Channel Estimation Using Subspace Decomposition for SC-FDMA Systems

Claude D'Amours*, Benoît Champagne†, Adel Omar Dahmane‡, and Ashraf Tahat§
*School of Electrical Engineering and Computer Science
University of Ottawa, Ottawa, Ontario, Canada, email: cdamours@uottawa.ca
†Department of Electrical and Computer Engineering
McGill University, Montreal, Quebec, Canada, email: benoit.champagne@mcgill.ca
‡Département de génie électrique et informatique
Université du Québec à Trois Rivières, Trois Rivières, Quebec, Canada, email:dahmane@uqtr.ca
§Department of Communications Engineering
Princess Sumaya University for Technology, PO Box 1438
Amman, Jordan, email: aat@ieee.org

***Abstract*—In SC-FDMA systems the bit error rate (BER) is very sensitive to channel estimation errors. We propose the use of blind (or semi-blind) subspace decomposition to estimate the channel frequency responses between the transmitter and the multiple antennas of the receiver in the uplink of a 4G system employing SC-FDMA. The proposed subspace-based channel estimation technique requires very little overhead in terms of pilot symbols dedicated to channel estimation. Furthermore, we show through simulations that, when applied to SC-FDMA, it can provide a BER performance comparable to a system with perfect channel estimates.**

***Index Terms*—SC-FDMA, Channel Estimation, Subspace Decomposition.**

I. Introduction

To provide the high data rates of the long term evolution (LTE) project, orthogonal frequency division multiple access (OFDMA) is now widely employed as it can provide high spectral efficiency by using higher order modulations over multiple narrowband subcarriers [1]. The main drawback of OFDMA is the high peak-to-average power ratio (PAR), which has motivated he development of single-carrier frequency division multiple access (SC-FDMA) systems for use in the uplink in LTE uplinks [2,3].

In OFDMA systems, a poor estimate of the channel gain on a particular carrier affects the detection of the symbol that is carried over that carrier. In SC-FDMA systems, due to the additional fast Fourier transform (FFT) at the transmitter and inverse FFT (IFFT) at the receiver, the equalizer operates in the frequency domain while the input/output data is specified in the time domain. Consequently, all symbols will be affected by a single error in the channel response at any given frequency. Therefore SC-FDMA systems are more sensitive to channel estimation errors than are OFDMA systems.

Significantly less research on channel estimation for SC-FDMA systems is available compared to that done in OFDMA. The limited work on this topic focuses mainly on using pilots in either the time or the frequency domain to estimate the frequency response of the channel [4,5]. In 4G uplinks, the use of pilots for channel estimation is proposed in which a frame made up entirely of pilots is periodically transmitted for this purpose [6]. However, in doing so, the spectral efficiency is reduced by the pilot symbol insertion ratio.

To reliably estimate the the subcarrier channel gains without a corresponding loss in spectral efficiency, one can use blind (or semi-blind) techniques, such as subspace decomposition [7]. While subspace decomposition may entail a higher computational complexity, little pilot data is required resulting in increased spectral efficiency. For multiple receive antenna systems, the correlation matrix of the observation is sufficient to determine the channel impulse responses between all transmit-receive antenna pairs, up to a constant [7,8]. This ambiguity can be resolved by the insertion of a small number of known data points, resulting in a semi-blind approach. While there has been many works on the use of subspace methods for blind channel estimation in the context of OFDM, to the best of our knowledge, these techniques have not yet been considered for channel estimation purposes in SC-FDMA systems.

In this letter, we propose the use of subspace-based channel estimation for a 4G wireless uplink employing SC-FDMA, where multiple users equipped with a single transmit antenna forward their signals to a base station (BS) equipped with multiple antennas. We develop the necessary formalism for the application of the subspace decomposition to the correlation matrix of the data array and show that the channel estimate reduces to the optimization of a simple quadratic form. We also propose a practical recursive scheme for the estimation of the required correlation matrix. We emphasize that since SC-FDMA transmission is proposed for the LTE uplink, the increased computational complexity is relegated to the BS, thus its use does not result in any increased complexity for the mobile units. In numerical simulations, the proposed subspace-based estimation approach provides a BER performance comparable to a system using perfect channel estimates.

