

# Group-based linear parallel interference cancellation for DS-CDMA systems

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**Abstract**—The increase in the demand for voice and data wireless services creates a need for a more efficient use of the available bandwidth. Current and future generations cellular systems based on DS-CDMA are known to be interference-limited. Several approaches exist to mitigate the multiple access interference including multiuser detection (MUD). Group-based techniques have been proposed to reduce the complexity of the MUD and have been shown to provide a performance-complexity tradeoff between match filtering and full MUD. In this work, we propose to reduce the inter-group interference (IGI), a limiting factor in group-based systems, using linear parallel interference cancellation (PIC). The complete equivalent matrix filter is derived and conditions for its convergence are discussed. The numerical results show that the proposed technique is effective against IGI, at a reduced computational cost.

## I. INTRODUCTION

Most of the current and future cellular wireless systems based on direct-spread code-division multiple access (DS-CDMA) are interference-limited. Several techniques exist for interference reduction; in particular for DS-CDMA systems, where multiple access interference (MAI) is known to limit the system capacity, multiuser detection (MUD) and beamforming (BF) with antenna arrays have been widely studied [1]–[3].

Optimal MUD takes the form of trellis decoding and is very complex due to the size of the search space which increases exponentially with the number of users and sequence length [4]. Solving with the Viterbi algorithm would represent a considerable challenge for real-time operations. Several reduced complexity suboptimal techniques for MUD have been proposed, including linear filtering approaches [5], and iterative techniques [6], [7].

Alternatively, to reduce the complexity of the MUD and at the same time reduce the co-channel interference, it has been proposed in [8], [9] to cluster users in mutually exclusive groups of spatial equivalence. The data symbols from each group are jointly detected using reduced dimension MUD while the inter-group interference (IGI), i.e. the interference created by the users outside of the group of interest, is reduced by using spatial filtering or *beamforming*, a concept illustrated in Fig. 1. The total complexity associated to the reduced dimension multiuser detectors is potentially significantly smaller than the full MUD complexity.

In the existing literature on group-based MUD, beamforming is generally used for separating the signal among the groups [10]–[13]. As indicated in [13], BF reduces the dimen-

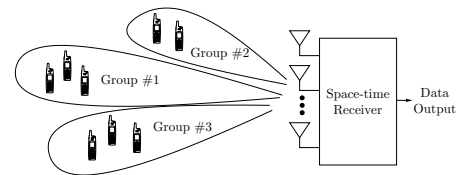


Fig. 1: Group-based space-time multiuser detection conceptual diagram.

sion of the observation space seen by the MUD filters. Thus the number of degrees of freedom for designing the filters is decreased, reducing at the same time their effectiveness. In this work, we propose to accomplish IGI reduction using parallel interference cancellation (PIC) among the groups, after MUD. The approach we propose provides significant advantages over the existing techniques. In particular, the independent BF units that are used for signal separation (e.g.: [10]) are removed, reducing the numerical complexity of the design. Also, since IGI is to be reduced at each PIC iteration, it can be ignored in the design of the MUD filter. Finally, the performance of this new structure is shown to be very close to that of the full MUD algorithm, at half its computational cost.

The remainder of the paper is organized as follows. Background information and system model are presented in Section II. In Section III the new structure is developed. Numerical simulation results are shown in Section IV, and finally some conclusions are drawn in Section V.

## II. BACKGROUND

### A. Signal model

Consider the uplink of a synchronous DS-CDMA communication system with  $K$  users transmitting blocks of  $N$  information symbols simultaneously through a dispersive channel to a common multi-antenna receiver. At each antenna, the received signal is converted to baseband, matched filtered to the transmission pulse and sampled at the “chip” rate of  $1/T_c$ , where  $T_c$  denotes the chip duration. The observed signal at the receiver therefore consists of a complex-valued vector of length  $NQ+W-1$ , where  $Q = T_s/T_c$  is the symbol expansion factor (or spreading factor),  $T_s$  is the symbol duration, and  $W$  is the finite impulse response channel length in units of  $T_c$ .

