Improved multiuser blind CCD-based vector channel estimation in colored noise

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Abstract— **In this paper, we propose and study a generalized criterion for the canonical correlation decomposition (CCD) based estimation of multiple vector channels in ambient colored noise. In this approach, it is assumed that the desired signals are received by two separated antenna clusters, so that the output noise is spatially uncorrelated. The proposed criterion exploits multiple signal codes through a weighted sum of projection errors, which incorporate the kernel matrices of the signals sharing the same target channel. Through computer simulations, we study the effects of using multiple codes in the blind CCD-based channel estimation and we show that the new criterion may indeed lead to quite significant performance improvements in the estimation of the multipath vector channels.**

I. Introduction

Channel estimation is a critical element in many wireless communications systems. Indeed, to optimally recombine signal components transmitted through multipath channels, knowledge of the relative delays, amplitudes and phases of the various propagation paths is needed. Examples include RAKE receivers and multiuser detectors in code division multiple access (CDMA) communications [1], as well as multiple-input multiple-output (MIMO) space-time coded systems [2]. In recent years, blind estimation of wireless channels has received considerable attention because of its advantages in terms of bandwidth efficiency. Blind algorithms only rely on the received signal to carry out estimation, so that precious bandwidth resources need not be used for the transmission of training sequences and/or pilot signals.

Of particular interest is the class of blind subspace-based methods for multipath vector channel estimation. These methods exploit the second-order statistics of the received signals to define a signal and a noise subspaces; the vector channel of interest is then estimated by exploiting the orthogonality property between the coded waveform signatures and the noise subspace. This way, the multi-user channel estimation problem is decomposed into a series of lower dimensional single user problems. Recent examples of blind subspace based methods can be found in [3]–[7].

In most of the literature on subspace channel estimation, it is generally assumed that the ambient noise is temporally white. In practice, this assumption is often violated due to, e.g., interference from narrowband sources. Wang and Poor [8] have proposed a blind method for the estimation of multiple channels in wireless CDMA systems in the presence of correlated ambient noise. Their approach is based on the practical assumption that the desired signals are received by two well separated antennas, so that the output noise is spatially uncorrelated. A new estimation algorithm is then proposed based on the concept of canonical correlation decomposition (CCD) [9]. The effectiveness of this approach is well supported by computer experiments.

The CCD approach proposed in [8] only makes use of a single independent signal (i.e. a single code) to estimate the target channel. However, the target channel is often shared by multiple simultaneous independent signals. This is the case for instance in a typical downlink environment, or on the uplink of some Third Generations cellular systems such as UTRA/TDD where mobiles are allowed to use multiple codes simultaneously for efficient bandwidth utilization. In a recent work, the use of multiple signal codes in subspace-based blind channel estimation has been studied [10]. In particular, it was shown that under very general conditions, increasing the number of codes leads to a more accurate channel estimation. Using multiple codes for the channel estimation may also bring performance improvements with the CCD approach in the presence of spatially correlated noise.

In this paper, we propose to extend the CCD approach in [8] to exploit the multiple codes for the channel estimation. We introduce and study a generalized criterion for the CCD-based estimation of multiple vector channels in ambient colored noise. The new criterion uses multiple signal codes through a weighted sum of projection errors, which incorporate the kernel matrices of the signals sharing the target channel. Through computer simulations, we study the effects of using multiple codes in the blind CCD-based multipath vector channel estimation under colored noise and we show that the criterion indeed leads to performance improvements that may be quite significant.

The paper is organized as follows: Section II describes the signal model and reviews the CCD method of [8]. The generalized criterion is presented in Section III, along with some motivations. The results of supporting computer experiments are presented in Section IV. This is followed by conclusions in Section V.