The rest of this letter is organized as follows: Section II presents the multi-antenna SC-FDMA system. Subspace decomposition for channel estimation in such systems is presented in Section III. The performance of the channel

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estimation technique in terms of BER and normalized mean square error (NMSE) is investigated in Section IV. In Section V we present our conclusions.

II. SC-FDMA System Model

The block diagrams of an SC-FDMA transmitter and teh corresponding multiantenna receiver is shown in Fig. 1. For a K user system, let

$$\mathbf{x}_k^{(i)} = [x_k^{(i)}(0), \cdots, x_k^{(i)}(N_1-1)]^T \tag{1}$$

denote the data transmitted by user k on the ith frame, where $x_k^{(i)}(n)$ is the nth independent identically distributed (iid) MQAM symbol with zero mean and unit variance and N_1 is the frame length. Let

$$\mathbf{X}_k^{(i)} = [X_k^{(i)}(0), \cdots, X_k^{(i)}(N_1-1)]^T \tag{2}$$

be the N_1-point FFT of $\mathbf{x}_k^{(i)}$. The entries of $\mathbf{X}_k^{(i)}$ are then mapped to N_1 of N_2 different subcarriers, where $N_2 > N_1$. This is equivalent to inserting $N_2 - N_1$ zeros into $\mathbf{X}_k^{(i)}$ to create an N_2 point FFT

$$\mathbf{S}_k^{(i)} = [S_k^{(i)}(0), \cdots, S_k^{(i)}(N_2-1)]^T. \tag{3}$$

This signal is then transformed back to the time domain by taking the N_2 point IFFT, resulting in

$$\mathbf{s}_k^{(i)} = [s_k^{(i)}(0), \cdots, s_k^{(i)}(N_2-1)]^T. \tag{4}$$

Finally, vector $\mathbf{s}_k^{(i)}$ is converted to serial format and a cyclic prefix (CP) is added prior to its transmission.

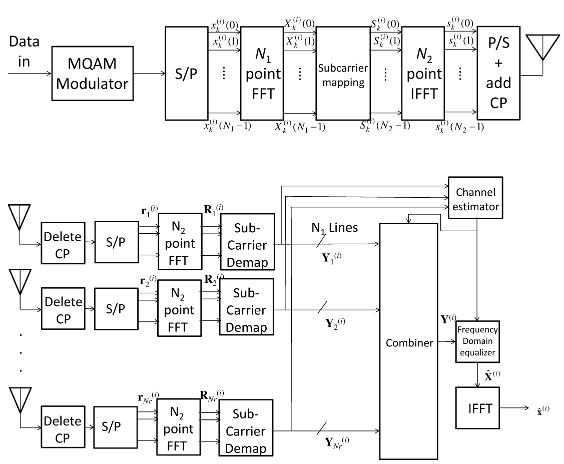

Fig. 1. Block diagram of SC-FDMA transmitter and multiantenna receiver.

In SC-FDMA, multiple signals can be separated at the receiver by performing subcarrier mapping so that a subcarrier is assigned to only one user. The total number of subcarriers, $N_2 = \sum_{k=1}^{K} N_{1,k}$, where $N_{1,k}$ is the number of subcarriers assigned to user k. In this paper we assume all users are assigned the same number of subcarriers, therefore $N_{1,k} = N_1$ for all k and we let $m_{k,n} \in \{0, 1, \ldots, N_2-1\}$ denote the subcarrier index to which $X_k^{(i)}(n)$ is mapped.

