Group optimal space-time MUD with beamforming

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***Abstract*— To improve the bandwidth efficiency of current and future generations of wireless cellular systems, several techniques exist such as beamforming (BF) and multiuser detection (MUD). To reduce the complexity of the MUD, it has been proposed to separate users within a cell into mutually exclusive groups and perform MUD independently in each group. Inter-group interference (IGI) is reduced by using beamforming. In this work, the group optimal MMSE linear receiver for BF based space-time multiuser detector (ST-MUD) receivers (BF-STMUD) is derived. The complexity of this structure is compared to that of the full ST-MUD. It is shown through computer simulations that with proper grouping, an important reduction in complexity can be achieved with almost insignificant loss of performance.**

I. INTRODUCTION

Current and future generations of wireless cellular systems require efficient use of the expensive and scarce available bandwidth. Multi-user detection (MUD) and the use of smart antennas (SA) to exploit the spatial domain are known to help reduce the interference significantly, and thus improve the system's efficiency.

To reduce the complexity of the MUD and at the same time reduce the co-channel interference, it has been proposed in [1], [2] to cluster users in mutually exclusive groups of spatial equivalence. The data symbols from each group are jointly detected using a reduced dimension MUD, while inter-group interference (IGI) is reduced by using spatial filtering or *beamforming* with SA, as illustrated in Fig. 1.

When implemented as a matrix inversion, the complexity associated to linear MUD is proportional to K^3, where K is the total number of users taken into account. Grouping has the potential to considerably reduce the total complexity compared to the full MUD complexity. Indeed, the total complexity when using MUD with grouping is proportional to $\sum_j K_j^3 < K^3$, where K_j is the number of users in group j and $K = \sum_j K_j$.

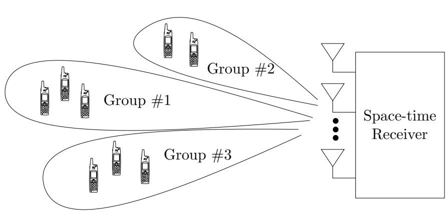

Fig. 1. User grouping

Although the idea of combining antenna arrays with multi-user detection is not new (e.g. [3]), it is only more recently that the concept of grouping closely located users in the spatial domain prior to joint detection has appeared [1], [2], [4]–[6].

In [1], [2], [5], [6], the groups are determined by thresholding on a normalized cross-correlation function based on the user spatial signature modeled as a single plane wave. A conventional beamformer is then used for each user in the system, prior to MUD in [1], [5] whereas [2], [6] use adaptive RLS beamforming weights for each user. In all those cases, a set of beamforming weights is assigned to every user. In contrast, [7] uses multiple switched beams and MUD is applied independently on each beam.

In the existing literature, it is generally assumed that the BF provides sufficient separation and the IGI is neglected in the development of the MUD expression; an assumption that may be violated in practice due to e.g. closely separated clusters. In this work, we develop and study a new group-optimal MMSE space-time multiuser detector (ST-MUD) receiver structure that incorporates the IGI in the derivation as a random signal contribution. Expressions for the signal to interference plus noise ratio (SINR) and the computational complexity in terms of number of complex floating point operations are also derived. Two different beamforming (BF) algorithms for spatial filtering are considered. Through computer simulations, it is shown that with proper grouping, the proposed structure has the potential to provide important complexity reduction with minimal performance loss.

The rest of the paper is organized as follows. The system model is presented in Section II. In Section III the optimum reduced complexity group-based BF-STMUD is derived and beamforming algorithms are discussed. Simulation results are shown in Section V. Finally, some conclusions are drawn in Section VI.

II. BACKGROUND

We consider the uplink of a time-division duplex synchronous CDMA system (TD-CDMA). Each of the K mobiles transmits a sequence $\mathbf{d}^{(k)} \in \{\pm 1\}^N$ of N data symbols, where the superscript k is the mobile index. The symbols are spread using a signature sequence $\mathbf{c}^{(k)} \in \mathbb{R}^Q$, $Q = T_s/T_c$ where T_s and T_c are the symbol and chip durations, respectively. The data is then synchronously transmitted through a dispersive channel, assumed fixed for the duration of the data block of N symbols. The space-time receiver at the base station is equipped with a standard linear antenna array of M elements.