Let  $M$  denote the number of antennas and  $\mathbf{x}^{(m)} \in \mathbb{C}^{NQ+W-1}$  for  $m = 1, \dots, M$ , be the received signal vector for the  $m^{\text{th}}$  antenna element. Following the linear model described in [5], it is convenient to represent the complete set of

observations in vector form as  $\mathbf{x} = \text{vec}([\mathbf{x}^{(1)} \dots \mathbf{x}^{(M)}]^T) \in \mathbb{C}^{M(NQ+W-1)}$ , where  $T$  denotes matrix transposition and  $\text{vec}(\cdot)$  is an operation that sequentially concatenates the columns of a matrix into a column vector of appropriate dimension. Similarly, the vector of  $NK$  information symbols transmitted by the  $K$  users can be represented in vector form as  $\mathbf{d} = \text{vec}([\mathbf{d}^{(1)} \dots \mathbf{d}^{(K)}]^T) \in \mathcal{A}^{NK}$ , where  $\mathbf{d}^{(k)} \in \mathcal{A}^N$  is the vector of information symbols for user  $k$  and  $\mathcal{A}$  is the symbol alphabet of  $N_{\mathcal{A}}$  elements (e.g.: for BPSK  $\mathcal{A} = \{\pm 1\}$ ). The symbols are assumed to be independent, identically distributed and normalized such that  $E[\mathbf{d}\mathbf{d}^H] = \mathbf{I}_{NK}$ , where  $H$  represents Hermitian transposition,  $\mathbf{I}_N$  is the identity matrix of dimension  $N$  and  $E$  denotes statistical expectation.

Let  $\mathbf{v}_k \in \mathbb{C}^{M(Q+W-1)}$  be the  $k^{\text{th}}$  user space-time *effective signature* vector, i.e. the space-time response to a unit pulse excitation sequence  $\delta = [1, 0, \dots, 0]$  as observed by the multi-antenna receiver after demodulation, sampling and vector formatting as described above. Define  $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_K] \in \mathbb{C}^{M(Q+W-1) \times K}$  to be the effective signature matrix for the set of  $K$  users. Then the total received vector may be conveniently expressed as

$$\mathbf{x} = \mathbf{T}\mathbf{d} + \mathbf{n}, \quad (1)$$

where  $\mathbf{T} \in \mathbb{C}^{M(NQ+W-1) \times NK}$  is a block-Toeplitz matrix. In particular, assuming a relatively short channel delay-spread so that symbols interfere only with their adjacent neighbors, i.e.  $W < Q$ , the matrix  $\mathbf{T}$  takes the special form [5]

$$\mathbf{T} = \begin{bmatrix} \mathbf{v} & & & \\ & \mathbf{v} & & \\ & & \ddots & \\ & & & \mathbf{v} \end{bmatrix} \quad (2)$$

(Note: The diagram shows a block-Toeplitz matrix with blocks of size  $MQ$  and  $(N-1)MQ$  on the diagonal.)

In this work the matrix  $\mathbf{T}$  is assumed to be known by the receiver, as it is commonly presumed. The vector  $\mathbf{n} \in \mathbb{C}^{M(NQ+W-1)}$  in (1) contains white circular complex Gaussian noise samples with covariance matrix  $E[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_{M(NQ+W-1)}$ , where  $\sigma^2$  is the noise power. The above model and ensuing results can be generalized as well to account for colored noise and the case  $W > Q$ .