The received signals consist of the superposition of the transmissions from the K users. For each antenna, following demodulation and sampling, the CP is discarded and the remaining symbols are serial-to-parallel converted. The ith received data block on antenna j is

$$\mathbf{r}_j^{(i)} = [r_j^{(i)}(0), \cdots, r_j^{(i)}(N_2-1)]^T = \sum_{k=1}^{K} (\mathbf{s}_k^{(i)} \circledast \mathbf{h}_{kj}^{(i)}) + \mathbf{z}_j^{(i)}, \tag{5}$$

where $\circledast$ denotes circular convolution, $\mathbf{h}_{kj}^{(i)} = [h_{kj,0}^{(i)}, \cdots, h_{kj,M}^{(i)}]^T$ is the impulse response of the wireless channel between the kth user's transmit antenna and the receiver's jth antenna, M is the channel order and $\mathbf{z}_j^{(i)} = [z_j^{(i)}(0), \cdots, z_j^{(i)}(N_2-1)]^T$ is the noise vector on the j antenna during block interval i. The noise is zero mean circularly symmetric white Gaussian with variance σ_z^2; therefore $E[\mathbf{z}_j^{(i)} \mathbf{z}_j^{(i)H}] = \sigma_z^2 \mathbf{I}$.

The N_2-point FFT of $\mathbf{r}_j^{(i)}$ is denoted as

$$\mathbf{R}_j^{(i)} = [R_j^{(i)}(0), \cdots, R_j^{(i)}(N_2-1)]^T = \sum_{k=1}^{K} diag(\mathbf{H}_{kj}^{(i)}) \mathbf{S}_k^{(i)} + \mathbf{Z}_j^{(i)} \tag{6}$$

where $\mathbf{H}_{kj}^{(i)}$ and $\mathbf{Z}_j^{(i)}$ are the N_2 point FFTs of $\mathbf{h}_{kj}^{(i)}$ and $\mathbf{z}_j^{(i)}$, respectively, and $\mathrm{diag}(\mathbf{H}_{kj}^{(i)})$ is a $N_2 \times N_2$ diagonal matrix with the entries of $\mathbf{H}_{kj}^{(i)}$ along its main diagonal. Examining a particular entry from $\mathbf{R}_j^{(i)}$, say $R_j^{(i)}(m) = \sum_{k=1}^{K} S_k^{(i)}(m) H_{kj}^{(i)}(m) + Z_j^{(i)}(m)$, we note that $S_k^{(i)}(m)$ is zero for all values of k except for the index of the user to which subcarrier m is assigned. Therefore, the receiver can separate each user's contribution to $\mathbf{R}_j^{(i)}$, producing $\mathbf{Y}_{kj}^{(i)} = [Y_{kj}^{(i)}(0), \cdots, Y_{kj}^{(i)}(N_1-1)]^T$ for $k \in \{1, 2, \cdots K\}$ where $Y_{kj}^{(i)}(n) = X_k^{(i)}(n) H_{kj}^{(i)}(m_{k,n}) + Z_j^{(i)}(m_{k,n})$.

For each user, the channel gains are estimated and the FFTs on each antenna are combined. Let $\hat{H}_{kj}^{(i)}(m_{k,n})$ denote the receiver's estimate of $H_{kj}^{(i)}(m_{k,n})$. The output of the combiner for user k is

$$\mathbf{Y}_k^{(i)} = [Y_k^{(i)}(0), \cdots, Y_k^{(i)}(N_1-1)]^T \tag{7}$$

where

$$Y_k^{(i)}(n) = \sum_{j=1}^{N_r} Y_{kj}^{(i)}(n) \hat{H}_{kj}^{(i)}(m_{k,n})^*. \tag{8}$$

