e.:

$$\begin{aligned} \max_{P_1, \dots, P_K} & \frac{1}{N} \sum_{i=1}^N \left[\prod_{j=1}^K \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right] \\ \text{s.t.} & : \sum_{j=1}^K P_j = P_T, P_j \geq 0. \end{aligned} \quad (14)$$

However, it is computationally challenging to find the global optima of (14) as its cost function is not necessarily concave in the feasible region. To ease this difficulty, we can average over $\rho_i^{1/K}$ instead, i.e.:

$$\rho'_{ave} = \frac{1}{N} \sum_{i=1}^N \rho_i^{\frac{1}{K}} \quad (15)$$

Since each $\rho_i^{1/K}$ is still an indication of the success probability at U_i , ρ'_{ave} is also a metric to represent the average reception reliability of the system. That is to say, we can alternatively formulate the problem as:

$$\begin{aligned} \max_{P_1, \dots, P_K} & \frac{1}{N} \sum_{i=1}^N \left[\prod_{j=1}^K \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right]^{\frac{1}{K}} \\ \text{s.t.} & : \sum_{j=1}^K P_j = P_T, P_j \geq 0. \end{aligned} \quad (16)$$

Proposition 1. *The cost function in (16) is concave in the closed and convex feasible region.*

Proof. Define function

$$g_j^{(i)}(P_j) = 1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma) \quad (17)$$

The second order partial derivative of (17) with respect to P_j is non-positive, i.e.:

$$\frac{\partial^2 g_j^{(i)}(P_j)}{\partial^2 P_j} = -\frac{r_{jj}^{(i)}}{\sqrt{\pi}\sigma} \left[\frac{(r_{jj}^{(i)})^2}{\sigma^2 \sqrt{P_j}} + \frac{1}{2} P_j^{-\frac{3}{2}} \right] e^{-\frac{P_j (r_{jj}^{(i)})^2}{\sigma^2}} \leq 0 \quad (18)$$

since $e^{-P_j (r_{jj}^{(i)})^2 / \sigma^2} \geq 0$ and $P_j \geq 0$. The Hessian of the function:

$$w_i(P_1, \dots, P_K) = \rho_i^{\frac{1}{K}} = \frac{1}{2} \left[\prod_{j=1}^K g_j^{(i)}(P_j) \right]^{\frac{1}{K}} \quad (19)$$

is accordingly given by

$$\nabla^2 w_i = -\frac{1}{2K^2} \left(\prod_{j=1}^K g_j^{(i)}(P_j) \right)^{\frac{1}{K}} \left[K \operatorname{diag}(\mathbf{d}) - \mathbf{q}\mathbf{q}^T \right], \quad (20)$$

where $\mathbf{d}, \mathbf{q} \in \mathbb{R}^{K \times 1}$ with respective entries:

$$d_l = \left(\frac{1}{g_l^{(i)}(P_l)} \frac{\partial g_l^{(i)}(P_l)}{\partial P_l} \right)^2 - \frac{1}{g_l^{(i)}(P_l)} \frac{\partial^2 g_l^{(i)}(P_l)}{\partial^2 P_l}, q_l = \frac{1}{g_l^{(i)}(P_l)} \frac{\partial g_l^{(i)}(P_l)}{\partial P_l}. \quad (21)$$

For any vector $\mathbf{v} \in \mathbb{R}^{K \times 1}$, we have:

$$\begin{aligned} \mathbf{v}^T \nabla^2 w_i \mathbf{v} = & -\frac{1}{2K^2} \left(\prod_{j=1}^K g_j^{(i)}(P_j) \right)^{\frac{1}{K}} \left[K \sum_{j=1}^K \left(\frac{1}{g_j^{(i)}(P_j)} \frac{\partial g_j^{(i)}(P_j)}{\partial P_j} \right)^2 v_j^2 \right. \\ & \left. - K \sum_{j=1}^K \frac{1}{g_j^{(i)}(P_j)} \frac{\partial^2 g_j^{(i)}(P_j)}{\partial^2 P_j} v_j^2 - \left(\sum_{j=1}^K \frac{1}{g_j^{(i)}(P_j)} \frac{\partial g_j^{(i)}(P_j)}{\partial P_j} v_j \right)^2 \right]. \end{aligned} \quad (22)$$