### B. Space-time MUD

In multi-user detection, the symbols transmitted from all  $K$  users are jointly estimated, based on the space-time observation vector  $\mathbf{x}$ . In a linear receiver, the soft symbols estimates are obtained from the output of the estimator  $\mathbf{M} \in \mathbb{C}^{NK \times NK}$ . For BPSK, the actual symbols estimates are taken as the sign of the real part of the soft estimates, i.e.:

$$\hat{\mathbf{d}} = \text{sgn}\{\Re(\mathbf{M}^H \mathbf{y})\}, \quad \mathbf{y} = \mathbf{T}^H \mathbf{x}, \quad (3)$$

where  $\mathbf{y}$  is the match filter (MF) output,  $\text{sgn}(\cdot)$  is a function that returns the sign of its argument and  $\Re(\cdot)$  is its real part. The linear filter minimizing the mean square error  $J^o(\mathbf{M}) = E\|\mathbf{d} - \mathbf{M}^H \mathbf{y}\|^2$  can be shown to take the form

$$\mathbf{M}_o = (\mathbf{T}^H \mathbf{T} + \sigma^2 \mathbf{I})^{-1}. \quad (4)$$

The complete operation thus consists of a match filter ( $\mathbf{T}^H$ ) followed by a minimum mean square error (MMSE) filter of dimension  $NK \times NK$ . If inverted using traditional techniques, the operation has complexity order  $\mathcal{O}(K^3)$ ; a considerable difficulty for real-time operations. We shall refer to the MUD filter in (4) as the *full space-time MUD (STMUD)*.

## III. ITERATIVE GROUP-BASED STMUD

### A. Group-based STMUD

In a group-based MUD receiver, the data symbols from each of the  $G$  groups are jointly detected using a reduced dimension MUD [8]. The grouping is based on spatio-temporal correlation; users with large correlation are said to be ‘‘close’’ and are placed in the same group for better detection.

Because beamforming reduces the dimension of the observation space through a non-invertible linear transformation, we choose here to apply the group-STMUD (GRP-STMUD) weights on the complete observation vector  $\mathbf{x}$ , as illustrated in the receiver block diagram of Fig. 2. Each pre-determined

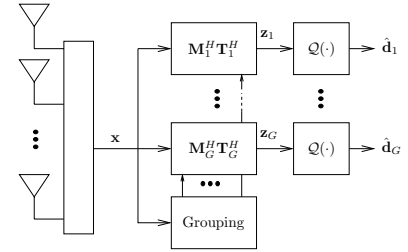


Fig. 2: Block diagram for the proposed GRP-STMUD receiver.

group has its own STMUD unit, represented by estimation matrices  $\mathbf{M}_j^H \mathbf{T}_j^H$ , and the symbols estimates are obtained through a non-linear decision device  $\mathcal{Q}(\cdot)$  (e.g. for BPSK  $\mathcal{Q}(\cdot) = \text{sgn}(\cdot)$ ).

We assume, without loss of generality, that the symbols in  $\mathbf{d}$  are ordered according to the grouping. Thus if we let  $\mathbf{d}_j$  be the  $NK_j$  symbols associated to users of group  $j$ , then we have  $\mathbf{d} = [\mathbf{d}_1^T, \dots, \mathbf{d}_G^T]^T$ , and the columns in  $\mathbf{T}$  are re-ordered accordingly. In addition, we define  $\mathbf{P}_j \in \mathbb{R}^{NK \times NK_j}$  as a matrix that selects the symbols associated to users of group  $j$  such that  $\mathbf{d}_j = \mathbf{P}_j^T \mathbf{d}$ . It is expressed as

$$\mathbf{P}_j = \begin{bmatrix} \mathbf{0}_{NK_j^- \times NK_j} \\ \mathbf{I}_{NK_j} \\ \mathbf{0}_{NK_j^+ \times NK_j} \end{bmatrix}, \quad (5)$$

where  $\mathbf{0}_{A \times B}$  is a matrix of dimension  $A \times B$  that contains only null entries,  $K_j^- \triangleq \sum_{l=1}^{j-1} K_l$  and  $K_j^+ \triangleq \sum_{l=j+1}^G K_l$  are the total number of users in the groups before and after group  $j$ , respectively. The complement of (5) is defined as  $\bar{\mathbf{P}}_j$ , so that  $\bar{\mathbf{d}}_j = \bar{\mathbf{P}}_j^T \mathbf{d}$  contains the  $N(K - K_j)$  symbols associated to the users outside of the group  $j$ . It takes the form

$$\bar{\mathbf{P}}_j = \begin{bmatrix} \mathbf{I}_{NK_j^-} & \mathbf{0}_{NK_j^- \times NK_j^+} \\ \mathbf{0}_{NK_j \times NK_j^-} & \mathbf{0}_{NK_j \times NK_j^+} \\ \mathbf{0}_{NK_j^+ \times NK_j^-} & \mathbf{I}_{NK_j^+} \end{bmatrix}. \quad (6)$$