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II. Signal Model

We consider the generalized received signal model in [10] for an L-dimensional received signal vector at one antenna or antenna cluster of a communication system:

$$\mathbf{r} = \sum_{i=1}^{N} \gamma_i b_i \mathbf{C}_i \mathbf{h}_i + \mathbf{v}, \tag{1}$$

where N is the number of individual symbols that comprise the received vector, γ_i is a real-valued received amplitude, b_i is the i-th information symbol, $\mathbf{C}_i$ is defined as a channel-independent kernel matrix with size $L \times P$, $\mathbf{h}_i$ is a $P \times 1$ normalized channel vector (i.e. $\|\mathbf{h}_i\| = 1$), and $\mathbf{v}$ is an $L \times 1$ additive noise vector. We assume that the information symbols b_i, for $i = 1, \ldots, N$, are independent and identically distributed with zero mean and unit variance. The additive noise vector $\mathbf{v}$ is circularly complex Gaussian with covariance matrix $\mathbf{\Sigma}$ and is independent from the information symbols b_i. We define

$$\mathbf{b} \triangleq [b_1, \ldots, b_N]^T, \tag{2}$$
$$\mathbf{\Gamma} \triangleq \mathrm{diag}[\gamma_1, \ldots, \gamma_N], \tag{3}$$
$$\mathbf{W} \triangleq [\mathbf{w}_1, \ldots, \mathbf{w}_N], \tag{4}$$

where $\mathbf{w}_i \triangleq \mathbf{C}_i \mathbf{h}_i$ for $i = 1, \ldots, N$ is the *effective* signature waveform for the i-th information symbol, i.e. combined effect of channel and kernel matrix as seen by the receiver. Using the above matrix notations, the signal model (1) can be expressed as

$$\mathbf{r} = \mathbf{W}\mathbf{\Gamma}\mathbf{b} + \mathbf{v}. \tag{5}$$

In the sequel, we refer to the individual products $\gamma_i b_i \mathbf{C}_i \mathbf{h}_i$ ($i = 1, \ldots, N$) in (1) as *signal components*. For generality, we assume that the N signal components experience M different channels, $1 \le M \le N$. Then we separate the N signal components into M groups, such that the signal components in each group share the same channel. We denote the number of signal components in the m-th group as K^m ($m = 1, \ldots, M$), so that $\sum_{m=1}^{M} K^m = N$. We use the superscript m to denote group affiliation, as in the common channel parameter $\mathbf{h}^m$, and we use the superscript l to further distinguish among the K^m signal components, as in $\gamma^{m,l}$, $b^{m,l}$, $\mathbf{C}^{m,l}$, and $\mathbf{w}^{m,l}$.

We now extend the model and assume that the signal is received at two different clusters of antennas, well separated so that the noise components from each cluster are mutually uncorrelated. A typical example for a specific case of this generalized formulation consists of the downlink of a DS-CDMA system, where the mobile terminals have two well separated received antennas, i.e. two clusters of one antenna.

Denote the individual antenna cluster noise covariance matrix $\mathbf{\Sigma}_j \triangleq E[\mathbf{v}_j \mathbf{v}_j^H]$, $j = (1, 2)$, where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the noise vector at antenna cluster 1 and 2, respectively. Subscripts in the sequel will denote antenna cluster index, unless indicated otherwise. Since the noise is spatially uncorrelated across antenna clusters, we also have $E[\mathbf{v}_1 \mathbf{v}_2^H] = \mathbf{0}$. Let the combined received signal vector include the signals from the two clusters of antennas:

$$\tilde{\mathbf{r}} \triangleq [\mathbf{r}_1^T \ \mathbf{r}_2^T]^T, \tag{6}$$

where $\mathbf{r}_j$ is the received signal as in (1) for antenna cluster j. The corresponding signal covariance matrix takes the form:

$$\tilde{\mathbf{R}} \triangleq E[\tilde{\mathbf{r}}\tilde{\mathbf{r}}^H] = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix}, \tag{7}$$

where the constituent covariance matrices are defined by

$$\mathbf{R}_{jj} \triangleq E[\mathbf{r}_j \mathbf{r}_j^H] = \mathbf{W}_j \mathbf{\Gamma}_j^2 \mathbf{W}_j^H + \mathbf{\Sigma}_j \tag{8}$$
$$\mathbf{R}_{jk} \triangleq E[\mathbf{r}_j \mathbf{r}_k^H] = \mathbf{W}_j \mathbf{\Gamma}_j \mathbf{\Gamma}_k \mathbf{W}_k^H, \tag{9}$$

and $\mathbf{W}_j$ and $\mathbf{\Gamma}_j$ are the effective signature and amplitude matrices for the j-th antenna cluster, respectively.