Considering (18) and Cauchy-Schwarz inequality $(\mathbf{a}^T \mathbf{a})(\mathbf{b}^T \mathbf{b}) - (\mathbf{a}^T \mathbf{b})^2 \geq 0$ where we apply

$$\mathbf{a} = \mathbf{1}^{K \times 1}, b_j = \frac{1}{g_j^{(i)}(P_j)} \frac{\partial g_j^{(i)}(P_j)}{\partial P_j} v_j, \quad (23)$$

we can show that $\mathbf{v}^T \nabla^2 w_i \mathbf{v} \leq 0$ for all $\mathbf{v} \in \mathbb{R}^{K \times 1}$, i.e., $\nabla^2 w_i \leq 0$. The function in (19) is thus concave. Since the concavity is preserved by non-negative weighted sum operations, the cost function in (16) is also concave. \square

B. Overall Success Probability (Geometric Mean)

Another goal of the design is to enhance the system's overall reception capability, i.e., when the integrity of all the received network chains by all user terminals is critical. To achieve this, the optimization problem can be formulated to maximize the geometric mean of the success probabilities at all user terminals. That is,

$$\rho_{all} = \left(\prod_{i=1}^N \rho_i \right)^{\frac{1}{N}} = \left(\prod_{i=1}^N \left[\prod_{j=1}^K \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right] \right)^{\frac{1}{N}}. \quad (24)$$

To maximize ρ_{all} , it is equivalent to maximize its logarithmic form, which is:

$$\log \rho_{all} = \frac{1}{N} \sum_{i=1}^N \left[\sum_{j=1}^K \log \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right]. \quad (25)$$

We can then formulate an optimization problem to maximize (25), i.e.:

$$\begin{aligned} \max_{P_1, \dots, P_K} & \frac{1}{N} \sum_{i=1}^N \left[\sum_{j=1}^K \log \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right] \\ \text{s.t.} & : \sum_{j=1}^K P_j = P_T, P_j \geq 0. \end{aligned} \quad (26)$$

Proposition 2. *The cost function*

$$f(P_1, \dots, P_K) = \frac{1}{N} \sum_{i=1}^N \left[\sum_{j=1}^K \log \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)}/\sigma)) \right] \quad (27)$$

is concave in the closed and convex feasible region.

Proof. The first order partial derivative of f to P_j is:

$$\frac{\partial f(P_1, \dots, P_K)}{\partial P_j} = \frac{1}{N} \sum_{i=1}^N \frac{r_{jj}^{(i)} e^{-P_j (r_{jj}^{(i)})^2 / \sigma^2}}{\ln 2 \sqrt{\pi} \sigma (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma)) \sqrt{P_j}}, \quad (28)$$

The second order partial derivative with respect to P_j is:

$$\frac{\partial^2 f(P_1, \dots, P_K)}{\partial^2 P_j} = \frac{r_{jj}^{(i)}}{N \ln 2 \sqrt{\pi} \sigma (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma))^2} \left[-\frac{r_{jj}^{(i)} e^{-P_j (r_{jj}^{(i)})^2 / 0.5 \sigma^2}}{\sqrt{\pi} \sigma P_j} - \frac{P_j^{-1.5} e^{-P_j (r_{jj}^{(i)})^2 / \sigma^2}}{2} - \frac{(r_{jj}^{(i)})^2 e^{-P_j (r_{jj}^{(i)})^2 / \sigma^2}}{\sigma^2 \sqrt{P_j}} \right] \leq 0. \quad (29)$$

The second order partial derivative with respect to P_j and P_k , where $k \neq j$, is:

$$\frac{\partial^2 f(P_1, \dots, P_K)}{\partial P_j \partial P_k} = 0. \quad (30)$$