Finally, let  $\mathbf{T}_j \triangleq \mathbf{TP}_j \in \mathbb{C}^{M(NQ+W-1) \times NK_j}$  and  $\bar{\mathbf{T}}_j \triangleq \mathbf{TP}_j^c \in \mathbb{C}^{M(NQ+W-1) \times N(K-K_j)}$  be the matrices containing the columns related to the users of group  $j$  and its complement, respectively.

To derive the group-based linear filter we begin with the group match filter output, given by  $\mathbf{y}_j = \mathbf{T}_j^H \mathbf{x}$ . Assuming a pre-determined and fixed grouping, the proposed optimal cost function for the MMSE linear estimator of group  $j$  becomes

$$J_j^o(\mathbf{M}) = E \|\mathbf{d}_j - \mathbf{M}^H \mathbf{y}_j\|^2, \quad (7)$$

where the dimension of the matrix  $\mathbf{M}$  is now  $NK_j \times NK_j$ , and the optimal MMSE linear weights for the so-called *GRP-STMUD-MMSE* receiver are obtained by solving

$$\mathbf{M}_{j,o} = \arg \min_{\mathbf{M}} J_j(\mathbf{M}). \quad (8)$$

Let  $\mathbf{R}_j \triangleq \mathbf{T}_j^H \mathbf{T}_j$  and  $\mathbf{C}_j \triangleq \mathbf{T}_j^H \bar{\mathbf{T}}_j$ , then it can be shown that the solution to the group MMSE linear weights optimality criterion of (8) is given by

$$\mathbf{M}_{j,o} = (\mathbf{R}_j \mathbf{R}_j^H + \mathbf{C}_j \mathbf{C}_j^H + \sigma^2 \mathbf{R}_j)^{-1} \mathbf{R}_j^H. \quad (9)$$

The complete group-based linear MUD filter, including the MF and the MMSE filter is given by  $\mathbf{T}_j \mathbf{M}_{j,o}$ , as shown in Fig. 2, and the soft output for group  $j$  is  $\mathbf{z}_j \triangleq \mathbf{M}_{j,o}^H \mathbf{T}_j^H \mathbf{x}$ .

The filter in (9) takes into consideration the interference from the other groups. In the context of group PIC, it is reasonable to expect that the IGI will be reduced after each step. Under this assumption, it is computationally advantageous to reduce the complexity of the filter in (9) and introduce a suboptimal filter that neglects IGI. The new cost function can be expressed as

$$\begin{aligned} J_j(\mathbf{M}) &= E \|\mathbf{d}_j - \mathbf{M}^H (\mathbf{y}_j - \mathbf{C}_j \bar{\mathbf{d}}_j)\|^2, \\ &= E \|\mathbf{d}_j - \mathbf{M}^H (\mathbf{R}_j \mathbf{d}_j + \mathbf{T}_j^H \mathbf{n})\|^2. \end{aligned} \quad (10)$$

Solving for the filter that minimizes (10) the same way as in (7)-(9), it can be shown that the new suboptimal filter becomes

$$\mathbf{M}_j = (\mathbf{T}_j^H \mathbf{T}_j + \sigma^2 \mathbf{I})^{-1}. \quad (11)$$

This linear filter, referred to here as *GRP-STMUD*, also takes the form of a MF followed by a MMSE filter of reduced dimensions  $NK_j \times NK_j$ . The cost for inverting the matrix in (11) is in the order of  $\mathcal{O}(K_j^3)$  so that the total cost becomes  $\sum_{j=1}^G \mathcal{O}(K_j^3)$  instead of  $\mathcal{O}(K^3)$  if (4) is used. Thus the proposed structure allows for potentially significant complexity reduction.