Based on the linear system model described in [8], the received signal at the antenna array for a block of N data

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symbols can be expressed in matrix-vector form. Let the channel coefficients from user k to the base station m^{th} antenna be $\mathbf{h}^{(k,m)} \in \mathbb{C}^W$, where W is the length of the channel impulse response in number of chips. Assuming a plane wave propagation through a FIR channel, the channel matrix for user k can be defined by

$$\begin{aligned}\mathbf{H}_k &= [\mathbf{h}^{(k,1)}\ \mathbf{h}^{(k,2)}\ \dots\ \mathbf{h}^{(k,M)}]^T \\ &= [\mathbf{v}(\theta_{k,1})\ \mathbf{v}(\theta_{k,2})\ \dots\ \mathbf{v}(\theta_{k,W})]\ \mathbf{A}_k, \quad 1 \le k \le K,\end{aligned} \tag{1}$$

where $\mathbf{v}(\theta)$ is the array manifold vector in direction θ, $\theta_{k,l}$ is the direction of arrival (DOA) for the l^{th} path of user k, $\mathbf{A}_k = \mathrm{diag}(a_{k,1}, \dots, a_{k,W})$ is the amplitude matrix where $a_{k,l}$ is the complex amplitude for user k's l^{th} path, and T denotes transposition.

Let the corresponding *effective signature* sequence $\mathbf{b}^{(k,m)} \in \mathbb{C}^{Q+W-1}$ be the convolution of the signature sequence and channel coefficient $\mathbf{b}^{(k,m)} = \mathbf{h}^{(k,m)} * \mathbf{c}_k$, $1 \le k \le K$, $1 \le m \le M$. Also let $\mathbf{b}^{(k)} \in \mathbb{C}^{M(Q+W-1)}$ be the combined effective signature for all antenna elements, i.e.:

$$\mathbf{b}^{(k)} \triangleq \mathrm{vec}\{[\mathbf{b}^{(k,1)}\ \mathbf{b}^{(k,2)}\ \dots\ \mathbf{b}^{(k,M)}]^T\}, \quad 1 \le k \le K, \tag{2}$$

where the $\mathrm{vec}(\cdot)$ operation forms a column vector from the elements of its argument matrix by concatenating the columns of that matrix, starting from the left. The effective signatures for all users can then be compactly combined in a matrix $\mathbf{V}$:

$$\mathbf{V} = [\mathbf{b}^{(1)}\ \mathbf{b}^{(2)}\ \dots\ \mathbf{b}^{(K)}] \in \mathbb{C}^{M(Q+W-1)\times K}. \tag{3}$$

Similarly, the data vectors for the K users are combined in one large vector of length NK such that data symbols corresponding to the same time interval are grouped together, i.e.:

$$\mathbf{d} \triangleq \mathrm{vec}\{[\mathbf{d}^{(1)}\ \mathbf{d}^{(2)}\ \dots\ \mathbf{d}^{(K)}]^T\} \in \{\pm 1\}^{NK}. \tag{4}$$

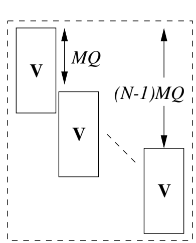
The observations measured from the antenna array for each time instant are also grouped in a vector, where data samples from the same time instant are grouped together:

$$\mathbf{x} = \mathrm{vec}\{[\mathbf{x}^{(1)}\ \mathbf{x}^{(2)}\ \dots\ \mathbf{x}^{(M)}]^T\} \in \mathbb{C}^{M(NQ+W-1)}, \tag{5}$$

where $\mathbf{x}^{(m)} \in \mathbb{C}^{NQ+W-1}$ is the received vector for antenna m. Combining equations (4) and (5), the total received vector becomes

$$\mathbf{x} = \mathbf{T}\mathbf{d} + \mathbf{n}, \tag{6}$$

where $\mathbf{T} \in \mathbb{C}^{M(NQ+W-1)\times NK}$ is a block-Toeplitz matrix containing the matrix V. Assuming $W < Q$, $\mathbf{T}$ takes the form:

$\mathbf{T} =$  . (7)

Finally, $\mathbf{n} \in \mathbb{C}^{M(NQ+W-1)}$ is the vector of independent circular complex Gaussian noise samples arranged the same way as $\mathbf{x}$ in (5) and $E[\mathbf{n}\mathbf{n}^H] = \sigma_w^2 \mathbf{I}_{M(NQ+W-1)}$, where σ_w^2 is the noise variance at each antenna element, H represents Hermitian transposition, and $\mathbf{I}_N$ is the identity matrix of dimension N.