Hence, the Hessian matrix $\nabla^2 f(P_1, \dots, P_K)$ is negative semi-definite. The concavity of the cost function is proved. \square

C. Minimal Success Probability (Maximin)

The goal of the optimization problem can also be set on maximizing the minimal success probability of all users. By doing this, the worst case scenario of the reception capability in the user groups will be improved. That is to say, the problem can be formulated as:

$$\begin{aligned} \max_{P_1, \dots, P_K} \min_i \sum_{j=1}^K \log \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma)) \\ \text{s.t. : } \sum_{j=1}^K P_j = P_T, P_j \geq 0, i = 1, \dots, N. \end{aligned} \quad (31)$$

Proposition 3. *The cost function*

$$f(P_1, \dots, P_K) = \min_{i=1, \dots, N} \sum_{j=1}^K \log \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma)) \quad (32)$$

is concave in the closed and convex feasible region.

Proof. Similarly to the proof in IV-B, it is easy to find that for each user i , the function

$$u^{(i)}(P_1, \dots, P_K) = \sum_{j=1}^K \log \frac{1}{2} (1 + \operatorname{erf}(\sqrt{P_j} r_{jj}^{(i)} / \sigma)) \quad (33)$$

is concave. Pick any $\mathbf{x}_1, \mathbf{x}_2 \in \operatorname{dom}(f)$, $\lambda \in [0, 1]$, and for some $m \in \{1, \dots, N\}$, we have

$$\begin{aligned} f(\lambda \mathbf{x}_1 + (1 - \lambda) \mathbf{x}_2) &= u^{(m)}(\lambda \mathbf{x}_1 + (1 - \lambda) \mathbf{x}_2) \\ &\geq \lambda u^{(m)}(\mathbf{x}_1) + (1 - \lambda) u^{(m)}(\mathbf{x}_2) \\ &\geq \lambda \min_{i=1, \dots, N} u^{(i)}(\mathbf{x}_1) + (1 - \lambda) \min_{i=1, \dots, N} u^{(i)}(\mathbf{x}_2) \\ &= \lambda f(\mathbf{x}_1) + (1 - \lambda) f(\mathbf{x}_2). \end{aligned} \quad (34)$$

The concavity of the cost function is thus proved. \square

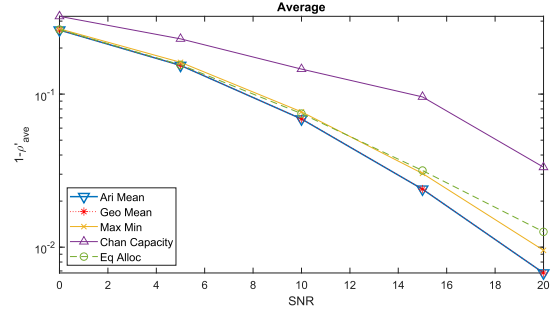


Fig. 2: Complementary values $(1 - \rho'_{ave})$ of the arithmetic mean of the success probabilities.

Based on the concavities of the cost functions, the global optima to problem (16), (26), and (31) hence can be found by using numerical programming tools respectively.

V. SIMULATIONS

This section presents simulation results of the proposed power allocation schemes for PNC in MWRC. BPSK signaling is adopted at both the relay and the user terminals, whose maximum transmitting power is normalized to $P_T = 1$. The network has 1 relay and 4 user terminals, both equipped with 6 antennas, i.e., $M = K = 6$. We assume the various radio links to be Rayleigh fading, i.e., the entries of the channel matrix \mathbf{H} are modeled as independent complex circular Gaussian random variables with zero mean and unit variance. The noise variance at receiving antennas is adjusted to obtain the desired SNR level. To simplify the discussion and minimize the effects of indirect factors in the performance comparison among the different estimation schemes, we consider uncoded systems as in, e.g., [16]. Five power allocation schemes are implemented for comparison in the following experiments, i.e., the arithmetic mean as in (16), the geometric mean as in (26), the maximin as in (31), the conventional channel capacity as in (12), and the equal power allocation where the total transmitting power is equally distributed to all signals.