An estimate of the FFT of the transmitted block of symbols,

$$\hat{\mathbf{X}}_k^{(i)} = [\hat{X}_k^{(i)}(0), \cdots, \hat{X}_k^{(i)}(N_1-1)]^T \tag{9}$$

is then obtained by applying a frequency domain equalizer (FDE), where

$$\hat{X}_k^{(i)}(n) = \frac{Y_k^{(i)}(n)}{\sum_{j=1}^{N_r} |\hat{H}_{kj}^{(i)}(m_{k,n})|^2 + \sigma_z^2} \tag{10}$$

Assuming perfect channel estimates, [4] shows that (10) minimizes the mean square error between $X_k^{(i)}(n)$ and $\hat{X}_k^{(i)}(n)$. The estimate of each user's ith data block is then obtained by taking the IFFT of $\hat{\mathbf{X}}_k^{(i)}$. Should the channel estimator produce a poor estimate of $H_{kj}^{(i)}(m_{k,n})$, the error is then spread to all data symbols through the IFFT function.

III. Subspace-Based Channel Estimation

The N_1-point IFFT of $\mathbf{Y}_{kj}^{(i)}$ is given by

$$\mathbf{y}_{kj}^{(i)} = \mathbf{x}_k^{(i)} \circledast \bar{\mathbf{h}}_{kj}^{(i)} + \bar{\mathbf{z}}_{kj}^{(i)}, \tag{11}$$

where $\bar{\mathbf{h}}_{kj}^{(i)} = [\bar{h}_{kj}(0)^{(i)} \cdots \bar{h}_{kj}(M_{\text{eff}})^{(i)}]^T$ and $\bar{\mathbf{z}}_{kj}^{(i)} = [\bar{z}_{kj}(0), \cdots, \bar{z}_{kj}(N_1-1)]^T$ are the equivalent impulse response between the kth user that the receiver's jth antenna and the equivalent received noise vector on antenna j affecting the kth user's signal respectively and M_{eff} is the effective channel order. Their N_1-point IFFTs are $\bar{\mathbf{H}}_{kj}^{(i)} = [H_{kj}^{(i)}(m_{k,0}), \cdots, H_{kj}^{(i)}(m_{k,N_1-1})]^T$ and $\bar{\mathbf{Z}}_{kj}^{(i)} = [Z_j^{(i)}(m_{k,0}), \cdots, Z_j^{(i)}(m_{k,N_1-1})]^T$ respectively. It can be shown that the variance of $\bar{z}_{kj}(n)$ is

$$\bar{\sigma}_z^2 = \frac{N_2}{N_1}\sigma_z^2. \tag{12}$$

In distributed SC-FDMA systems, each user's subcarriers are uniformly spaced throughout the N_2 subcarriers, i.e. $m_{k,n} = m_{k,n-1}+K$. In this case $\bar{\mathbf{h}}_{kj}^{(i)}$ is simply a frequency modulated version of $\mathbf{h}_{kj}^{(i)}$ and therefore has order M provided $M < N_1$. When the subcarriers are not uniformly spaced, $\bar{\mathbf{h}}_{kj}^{(i)}$ has N_1 non-zero elements in general. However, the effective order of this impulse response is $M_{\text{eff}} < M$ as most of the elements in $\bar{\mathbf{h}}_{kj}^{(i)}$ are close to zero in magnitude for $n > M_{\text{eff}}$ [9]. Accordingly, we assume that $\bar{\mathbf{h}}_{kj}^{(i)}$ is a $M \times 1$ vector.

For each user on each antenna, let $L < N_1$ be the length of data blocks used for the purpose of channel estimation, which we define as $\mathbf{a}_{kj}^{(i)}(n) = [y_{kj}^{(i)}(n), \cdots, y_{kj}^{(i)}(n-L+1)]^T$. Assuming that the channel varies slowly enough for it to be considered static over one data frame, then

$$\mathbf{a}_{kj}^{(i)}(n) = \mathcal{H}_{kj}^{(i)}\mathbf{x}_k^{(i)}(n) + \mathbf{w}_{kj}^{(i)}(n) \tag{13}$$