In multi-user detection, the symbols transmitted from all K users are jointly estimated, based on the observations vector $\mathbf{x}$. For a linear detector, the estimated binary symbols are obtained by taking the sign of the real part of the output of the estimator $\mathbf{M} \in \mathbb{C}^{NK\times M(NQ+W-1)}$, i.e.:

$$\hat{\mathbf{d}} = \mathrm{sgn}\{\Re(\mathbf{M}^H \mathbf{x})\}, \tag{8}$$

where $\mathrm{sgn}(\cdot)$ is a function that returns the sign of its argument and $\Re(x)$ is the real part of x. Based on the matrix model (6), it can be shown that the minimum mean square error (MMSE) linear estimators $\mathbf{M}^H_{\text{MMSE}}$ takes the form:

$$\mathbf{M}^H_{\text{MMSE}} = (\mathbf{T}^H\mathbf{T} + \sigma_w^2 \mathbf{I})^{-1}\mathbf{T}^H. \tag{9}$$

Notice that the estimator consists of a match filter ($\mathbf{T}^H$) followed by a decorrelator. The decorrelator matrix has dimensions $NK \times NK$. If inverted using traditional techniques, the operation has complexity in terms of number of users of order $\mathcal{O}(K^3)$; a considerable difficulty for real-time operations.

III. Optimal group-based BF-STMUD

The receiver structure illustrated in Fig. 2 is considered. It consists of G independent parallel space-time multiuser processor "cards" ($j \in \{1, \dots, G\}$) containing one beamformer with weight vector $\mathbf{w}_{j,i} \in \mathbb{C}^M$ for each user $i \in \{1, \dots, K_j\}$, and a group ST-MUD for data symbols estimation. Let $\mathbf{W}^{(j)} \triangleq [\mathbf{w}_{j,1}\ \dots\ \mathbf{w}_{j,K_j}]$, then the input to the group ST-MUD becomes $\mathbf{y}_j = \mathbf{W}_j^H \mathbf{x}$, where $\mathbf{W}_j = \mathbf{I}_{NQ+W-1} \otimes \mathbf{W}^{(j)}$, and $\otimes$ denotes the Kronecker product. Next we derive the optimal MMSE ST-MUD weights for a given group and set of beamformer weights. We then discuss two different approaches for obtaining the beamforming weights.

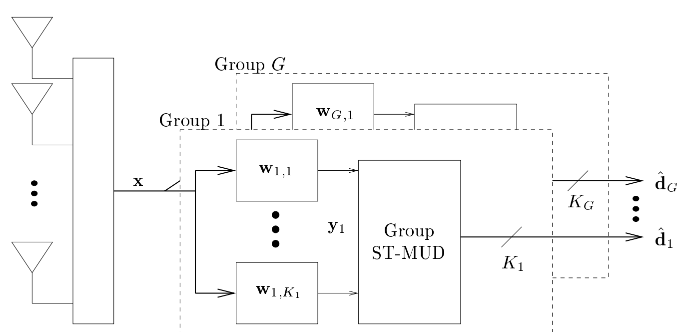


Fig. 2. Group-based BF-STMUD receiver diagram

A. Optimal BF-based receiver

To derive the optimal MMSE group ST-MUD weights, the received vector after beamforming is first expressed as a sum of three signal contributions: the signal from the users within the group of interest, the so-called inter-group interference (IGI) which comes from the users outside of the group and the additive white Gaussian noise. The IGI is considered random for the development of the MMSE estimator.

Let $\mathbf{P}^{(j)} \in \mathbb{R}^{K\times K_j}$ be the matrix that consists of K_j elementary vectors $\{\mathbf{e}_i\}$ of dimension K, where $\mathbf{e}_i$ is a vector containing zeros except at position i, where it contains the value 1. Also, let $\mathbf{P}_j \triangleq (\mathbf{I}_N \otimes \mathbf{P}^{(j)}) \in \mathbb{R}^{NK\times NK_j}$ such that

$$\mathbf{d}_j = \mathbf{P}_j^T \mathbf{d} \tag{10}$$

represents the NK_j data symbols transmitted by users in group j only. Similarly, define $\mathbf{P}_{\bar{j}}$ as the complement of $\mathbf{P}_j$ such that $\mathbf{P}_{\bar{j}}^T \mathbf{P}_j = \mathbf{0}$ and $\mathbf{d}_{\bar{j}} = \mathbf{P}_{\bar{j}}^T \mathbf{d} \in \mathbb{R}^{N(K-K_j)}$ is the vector of all symbols transmitted from the users outside the group j. The beamformer output $\mathbf{y}_j$ can now be expressed in this form:

$$\begin{aligned}
\mathbf{y}_j &= \mathbf{W}_j^H \mathbf{x} && (11)\\
&= \mathbf{W}_j^H (\mathbf{T}\mathbf{P}_j\mathbf{P}_j^T\mathbf{d} + \mathbf{T}\mathbf{P}_{\bar{j}}\mathbf{P}_{\bar{j}}^T\mathbf{d} + \mathbf{n}) && (12)\\
&= \mathbf{T}_{jj}\mathbf{d}_j + \mathbf{T}_{j\bar{j}}\mathbf{d}_{\bar{j}} + \mathbf{n}_j, && (13)
\end{aligned}$$

where $\mathbf{T}_{jj} \equiv (\mathbf{W}_j^H \mathbf{T}\mathbf{P}_j) \in \mathbb{C}^{K_j(NQ+W-1)\times NK_j}$ contains only the columns related to the users in the group j, $\mathbf{T}_{j\bar{j}} \equiv (\mathbf{W}_j^H \mathbf{T}\mathbf{P}_{\bar{j}}) \in \mathbb{C}^{K_j(NQ+W-1)\times N(K-K_j)}$ contains only the columns related to the users outside group j, and $\mathbf{n}_j \equiv \mathbf{W}_j^H \mathbf{n}$ is the correlated noise vector (after beamforming) with $E[\mathbf{n}_j\mathbf{n}_j^H] = \sigma_w^2 \mathbf{W}_j^H \mathbf{W}_j \equiv \mathbf{R}_n$. Notice that the first term in (13) corresponds to the signal part for group j, the second and third terms to the IGI and noise, respectively.

Following the method in [9], it can be shown that the MMSE linear estimator, solution to the linear optimization problem

$$\mathbf{M}^{(j)}_{\text{BF-MMSE}} = \arg\min_{\mathbf{M}} \; E\|\mathbf{d}_j - \mathbf{M}^H \mathbf{y}_j\|^2, \tag{14}$$

can be expressed as

$$\mathbf{M}^{(j)H}_{\text{BF-MMSE}} = \mathbf{R}(\mathbf{R}\mathbf{R}^H + \mathbf{C}\mathbf{C}^H + \mathbf{T}_{jj}^H \mathbf{R}_n \mathbf{T}_{jj})^{-1} \mathbf{T}_{jj}^H \tag{15}$$

where $\mathbf{R} \equiv \mathbf{T}_{jj}^H \mathbf{T}_{jj}$ and $\mathbf{C} \equiv \mathbf{T}_{jj}^H \mathbf{T}_{j\bar{j}}$. Notice that the rightmost element in (15) corresponds to a match filter to the users in group j and that the matrix inversion has now reduced dimension $NK_j \times NK_j$. Depending on the grouping, this may represent a considerable reduction in complexity.

It is important to realize here that the optimality of (15) is only with respect to the pre-determined grouping and beamforming weights. Joint optimization for grouping, beamforming, and linear weight design for data estimation would be very costly and is not considered in this work.

B. Beamforming algorithms

The linear estimator in (15) is optimal given the weight vectors for each user in the group ($\mathbf{W}^{(j)}$). Several beamforming algorithms exist and two of the common approaches are presented next.

1) Conventional BF: In this method, the beamforming weight vectors are designed based on the estimated DOA of the user of interest's main or strongest path. Thus

$$\mathbf{w}_{j,i} = \mathbf{v}(\hat{\theta}_i^{(j)}), \tag{16}$$

where $\hat{\theta}_i^{(j)}$ is the estimated main path DOA for the user i in group j. The advantage of this technique is its simplicity; it only requires the DOA estimate for the main path of each user. The underlying assumption here and for the next beamforming algorithm also is that the multipaths are closely concentrated around the main path.

2) MPDR: Conventional beamforming is simple but is not optimal with respect to the interference and noise. A better choice in this case is to use the well known minimum power distortion-less response (MPDR) beamformer, since it only requires the array covariance matrix $\mathbf{S}_\mathbf{x}$ in addition to the users' DOA [10]. The array covariance matrix is given by $\mathbf{S}_\mathbf{x} = E[\mathbf{x}(n)\mathbf{x}^H(n)]$, where $\mathbf{x}(n) \in \mathbb{C}^M$ is the array output at time n, and it can be estimated using the sample matrix:

$$\hat{\mathbf{S}}_\mathbf{x} \triangleq \frac{1}{N_o} \sum_{n=0}^{N_o-1} \mathbf{x}(n)\mathbf{x}^H(n), \tag{17}$$

where N_o is the number of data observations. The beamforming weight vector for the user i in group j then becomes:

$$\mathbf{w}_{j,i} = \frac{\hat{\mathbf{S}}_\mathbf{x}^{-1}\mathbf{v}(\hat{\theta}_i^{(j)})}{\mathbf{v}^H(\hat{\theta}_i^{(j)})\hat{\mathbf{S}}_\mathbf{x}^{-1}\mathbf{v}(\hat{\theta}_i^{(j)})}. \tag{18}$$

As in the case of conventional beamforming, the DOA for the main path must be estimated from the received signal.