Within the above framework, the goal of blind channel estimation is to determine the target common channel vector for group m to antenna cluster j denoted here by $\mathbf{h}_j^m$, using T observations of the combined received signal vector in (6). In blind channel estimation, the transmitted information symbols, as represented by vector $\mathbf{b}$ in (5), are unknown. To estimate the target channel vector $\mathbf{h}_j^m$, at least one kernel matrix in the m-th group needs to be known by the estimating algorithm. In practice, the specific available knowledge of the kernel matrices depends on the particular system under consideration.

In the next section, we formulate a cost function for a blind CCD-based channel estimation for the communication system model described above. The cost function incorporates the set of kernel matrices of the signal components sharing the same target channels.

III. Blind CCD-based Channel Estimation with Multiple Signature Waveforms

A. Classical single signature approach

Consider classical subspace-based blind channel estimation in pure white noise, which only requires a single antenna cluster and does not exploit common channels. Let us temporarily denote the covariance matrix in (8) as $\mathbf{R} = \mathbf{W}\mathbf{\Gamma}^2\mathbf{W}^H + \mathbf{\Sigma}$ where $\mathbf{\Sigma} \equiv \mathbf{\Sigma}_j = \sigma^2 \mathbf{I}_L$. Its eigenvalue decomposition (EVD) can be expressed in the form

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H \tag{10}$$

where $\mathbf{\Lambda} = \mathrm{diag}[\lambda_1, \ldots \lambda_L]$ is a diagonal matrix with the eigenvalues in a non-increasing order, and $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_L]$ is a unitary matrix that contains the corresponding eigenvectors. Since the rank of signal matrix $\mathbf{W}\mathbf{\Gamma}^2\mathbf{W}^H$ is N, we can separate the eigenvalues into two distinct groups: the signal and noise eigenvalues. The corresponding signal and noise eigenvalues matrices are given by $\mathbf{\Lambda}_s = \mathrm{diag}[\lambda_1, \ldots, \lambda_N]$ and $\mathbf{\Lambda}_n = \mathrm{diag}[\lambda_{N+1}, \ldots, \lambda_L]$, respectively. Similarly, the eigenvectors can also be separated into the signal and noise eigenvectors, represented by $\mathbf{U}_s = [\mathbf{u}_1, \ldots, \mathbf{u}_N]$ and $\mathbf{U}_n = [\mathbf{u}_{N+1}, \ldots, \mathbf{u}_L]$, respectively. The EVD in (10) can then be conveniently expressed as

$$\mathbf{R} = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}. \tag{11}$$

The columns of $\mathbf{W}$ span the signal subspace, and consequently they are orthogonal to the columns of $\mathbf{U}_n$, which span the noise subspace, i.e.:

$$\mathrm{Span}[\mathbf{W}] = \mathrm{Span}[\mathbf{U}_s] \perp \mathrm{Span}[\mathbf{U}_n]. \tag{12}$$

Consequently, the noise subspace is orthogonal to the effective signature of the individual signal components. Therefore the following equation holds for $\mathbf{h}_i$, the vector channel associated to the i^{th} signal component:

$$\mathbf{U}_n^H \mathbf{w}_i = \mathbf{U}_n^H \mathbf{C}_i \mathbf{h}_i = \mathbf{0}. \tag{13}$$

The normalized channel estimate can be obtained up to a phase ambiguity by solving (13) for $\mathbf{h}_i$, given the kernel matrix $\mathbf{C}_i$ and the noise subspace $\mathbf{U}_n$, which is usually estimated from the received signal.