where $\mathbf{w}_{kj}^{(i)}(n) = [\bar{z}_{kj}^{(i)}(n), \cdots, \bar{z}_{kj}^{(i)}(n-L+1]^T$ and $\mathcal{H}_{kj}^{(i)}$ is an $L \times (L+M)$ matrix given by

$$\mathcal{H}_{kj}^{(i)} = \begin{bmatrix} \bar{\mathbf{h}}_{kj}^{(i)} & 0 & 0 & \cdots & 0 \\ 0 & \bar{\mathbf{h}}_{kj}^{(i)} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \bar{\mathbf{h}}_{kj}^{(i)} \end{bmatrix} \tag{14}$$

Next, let us define the kth user's data matrix $\mathbf{A}_k^{(i)} = [\mathbf{A}_{k1}^{(i)} \; \mathbf{A}_{k2}^{(i)} \cdots \; \mathbf{A}_{kN_r}^{(i)}]$ where the $(N_1-L+1) \times L$ matrix $\mathbf{A}_{kj}^{(i)H} = [\mathbf{a}_{kj}^{(i)}(L) \;\; \mathbf{a}_{kj}^{(i)}(L+1) \;\; \cdots \;\; \mathbf{a}_{kj}^{(i)}(N_1)]$. The sample correlation matrix of block i, $\mathbf{\Phi}_k^{(i)}$ is then obtained by

$$\mathbf{\Phi}_k^{(i)} = \mathbf{A}_k^{(i)H}\mathbf{A}_k^{(i)}/(N_1-L+1). \tag{15}$$

It is shown in [10,11] that $\mathbf{\Phi}_k^{(i)}$ is an unbiased estimator of the ensemble averaged correlation matrix, i.e. $E[\mathbf{\Phi}_k^{(i)}] = \mathcal{R}_k^{(i)} = \mathcal{H}_k^{(i)}\mathcal{H}_k^{(i)H} + \bar{\sigma}_z^2\mathbf{I}_{N_rL}$ where $\mathcal{H}_k^{(i)} = [\mathcal{H}_{k1}^{(i)T} \; \mathcal{H}_{k2}^{(i)T} \; \cdots \; \mathcal{H}_{kN_r}^{(i)T}]^T$. The signal induced component of $\mathcal{R}_k^{(i)}$ is $\mathcal{H}_k^{(i)}\mathcal{H}_k^{(i)H}$ which, assuming that $N_rL > L+M$ and $\mathcal{H}_k^{(i)}$ is full column rank, has rank $L+M$ while the noise induced part of $\mathcal{R}_k^{(i)}$ is $\bar{\sigma}_z^2\mathbf{I}_{N_rL}$.

Let $\lambda_{k0}^{(i)} \le \lambda_{k1}^{(i)} \le \cdots \le \lambda_{kN_r-1}^{(i)}$ be the N_rL eigenvalues of $\mathcal{R}_k^{(i)}$ with corresponding normalized eigenvectors defined as $\mathbf{q}_{kl}^{(i)}$. Since the rank of $\mathcal{H}_k^{(i)}\mathcal{H}_k^{(i)H}$ is less than the rank of $\mathcal{R}_k^{(i)}$, we can divide these eigenvalues into two groups [10]:

1. $\lambda_{kl}^{(i)} = \sigma_z^2, \quad 0 \le l \le D_n - 1,$
2. $\lambda_{kl}^{(i)} > \sigma_z^2, \quad D_n \le l \le N_rL - 1.$

where we define $D_n = N_rL - M - L$. Accordingly the space spanned by the eigenvectors of $\mathcal{R}_k^{(i)}$ can be separated into two subspaces, namely: the noise subspace is spanned by the eigenvectors associated to the first group and the signal subspace spanned by those associated to the second group.