### B. Parallel interference cancellation with GRP-STMUD

Inter-group interference can be a factor for significant performance degradation in group-based receivers. The proposed parallel interference cancellation structure reduces the IGI at each stage, based on successive symbol estimates, starting with the output of the GRP-STMUD.

Let  $s$  represent the PIC stage index and define  $\hat{\mathbf{i}}_j^{(s)}$  as the inter-group interference term estimate at stage  $s$  for group  $j$ . Furthermore, let  $\mathbf{z}^{(s)} \in \mathbb{C}^{NK}$  be the vector symbol soft estimate at stage  $s$  and  $\mathbf{z}_j^{(s)} \triangleq \mathbf{P}_j^T \mathbf{z}^{(s)}$  be the symbol vector

symbol soft estimate of dimension  $NK_j$  for group  $j$ . Then the vector symbol estimate after interference cancellation for group  $j$  at stage  $s$  can be expressed for  $1 \leq s \leq S$ , where  $S$  is the total number of PIC stages, as

$$\begin{aligned} \mathbf{z}_j^{(s)} &= \mathbf{M}_j^H \mathbf{T}_j^H \mathbf{x} - \hat{\mathbf{i}}_j^{(s)} \\ &= \mathbf{M}_j^H \mathbf{T}_j^H \mathbf{x} - \mathbf{M}_j^H \mathbf{T}_j^H \bar{\mathbf{T}}_j \bar{\mathbf{z}}_j^{(s-1)} \end{aligned} \quad (12)$$

where  $\bar{\mathbf{z}}_j^{(s)} = \bar{\mathbf{P}}_j^T \mathbf{z}^{(s)}$  is the vector symbol soft estimate for the users outside of group  $j$  at stage  $s$ .

Figure 3 illustrates the *GRP-STMUD-PIC* receiver in block diagram form. As shown, the first stage takes its input,  $\mathbf{z}_j^{(0)}$  for  $j = 1, \dots, G$ , directly from the (soft) output of a GRP-STMUD receiver. So the GRP-STMUD-PIC receiver can be interpreted as an extension of the GRP-STMUD receiver shown in Fig. 2 above. In short, the  $S$ -stages GRP-STMUD-PIC receiver can be summarized by the following two equations: firstly, the soft symbol estimation update equation, given here for  $s \geq 1$  by

$$\mathbf{z}^{(s)} = \underbrace{\begin{bmatrix} \mathbf{M}_1^H \mathbf{T}_1^H \\ \vdots \\ \mathbf{M}_G^H \mathbf{T}_G^H \end{bmatrix}}_{\mathbf{F}^H} \mathbf{x} - \underbrace{\begin{bmatrix} \mathbf{M}_1^H \mathbf{T}_1^H \bar{\mathbf{T}}_1 \bar{\mathbf{P}}_1^T \\ \vdots \\ \mathbf{M}_G^H \mathbf{T}_G^H \bar{\mathbf{T}}_G \bar{\mathbf{P}}_G^T \end{bmatrix}}_{\mathbf{G}} \mathbf{z}^{(s-1)}, \quad (13)$$

with  $\mathbf{z}^{(0)} = \mathbf{F}^H \mathbf{x}$ , and secondly, the decision equation following stage  $S$ , that can be expressed as

$$\hat{\mathbf{d}} = \mathcal{Q}(\mathbf{z}^{(S)}). \quad (14)$$

In practice, the number of stages  $S$  is fixed by the hardware or determined in real-time by some convergence measure.

### C. PIC Convergence

In this section, the convergence of the GRP-STMUD-PIC is studied. It can be shown that the recursive equation (13) can be expressed compactly as

$$\mathbf{z}^{(S)} = \sum_{s=1}^S (-\mathbf{G})^{(s-1)} \mathbf{F}^H \mathbf{x}. \quad (15)$$

Because of the exponential, the structure of  $\mathbf{G}$  determines the convergence properties of (15).