B. Blind CCD-based channel estimation in colored noise

We now consider the case of a signal corrupted by ambient colored noise, received by two separated clusters of antennas with mutually uncorrelated noise, i.e.: the situation described by equations (6) to (9) above. Again here, the common channels are not exploited.

In [8], it was proposed to use the canonical correlation decomposition of the covariance matrix $\mathbf{R}_{12}$ in (9) to properly estimate the null subspace $\mathrm{Null}(\mathbf{W}_j^H)$ which is orthogonal to the signal subspace $\mathrm{Range}(\mathbf{W}_j)$. It is argued that the CCD provides superior estimate of the noise subspace compared to the singular value decomposition of $\mathbf{R}_{12}$ alone since the CCD uses the information in both $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$ together with $\mathbf{R}_{12}$, and creates the maximum correlation between the two data sets.

Assume that the matrices $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$ are both positive definite. Then the CCD of the matrix $\mathbf{R}_{12}$ leads to [9]:

$$\mathbf{R}_{11}^{-1}\mathbf{R}_{12}\mathbf{R}_{22}^{-1} = \underbrace{\mathbf{R}_{11}^{-1/2}\mathbf{V}_1}_{\mathbf{L}_1}\,\mathbf{\Pi}\,\underbrace{\mathbf{V}_2^H\mathbf{R}_{22}^{-1/2}}_{\mathbf{L}_2^H}, \tag{14}$$

where the $L \times L$ matrix $\mathbf{\Pi} = \mathrm{diag}(\pi_1, \ldots, \pi_N, 0, \ldots, 0)$ with $\pi_1 \geq \ldots \geq \pi_N > 0$ and $\mathbf{V}_j$ is a unitary matrix. We further partition the matrix $\mathbf{L}_j = [\mathbf{L}_j^s\ \mathbf{L}_j^n]$ where $\mathbf{L}_j^s$ and $\mathbf{L}_j^n$ contain the N first and $L-N$ last columns of $\mathbf{L}_j$, respectively. We then have that $\mathbf{L}_j^n$ spans the noise subspace of $\mathbf{R}_{jj}$ [9], i.e.:

$$\mathrm{Null}(\mathbf{W}_j^H) = \mathrm{Range}(\mathbf{L}_j^n), \quad j = 1, 2. \tag{15}$$

Finally, the channel coefficient for each signal component can be estimated as in (13) by solving

$$\mathbf{L}_j^{n\,H}\mathbf{C}_i\mathbf{h}_{i,j} = \mathbf{0}, \quad j = 1, 2, \tag{16}$$

where $\mathbf{h}_{i,j}$ represents the channel vector for the i^{th} signal component to antenna cluster j.

C. Multiple signatures in CCD-based channel estimation

In a recent work [10], a generalized subspace-based channel estimation model that exploits all the signal components that are subject to the same channel was proposed. It was shown that the channel estimates are more reliable when multiple kernel matrices of the same group are used for the estimation.

Based on this work, we extend the CCD algorithm in [8] for the case of multiple signatures. To this end, we propose a new cost function that includes a weighted sum of projection errors for the case of ambient colored noise vector channel estimation. This is in contrast to the suboptimum algorithm of [8] presented in section III-B, where common channels are not exploited and all signal components and their corresponding channels are processed independently.