For $l = 0, 1, \cdots, D_n - 1$ we can express a $N_rL \times 1$ eigenvector from the noise subspace as $\mathbf{q}_{kl}^{(i)} = [\mathbf{q}_{kl1}^{(i)} \; \cdots \; \mathbf{q}_{klN_r}^{(i)T}]^T$ where $\mathbf{q}_{klj}(i) = [q_{klj,0}^{(i)} \; q_{klj,1}^{(i)} \; \cdots \; q_{klj,L-1}^{(i)}]$. Let $\mathcal{G}_{kl}^{(i)} = [\mathbf{G}_{kl1}^{(i)T} \; \cdots \; \mathbf{G}_{klN_r}^{(i)T}]^T$, where $\mathbf{G}_{klj}^{(i)}$ is the $N_1 \times (N_1+L-1)$ matrix given by:

$$\mathbf{G}_{klj}^{(i)} = \begin{bmatrix} q_{klj,0}^{(i)} & \cdots & q_{klj,L-1}^{(i)} & 0 & \cdots & 0 \\ 0 & q_{klj,0}^{(i)} & \cdots & q_{k0j,L-1}^{(i)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{klj,0}^{(i)} & \cdots & q_{klj,L-1}^{(i)} \end{bmatrix} \tag{16}$$

Also let $\bar{\mathbf{h}}_k^{(i)} = [\bar{\mathbf{h}}_{k1}^{(i)T} \; \cdots \; \bar{\mathbf{h}}_{kN_r}^{(i)T}]^T$. Following [10], it can be shown that $\bar{\mathbf{h}}_k^{(i)H}\mathcal{G}_{kl}^{(i)}\mathcal{G}_{kl}^{(i)H}\bar{\mathbf{h}}_k^{(i)} = 0$ for $l = 0, 1, \cdots D_n - 1$. Therefore $\bar{\mathbf{h}}_k^{(i)H}\mathcal{Q}_k^{(i)}\bar{\mathbf{h}}_k^{(i)} = 0$, where $\mathcal{Q}_k^{(i)} = \sum_{l=0}^{D_n-1}\mathcal{G}_{kl}^{(i)}\mathcal{G}_{kl}^{(i)H}$.

In practice, only estimates of $\mathcal{Q}_k^{(i)}$ and $\mathcal{R}_k^{(i)}$, respectively denoted as $\hat{\mathcal{Q}}_k^{(i)}$ and $\hat{R}_k^{(i)}$, are available for performing the channel estimation. In particular, we may use $\mathbf{\Phi}_k^{(i)}$ to approximate $\mathcal{R}_k^{(i)}$; however, this approximation may not be accurate enough due to insufficient averaging to reduce the effect of noise. Therefore, we propose to find $\hat{R}_k^{(i)}$ by employing time-averaging over a limited number of data frames, where an exponential window is used to account for the time-varying nature of the unknown channels. Accordingly, we let $\hat{\mathcal{R}}_k^{(0)} = \mathbf{\Phi}_k^{(0)}$ and

$$\hat{\mathcal{R}}_k^{(i)} = \mu\mathbf{\Phi}_k^{(i)} + (1-\mu)\hat{\mathcal{R}}_k^{(i-1)}, \tag{17}$$

where $0 < \mu \le 1$. The eigenvectors associated with the D_n smallest eigenvalues of $\hat{\mathcal{R}}_k^{(i)}$ are then selected to approximate the spanning vectors of the noise subspace of $\mathcal{R}_k^{(i)}$. This step implies that the receiver knows the effective channel order, M_{eff}. Methods for determining M_{eff} are discussed in [11]. We employ these eigenvectors in (16) to produce the desired estimate.