The term  $\mathbf{M}_j^H \mathbf{T}_j^H \bar{\mathbf{T}}_j$  in (15) consists of the response of the interfering users effective signatures to the linear GRP-STMUD filter for group  $j$  and is referred to as the *attenuated interference* term. It can also be seen that  $\bar{\mathbf{P}}_j^T$  simply transforms the product from a  $NK_j \times N(K-K_j)$  matrix to a full  $NK_j \times NK$  matrix, by inserting columns of zeros at the location where the groups of interest's response should be located. Thus the matrix  $\mathbf{G}$  is Hermitian and takes the following form:

$$\mathbf{G} = \begin{array}{c} \begin{array}{ccc} \xrightarrow{NK_1} & \dots & \xrightarrow{NK_G} \\ \begin{array}{|c|} \hline \mathbf{0} \\ \hline \end{array} & & \\ & \ddots & \\ & & \begin{array}{|c|} \hline \mathbf{0} \\ \hline \end{array} \end{array} \end{array}. \quad (16)$$

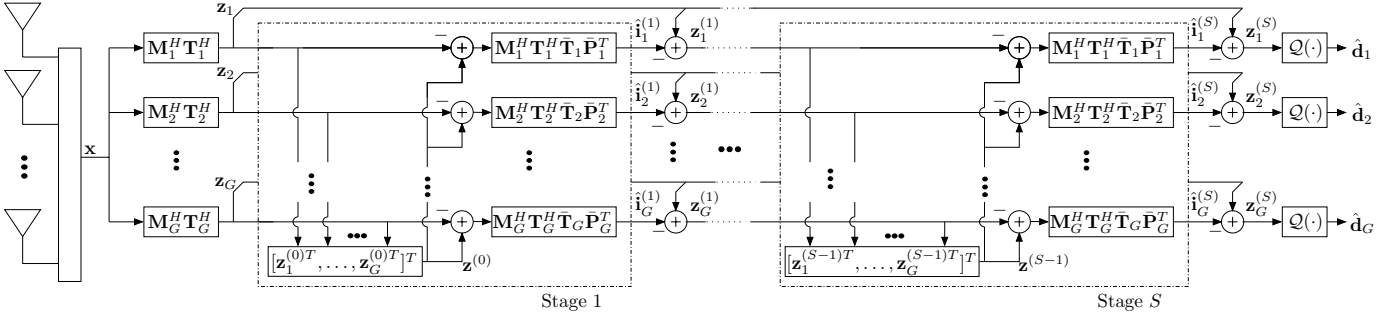


Fig. 3: Block diagram for the proposed GRP-STMUD-PIC receiver.

As  $S \rightarrow \infty$ , the convergence of the sum in (15) depends entirely on the eigenvalues of  $\mathbf{G}$ . If  $\lambda_p$  denotes the  $p^{\text{th}}$  eigenvalue of  $\mathbf{G}$ , then it can be shown that the geometric sum will converge if and only if  $|\lambda_p| < 1, \forall p$  [14].

To bound the eigenvalues in  $\mathbf{G}$ , we make use of the Geršgorin discs. According to this theorem, each eigenvalues of  $\mathbf{G}$  satisfies at least one of the inequalities

$$|\lambda - g_{pp}| \leq r_p, \quad \text{where } r_p = \sum_{\substack{q=1 \\ q \neq p}}^{NK} |g_{pq}|, \quad (17)$$

where  $g_{pq}$  is the element at position  $(p, q)$  in matrix  $\mathbf{G}$  and  $(p = 1, \dots, NK)$ . Notice that  $r_p$  can be interpreted as the sum of the attenuated interference magnitudes for a given symbol. Since  $g_{pp} = 0$  in (16), to guarantee the convergence of (15) we require  $r_p < 1, \forall p$ .