The channel coefficient $\mathbf{h}_j^m$ is shared by signal components of the m^{th} group. Let $\mathbf{w}_j^{m,l}$ be the effective signature for the l^{th} signal components ($l = 1, \ldots, K$, where $K \equiv K^m$ for notational convenience) of group m to antenna cluster j. Without loss of generality, let $\bar{\mathbf{W}}_j \triangleq [\mathbf{w}_j^{m,1}, \ldots, \mathbf{w}_j^{m,K}]$ and thus

$$\mathrm{Span}[\bar{\mathbf{W}}_j] \subseteq \mathrm{Span}[\mathbf{W}_j] \perp \mathrm{Span}[\mathbf{L}_j^n], \tag{17}$$

and consequently $\mathbf{L}_j^{n\,H}\bar{\mathbf{W}}_j = \mathbf{0}$. Defining

$$\mathcal{L}_j^n \triangleq \mathbf{I}_K \otimes \mathbf{L}_j^n \tag{18}$$

$$\mathcal{C}^T \triangleq [(\mathbf{C}^{m,1})^T, \ldots, (\mathbf{C}^{m,K})^T] \tag{19}$$

where $\otimes$ represents the Kronecker product, and applying vectorization operation on $\mathbf{L}_j^{n\,H}\bar{\mathbf{W}}_j$, we finally obtain

$$\mathrm{vec}[\mathbf{L}_j^{n\,H}\bar{\mathbf{W}}_j] = \mathcal{L}_j^{n\,H}\,\mathrm{vec}[\bar{\mathbf{W}}_j] = \mathcal{L}_j^{n\,H}\mathcal{C}\mathbf{h}_j^m = \mathbf{0}. \tag{20}$$

In practice, the covariance matrix $\mathbf{R}$ from which the noise subspace is obtained is unknown and must be estimated from the observed data via time averaging. Assuming a locally stationary environment, one such estimate based on a rectangle window of T samples is given by $\widehat{\mathbf{R}} = \frac{1}{T}\sum_{k=1}^{T}\tilde{\mathbf{r}}[k]\tilde{\mathbf{r}}[k]^H$, where $\tilde{\mathbf{r}}[k]$ is the combined received vector at time instant k. The CCD in (14) applied to the estimates of the submatrices of $\widehat{\mathbf{R}}$ will also in practice give noisy estimates of the subspaces. Consequently, we define the noisy estimate of $\mathcal{L}_j^n$ in (18) by $\widehat{\mathcal{L}}_j^n \triangleq \mathbf{I}_K \otimes \widehat{\mathbf{L}}_j^n$.

In this work, and because of the presence of noisy estimates, we consider the following optimization criterion to obtain the blind channel estimates in (20):

$$\hat{\mathbf{h}}_j^m = \arg\min_{\|\mathbf{t}\|=1} \mathbf{t}^H\mathbf{D}\mathbf{t}, \tag{21}$$

where $\mathbf{D} \triangleq \mathcal{C}^H\widehat{\mathcal{L}}_j^n\widehat{\mathcal{L}}_j^{n\,H}\mathcal{C} = \sum_{l=1}^{K}(\mathbf{C}^{m,l})^H\widehat{\mathbf{L}}_j^n\widehat{\mathbf{L}}_j^{n\,H}\mathbf{C}^{m,l}$. For the noiseless case $\widehat{\mathcal{L}}_j^n = \mathcal{L}_j^n$ and if the identifiability condition (see [5]) is met, all of the eigenvalues of $\mathbf{D}$ are positive but the smallest one, which is zero. However in practice, the estimation errors may result in a positive perturbation in the smallest eigenvalue so that the matrix $\mathbf{D}$ is positive definite and no (non-trivial) solution of (20) exists. In this case, the target channel vector can still be estimated by minimizing the cost function in (21). Thus, the optimization criterion in (21) is more robust to perturbations in $\mathcal{L}_j^n$ than (20).