Next, let $\hat{\mathbf{h}}^{(i)}$ be the vector that minimizes the cost function $\mathcal{E}[\mathbf{h}] = \mathbf{h}^H\hat{\mathcal{Q}}_k^{(i)}\mathbf{h}$ subject to $\|\hat{\mathbf{h}}^{(i)}\|^2 = 1$. This is equivalent to finding the eigenvector associated to the smallest eigenvalue of $\hat{\mathcal{Q}}_k^{(i)}$. It can be shown that when $\hat{\mathcal{Q}}_k^{(i)} = \mathcal{Q}_k^{(i)}$ then $\hat{\mathbf{h}}^{(i)} = c_k(\mathbf{h}_k^{(i)}$, where $\mathbf{h}_k^{(i)}$ is the kth user's desired channel impulse response and c_k is a complex scalar representing the ambiguity of the kth user's channel estimate. The actual channel estimate is produced by resolving the ambiguity, i.e. $\hat{\mathbf{h}}_k^{(i)} = \alpha_k \hat{\mathbf{h}}^{(i)}$ where here α_k is chosen to minimize the mean square error between $\hat{\mathbf{h}}_k^{(i)}$ and $\mathbf{h}_k^{(i)}$. In practice, the ambiguity can be resolved by inserting one or more known symbols into the transmitted data frame [12]. In this respect, we emphasize that the pilot symbol insertion rate of the proposed approach is much lower than that of a non-blind, training based estimator, resulting in increased spectral efficiency. Estimates of $\bar{\mathbf{h}}_{kj}^{(i)}$ are then extracted from $\hat{\mathbf{h}}_k^{(i)}$, their N_1-point FFTs are computed and then they are used in the combining and equalization process discussed in Section II.

IV. Simulation Results

We use Monte Carlo simulations to evaluate the performance of the proposed method in terms of normalized mean square error (NMSE) and bit error rate (BER) where here the NMSE is defined as $\sum_{n=0}^{N_1-1} |\hat{H}_{kj}^{(i)}(m_{k,n}) - H_{kj}^{(i)}(m_{k,n})|^2 / \sum_{n=0}^{N_1-1} |H_{kj}^{(i)}(m)|^2$. We consider the performance of our system operating on two different frequency selective Rayleigh fading channels:

- CH1: 5 resolvable paths of equal strength
- CH2: 5 resolvable paths with power profile of [0dB, -1dB, -2dB, -3dB , -4dB].

Additionally the channel models used in the simulation are quasi-static and remain invariant for the duration of one FFT frame. The time evolution of $\mathbf{h}_k^{(i)}$ from one frame to the next is controlled by a first order autoregressive model with fading rate $B_d T = 0.025$, where B_d is the Doppler spread and T is the data block interval.

The SC-FDMA system under study uses 16QAM modulation with the following parameter values: $K = 2$ users, $N_r = 4$ receive antennas, FFT lengths $N_1 = 64$ and $N_2 = 128$, uniform carrier spacing, CP length set to 8, $L = 6$. We also define E_b/N_o as the energy per bit to single sided noise spectral density ratio. We assume that ambiguity and channel order determination are performed perfectly.

Fig. 2 shows the BER versus E_b/N_o for the different channels under consideration and for different values of the parameter μ in the recursive computation of $\hat{\mathcal{R}}_k^{(i)}$. This is compared to the ideal case where the channel gains are known to the receiver. We see that the SC-FDMA system employing subspace decomposition for blind channel estimation performs to within roughly 1.2 and 1.7 dB of the SC-FDMA system with perfect channel knowledge at a BER of 10^{-3} when operating on CH1 and CH2 respectively. The BER of 10^{-3} is achieved at the lowest E_b/N_o when $\mu = 0.4$ on both channels. However the blind channel estimation technique improves spectral efficiency by not requiring as many additional pilot symbols as a training-based channel estimator. These additional pilot symbols can increased the channel load by up to 15% [3].