Provided that the convergence condition is met and using a known fact of geometric series for matrices (see e.g. [14]), the sum in (15) can be shown to converge as  $S \rightarrow \infty$  to

$$\mathbf{z}^{(\infty)} = (\mathbf{I} + \mathbf{G})^{-1} \mathbf{F}^H \mathbf{x}. \quad (18)$$

Asymptotically as the noise power decreases, i.e.  $\sigma^2 \rightarrow 0$ ,

$$\begin{aligned} \mathbf{F}^H \mathbf{x} &= \mathbf{F}^H \mathbf{T} \mathbf{d} \\ &= (\text{diag}(\mathbf{M}_1^H \mathbf{T}_1^H \mathbf{T}_1, \dots, \mathbf{M}_G^H \mathbf{T}_G^H \mathbf{T}_G) + \mathbf{G}) \mathbf{d}. \end{aligned} \quad (19)$$

Combining (11) with  $\sigma^2 = 0$  into (19) and replacing the result in (18), we finally find

$$\lim_{\substack{S \rightarrow \infty \\ \sigma \rightarrow 0}} \mathbf{z}^{(S)} = \mathbf{d}. \quad (20)$$

What we conclude from these observations, and particularly from the structures of (16) and (18), is that the PIC implements decorrelation among groups only, and leaves the filtering within the groups to the individual GRP-STMUD filters.

#### IV. COMPUTER EXPERIMENTS

We consider the received signal model of (1) for the uplink of a DS-CDMA system. The  $K = 12$  users have orthogonal spreading codes of length  $Q = 16$  and transmit BPSK data symbols in blocks of  $N = 50$ . The signals are received by  $M = 6$  antennas in a standard linear array configuration. The channel consists of  $W = 6$  equal power multipaths, with the

main path having DOA  $\theta_0$  uniformly distributed within the sector width of  $120^\circ$ , and all other paths uniformly distributed within  $[\theta_0 + \Delta\theta, \theta_0 - \Delta\theta]$ , with  $\Delta\theta = 30^\circ$ . The hardware resources are assumed limited so that the grouping structures can support up to  $G_{\text{max}} = 4$  groups of a maximum of  $K_{\text{max}} = 4$  users each.

#### A. Performance results

Figure 4 shows the BER for a given typical user distribution and channel condition scenario. Ideal power control is assumed and the SNR is thus given by  $P/\sigma^2$ , where  $P \equiv P_j = \|\mathbf{v}_j\|^2$  is the received power of each user. As expected, the full STMUD outperforms the other algorithms and the MF or conventional receiver performs poorly; at a BER of  $10^{-3}$ , there is a 3dB difference between the two approaches.

We also observe in this scenario an improvement over the MF of approximately 2dB at BER of  $10^{-3}$  when using the grouping approaches (without PIC). The GRP-STMUD-MMSE performs slightly better than GRP-STMUD but the difference is negligible. When using PIC, the difference at BER of  $10^{-3}$  between GRP-STMUD and the full STMUD reduces to a negligible 0.3dB, approximately.

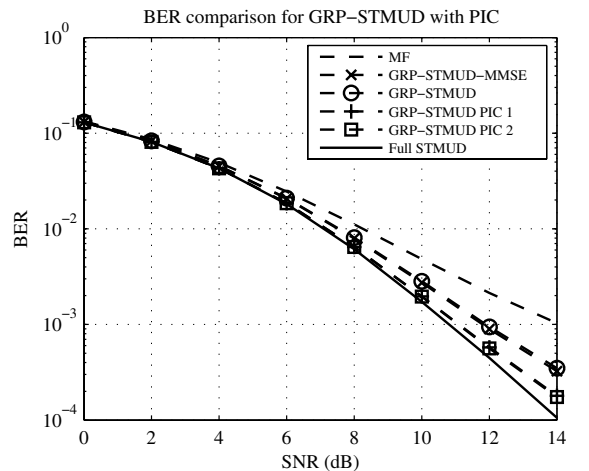


Fig. 4: BER comparison between match filter (MF), full STMUD, GRP-STMUD-MMSE, GRP-STMUD, and GRP-STMUD-PIC.