A further modification to the optimization criterion is motivated by the development of the generalized subspace based blind channel estimator in [11]. Specifically, we allow the assignment of different real-valued weights for each of the different terms $(\mathbf{C}^{m,l})^H \widehat{\mathbf{L}}_j^n \widehat{\mathbf{L}}_j^{n\,H} \mathbf{C}^{m,l}$, i.e.

$$\hat{\mathbf{h}}_j^m = \arg \min_{\|\mathbf{t}\|=1} \ \mathbf{t}^H [\sum_{l=1}^{K} \alpha_{j,l} (\mathbf{C}^{m,l})^H \widehat{\mathbf{L}}_j^n \widehat{\mathbf{L}}_j^{n\,H} \mathbf{C}^{m,l}] \mathbf{t}, \quad (22)$$

where $\alpha_{j,l} > 0$ are user-specified weight parameters. Let $\mathcal{A}_j \triangleq \mathrm{diag}[\sqrt{\alpha_{j,1}}, \ldots, \sqrt{\alpha_{j,K}}\ ] \otimes \mathbf{I}_{L-N}$, so that the optimization criterion in (21) can be expressed in matrix form as

$$\hat{\mathbf{h}}_j^m = \arg \min_{\|\mathbf{t}\|=1} \ \mathbf{t}^H \mathcal{C}^H \widehat{\mathcal{L}}_j^n \mathcal{A}_j \mathcal{A}_j^H \widehat{\mathcal{L}}_j^{n\,H} \mathcal{C} \mathbf{t}. \quad (23)$$

The solution of (23) can be calculated as the eigenvector corresponding to the smallest eigenvalue of $\mathcal{C}^H \widehat{\mathcal{L}}_j^n \mathcal{A}_j \mathcal{A}_j^H \widehat{\mathcal{L}}_j^{n\,H} \mathcal{C}$. The multiple signature waveforms are thus incorporated directly in the optimality criterion. The overall "CCD-MS" algorithm presented here is detailed in Table 1.

Table 1 Blind CCD-based channel estimation algorithm in colored noise using multiple signature waveforms (CCD-MS)

K is user specified
$\alpha_{j,l}$, $l = 1, \ldots, K$, $j = 1, 2$, are user specified
$\mathbf{A}_j = \mathrm{diag}[\sqrt{\alpha_{j,1}}, \ldots, \sqrt{\alpha_{j,K}}\ ], \quad j = 1, 2$
$\mathcal{A}_j = \mathbf{A}^j \otimes \mathbf{I}_{L-N}, \quad j = 1, 2$
$\mathcal{C} = [(\mathbf{C}^{m,1})^T, \ldots, (\mathbf{C}^{m,K})^T]^T$

$\widehat{\mathbf{R}} = \frac{1}{T} \sum_{k=1}^{T} \tilde{\mathbf{r}}[k] \tilde{\mathbf{r}}[k]^H$
$\widehat{\mathbf{R}} = \begin{bmatrix} \widehat{\mathbf{R}}_{11} & \widehat{\mathbf{R}}_{12} \\ \widehat{\mathbf{R}}_{21} & \widehat{\mathbf{R}}_{22} \end{bmatrix}$
$\widehat{\mathbf{R}}_{11}^{-1} \widehat{\mathbf{R}}_{12} \widehat{\mathbf{R}}_{22}^{-1} = \widehat{\mathbf{L}}_1 \ \widehat{\mathbf{\Pi}} \ \widehat{\mathbf{L}}_2^H$ (see CCD alg. in [9])

For $j = 1, 2$:
$\widehat{\mathbf{L}}_j = [\widehat{\mathbf{L}}_j^s \ \widehat{\mathbf{L}}_j^n]$, where $\widehat{\mathbf{L}}_j^s$ is $L \times N$ and $\widehat{\mathbf{L}}_j^n$ is $L \times (L - N)$
$\widehat{\mathcal{L}}_j^n = \mathbf{I}_K \otimes \widehat{\mathbf{L}}_j^n$
Construct the matrix $\mathcal{C}^H \widehat{\mathcal{L}}_j^n \mathcal{A}_j \mathcal{A}_j^H \widehat{\mathcal{L}}_j^{n\,H} \mathcal{C}$
$\hat{\mathbf{h}}_j^m$ is the smallest eigenvector of $\mathcal{C}^H \widehat{\mathcal{L}}_j^n \mathcal{A}_j \mathcal{A}_j^H \widehat{\mathcal{L}}_j^{n\,H} \mathcal{C}$