A. Average Success Probabilities

In Fig. 2, we present the test result of the proposed power allocation scheme of (16). The experiment evaluates the average probability ρ'_{ave} among all user terminals, indicating the system's average reliability of successfully receiving the network codes in the downlink phase. Fig. 2 compares the complementary values of the arithmetic mean of the success probability at all terminals under five power allocation schemes. Based on the result, we see that the proposed arithmetic mean scheme has the best performance among all in this case. The geometric mean scheme has a slight disadvantage to the arithmetic mean scheme. The maximin, the equal power allocation, and conventional channel capacity schemes have obvious disadvantages in such a scenario. The result herein shows the effectiveness of the proposed arithmetic mean scheme in improving the average reception capability of the system.

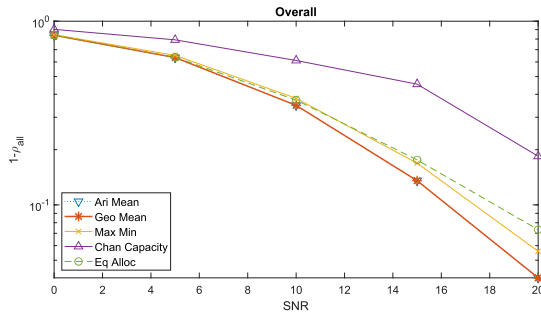


Fig. 3: Complementary values ($1 - \rho_{all}$) of the geometric mean of the success probabilities.

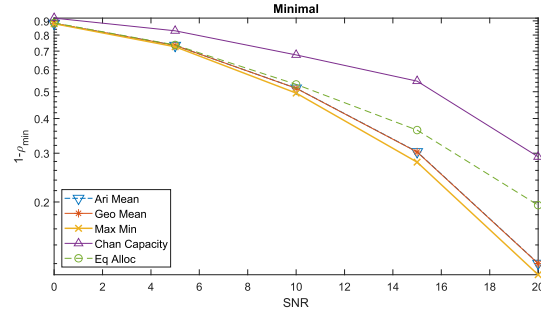


Fig. 4: Complementary values ($1 - \rho_{min}$) of the minimal success probability.

B. Overall Success Probabilities

In Fig. 3, we present the test result of the proposed power allocation scheme of (26). This experiment evaluates the overall success probability of all user terminals, indicating the system's capability of correctly receiving every network code at every user terminal in the downlink phase. Fig. 3 compares the complementary values of the geometric mean of the success probability when all five power allocation schemes are implemented. From the result, we observe that the proposed geometric mean scheme has a slight advantage over the arithmetic mean scheme while both schemes have obvious advantage over the rest schemes. The proposed geometric mean scheme hence improves the overall reception capability.

C. Minimal Success Probabilities

In Fig. 4, we present the test result of the proposed power allocation scheme of (31). In this experiment, the scenario is considered for a system where the worst user reception capability is critical, e.g., a collaborative file sharing process where the worst node in the network slows down the overall progress. Fig. 4 shows the complementary values of the lowest success probability of the terminal in the user group under five power allocation schemes. Comparing to all other schemes, we observe that the maximin scheme provides the best success probability for the worst user terminal in this case. This demonstrates the effectiveness of the proposed maximin scheme in helping enhancing the worst reception capability of the user terminals in the system.

VI. CONCLUSION

In this paper, we propose a novel power allocation scheme for PNC in downlink MWRC. The power allocation is formulated as a constrained optimization problem, maximizing the success probability under a total power constraint when using Babai estimation for signal detection. To meet the different requirements for transmission quality in applications, we consider the arithmetic mean, the geometric mean, and the maximin of success probability over the participating user terminals. The constrained optimization is formulated for each measure and the concavity of the objective is demonstrated. The performance of the proposed power allocation schemes for downlink PNC in MWRC is evaluated by means of computer simulations. The results demonstrate the effectiveness of the proposed schemes in improving the success probability in the reception of a chain of network codes.

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