As shown in Fig. 5, only a few PIC iterations are necessary to nearly reach the lower bound for PIC (achieved by GRP-



STMUD-PIC  $\infty$ ), making this approach very appealing. Recall that the convergence rate will depend on the interference seen from each group and thus different grouping may lead to different convergence rates.

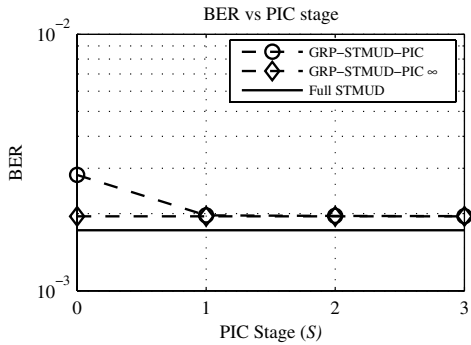


Fig. 5: BER convergence of GRP-STMUD-PIC versus  $S$  at 12dB SNR.

### B. Complexity

The expressions for the optimal MMSE linear estimators in (4), (9) and (11) include a matrix inversion and several matrix multiplications. Fortunately, the structure of the data matrix  $\mathbf{T}$  in (2) can be exploited extensively, leading to significant complexity reduction. The most important reduction results from the structure in the  $\mathbf{T}^H \mathbf{T}$  matrix product [13].

To compare the complexity between the two approaches, the number of complex floating point operations (CFLOPS) is counted for the different parts of equations (4), (11) and (13) by taking advantage of the symmetries.

The total complexity is classified in three distinct parts: overhead (the different matrix products and sums to obtain the matrices to invert in (4) and (9)), linear system solution (lss) and the cost for each stage (PIC). Figure 6 shows the numerical complexity in terms of CFLOPS for solving a system with  $N = 50$  data symbols. The STMUD hardware can support  $K = 16$  users simultaneously and the GRP-STMUD-PIC structure has  $G_{\max} = 4$  groups of  $K_{\max} = 4$  users with  $S \in \{0, 1, 2\}$  stages or iterations ( $S = 0$  corresponds to the GRP-STMUD case). The results show that the total complexity associated to the full STMUD for  $K = 12$  is approximately two times that of the proposed GRP-STMUD-PIC system with  $S = 2$ , for approximately the same BER performance.

The results also demonstrate that due to the added complexity in the system solution and overhead associated to equation (9), the GRP-STMUD-MMSE structure is more complex than GRP-STMUD. Since the BER performances are essentially the same, GRP-STMUD-MMSE is not advantageous compared to GRP-STMUD.

### V. CONCLUSION

In this work, we have proposed a new group-based space-time receiver structure for DS-CDMA systems that uses parallel interference cancellation to reduce the problematic interference among groups. We have shown that the new

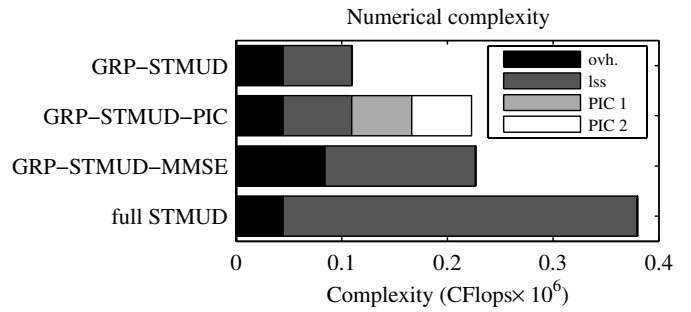


Fig. 6: Numerical complexity of the full STMUD, GRP-STMUD-MMSE and GRP-STMUD-PIC for  $K = 12$ ,  $G_{\max} = 4$ , and  $K_{\max} = 4$ .

structure provides BER performance close to the full STMUD at a fraction of the complexity.

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