Next we investigate the NMSE performance of the proposed blind estimator under simliar conditions. We note that from Fig. 3 the system with the lowest NMSE does not necessarily provide the lowest BER. For example, from Fig. 2 we see that $\mu = 0.4$ has the lowest BER at $E_b/N_o = 14$dB on both channels while from Fig. 3 $\mu = 0.6$ provides the lowest NMSE at the same E_b/N_o. Channel estimation error can be divided into noise error and lag error. Higher values of μ experience higher noise errors due to insufficient averaging, while lower values of μ suffer from lag errors. The results show that it is sometimes favorable in terms of BER to trade noise error for an equivalent or slightly greater amount of lag error.

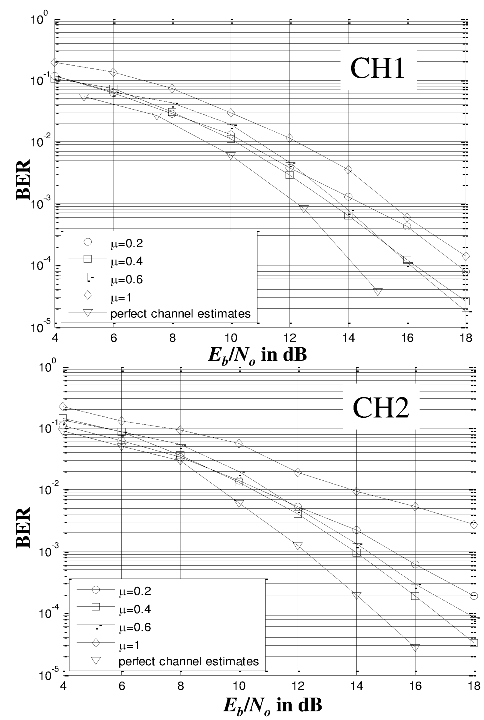

Fig. 2. BER of proposed SC-FDMA system operating on CH1 and CH2.

V. Conclusions

We proposed the use of subspace decomposition with recursive covariance matrix estimation for blind (or semi-blind) channel estimation in SC-FDMA uplink transmissions with multiple receive antennas. Specifically, the proposed technique requires the transmission of much less pilot symbols compared to a training-based channel estimation approach. For the cases presented in the paper we show that, at a BER of 10^{-3}, the proposed SC-FDMA receiver with blind channel estimation experiences a 1dB power loss compared to SC-FDMA systems with perfect channel estimates. However, a full-fledged training-based estimation approach requires that some power be allocated to pilot symbols, which is reflected in increased energy per bit. For instance, a typical 15% pilot insertion rate translates into a 0.7 dB loss compared to the ideal case. Therefore our proposed system will have similar BER performance compared to a training based system while having

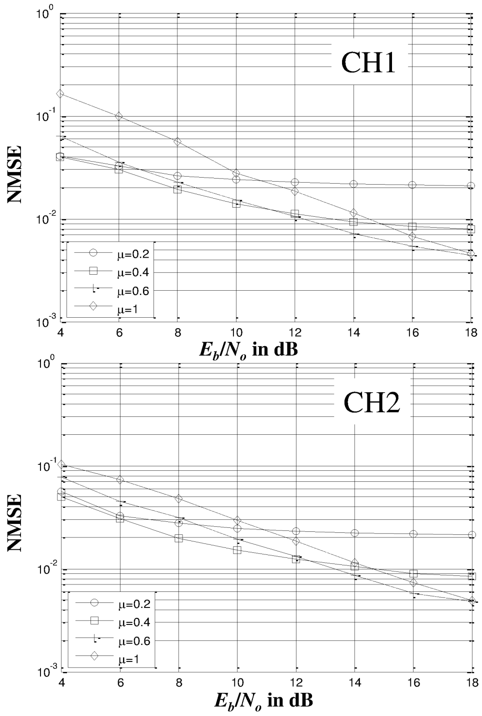

Fig. 3. NMSE of channel estimates of proposed system operating on CH1 and CH2.

an improved spectral efficiency. We additionally demonstrated that the technqiue can estimate the unknown channel frequency responses with NMSEs below 1% with E_b/N_o around 12dB.

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