IV. Computer experiments

Consider a DS-CDMA downlink connection from a base station to N users. The information symbol $b_i \in \{\pm 1\}$ for user i is spread by a unique complex spreading code $\mathbf{c}_i = [c_1^i, \ldots, c_{L_c}^i]^T$, where L_c is the processing gain. The frequency selective channel is modeled as a FIR filter. The normalized coefficient vector $\mathbf{h}$ is of dimension $P \times 1$. Adopting the model in [5], the kernel matrix $\mathbf{C}_i$ of the i-the user is a $(L_c - P + 1) \times P$ Toeplitz matrix with the first column $[c_P^i, \ldots, c_{L_c}^i]^T$ and first row $[c_P^i, \ldots, c_1^i]$. The effective signature corresponding to this kernel matrix consists of the part of the received signal free from the ISI caused by previous symbols and since in general $P \ll L_c$, this ISI can be neglected. The received signal amplitude for the i-th user is γ_i at both antennas and the signal of all the users are synchronized, a common assumption for downlink transmission. The received signal at the mobile is expressed as in (1), for each antenna.

In the simulations, the following specific values are used: number of active users $N = 6$, processing gain $L_c = 12$ and length of channel vector $P = 4$. The binary signature sequences are randomly generated and constant throughout the simulation. For each antenna cluster, the colored noise vector $\mathbf{v}_j$ is obtained by applying a normalized correlation matrix to a complex white Gaussian noise vector.

The performance metric is the mean square error (MSE) obtained by averaging over 10^3 independent runs. It is assumed that the phase ambiguity has been resolved so that the MSE for the channel to antenna j becomes:

$$\overline{\mathrm{MSE}}_j = \frac{1}{K_o} \sum_{k=1}^{K_o} \frac{2 - 2|\hat{\mathbf{h}}_j^H(k) \mathbf{h}_j|^2}{\|\hat{\mathbf{h}}_j(k)\| \|\mathbf{h}\|}, \quad (24)$$

where $\hat{\mathbf{h}}_j^H(k)$ is the k^{th} sample vector channel estimate and K_o is the number of sample estimates or independent runs.

For the first two experiments, the covariance matrix is estimated for each run from the received signal of 10^4 transmitted symbols. The channel is estimated for different set of kernel matrices. The first set consists of $\mathcal{S}^1 = \mathbf{C}_1$, $\mathcal{S}^2 = \{\mathbf{C}_1, \mathbf{C}_2\}$ and so on up to $\mathcal{S}^6 = \{\mathbf{C}_1, \ldots, \mathbf{C}_6\}$.

Figure 1 shows the MSE of the channel estimates for different values of signal to noise ratio (SNR). For that first experiment, it is assumed that power control is used so that the received amplitude for all users are equal to unity, i.e. $\gamma_1 = 1, \ldots, \gamma_6 = 1$. Similarly, the user specified weights, are also all set to one ($\alpha_{j,i} = 1, \ \forall i, \ j = 1, 2$).

The results clearly shows the advantage of the CCD-MS method over the EVD approach of section III-A and the single waveform CCD approach of section III-B; the MSE decreases as we increase the number of kernel matrices in the estimation algorithm, indicated by the number in the legend. There is a gain of approximately 10dB when using only two kernel matrices (CCD-MS 2) and 20dB when using six kernel matrices (CCD-MS 6), instead of a single one (CCD) for the channel to antenna 1. The gain for the channel to antenna 2 when using two kernel matrices is approximately 5dB for antenna1 and more than 12dB when using all six kernel matrices (CCD-MS 6). The improvement varies with channel realization but in general, it can be observed that the gain increment is significant at first, and seems to taper as we increase the number of kernel matrices.

For the second experiment, the users' received amplitudes are set to be proportional to their respective index, i.e. $\gamma_1 = 1, \gamma_2 = 2, \ldots, \gamma_6 = 6$. Two sets of user-specified weights are used in (23) for the channel estimation. The first set, named *unity*, consists of all ones i.e. $\alpha_{j,l} = 1, \ \forall l, \ j = 1, 2$, as in the first experiment. The second set, named *proportional*, consists of weights proportional to the power of the received contributions, i.e. $\alpha_{j,l} = \gamma_l^2, \ \forall l, \ j = 1, 2$. In Fig. 2, the MSE

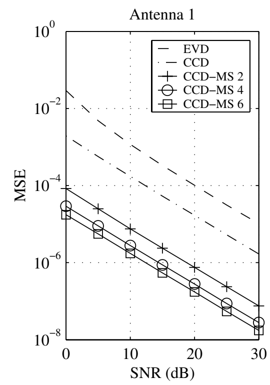

Fig. 1. MSE of channel estimates for equal amplitude signals ($\gamma_l = 1,\ \alpha_{j,l} = 1,\ \forall l,\ j = 1, 2$).

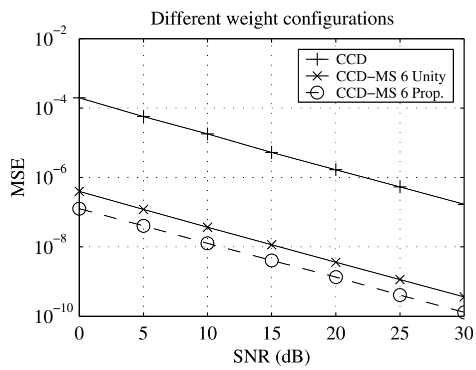

Fig. 2. MSE of channel estimates for different amplitude signals ($\gamma_l = i,\ \alpha_{j,l} = \gamma_l^2,\ \forall l,\ j = 1$).

for the channel estimates obtained using CCD-MS 6 with a unity and proportional set of user specified weights are shown along with the MSE for the traditional CCD approach for reference purposes. It can be observed that the "proportional" choice of weights improves the MSE with a measured gain of approximately 5dB. This choice of weights is motivated by the findings in [11].

Finally, we study the effect of imperfect covariance matrix estimation by varying the length of the observation window T. Figure 3 shows the MSE for different observation time. The MSE improves rapidly at first and tapers at larger values of T. More importantly, the gain in MSE for using CCD-MS over CCD and SVD estimation algorithms is actually more important in the presence of errors in the covariance matrix estimate. For antenna 2 it can be seen for example that for the same MSE, CCD requires an observation window of approximately 60 samples whereas CCD-MS 2 only requires 20 samples; a considerable improvement. This gain is even larger for antenna 1.

V. Conclusion

We have presented a generalized criterion for the CCD-based estimation of multiple vector channels in ambient colored noise. The proposed method, which uses multiple signature waveforms for the channel estimation shows significant improvement when compared to the method in [8] that

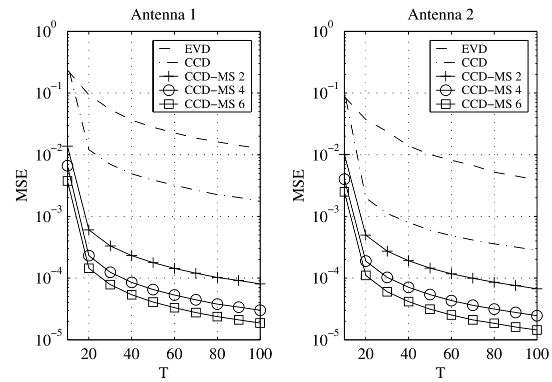

Fig. 3. MSE vs. Covariance estimation length (T) ($\gamma_l = 1,\ \alpha_{j,l} = 1,\ \forall l,\ j = 1, 2$).

uses a single signature. In future work, we will investigate the optimization of the MSE as a function of the user-specified weights and develop the Cramér-Rao bound (CRB) to complete the study.

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