IV. Complexity Analysis

The expressions for the optimal MMSE linear estimators in (9) and (15) include a matrix inversion and several matrix multiplications; a challenge for real-time operations. Fortunately, the structure of the data matrix $\mathbf{T}$ in (7) can be exploited extensively, leading to significant complexity reduction.

The most important complexity reduction results from the inherent structure in the $\mathbf{T}^H\mathbf{T}$ matrix product. Indeed, for $W \le Q$, and because of Hermitian symmetry, only two $K\times K$ blocks need to be computed for the complete matrix product. This observation can be extended to the other matrix products in (15). Also, because of the structure of $\mathbf{T}$, the Hermitian matrix to be inverted in (9) is block multi-diagonal. This can be exploited in the Cholesky factorization to solve the exact inverse problem. Similarly, the Hermitian matrix to be inverted in (15) also has a block multi-diagonal structure that can also be exploited in the same way.

To compare the complexity between the two approaches, the number of complex floating point operations (flops) is counted for the different parts of equations (9) and (15). Table I shows the dominant terms for the match filtering (MF), overhead (OH), linear system solution operations (Sol.) and the beamforming (BF) (not including weight design) complexity expressions. The overhead includes matrix products that do not belong to any of the previously mentioned category.

Notice that the complexity for solving the linear system is $\mathcal{O}(NK^3)$ compared to $\mathcal{O}(N^3K^3)$ if the Cholesky algorithm did not take into consideration the structure in $\mathbf{T}$. For large N, most of the saving in complexity comes from linear system solution; indeed, for $K_j \ll K$, $\sum_j K_j^3 \ll K^3$. The total complexity for the cases $K = 16$, $M = 12$, $Q = 16$, $W = 10$ for groups of $K_j = 4, \forall j$ is shown in Fig. 3, for $N = 100$ and

Operation	ST-MUD	BF-STMUD (per group)
Match Filtering	$2NKMQ$	$2NK_j^2Q$
Beamforming	N/A	$2K_jMQ$
Overhead	$2K^2MQ$	$2KQK_j(M+K)$ $+12KK_j^2+10NK_j^2$
System solution	NK^3	$8NK_j^3$

TABLE I

COMPLEXITY (IN COMPLEX FLOPS)

$N = 25$. As illustrated, for large N important computational savings can be obtained when using BF-STMUD. For smaller N, the overhead associated to computing the various matrix products in (15) becomes the dominant term and the reduction of complexity is modest.

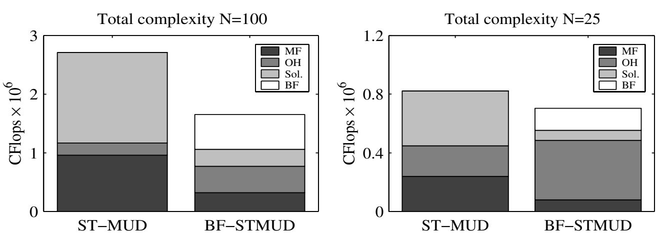


Fig. 3. Complexity (in complex flops)

V. COMPUTER EXPERIMENTS

The BF-STMUD system proposed with the different algorithms for beamforming in section III B was implemented and tested along with the more traditional ST-MUD system of (9) for comparison. The system consists of $K = 8$ users, assumed to be located within a $120°$ sector. For simplicity and also to study the effects of IGI independently from the effects of ISI, a $W = 1$ path channel model corresponding to the direct line of sight path is used with equal amplitude for each user, representing a perfect power control situation. The DOAs are fixed for the entire simulation time and are given in Table II along with the associated grouping. The CDMA codes are

User #	1	2	3	4	5	6	7	8
Group #	1	1	2	2	3	3	4	4
DOA (in deg.)	0	10	45	55	-25	-20	-45	-50

TABLE II

USER DOA AND GROUPING

chosen randomly and the processing gain is $Q = 16$. The standard linear array (SLA) has $M = 12$ antenna elements assumed ideal and isotropic. The block size is $N = 10$ data symbols for all simulations for faster computations; since there is no ISI in these simulations, this smaller choice of N has no impact on the performance results. The estimate for the covariance matrix on the other hand is obtained by using the sample matrix over blocks equivalent to 150 data symbols.

In the first part of these experiments, the average SINR is measured with the help of (21) and (22) developed in the appendix. As (21) suggests, at high input SNR (i.e.: $\sigma_w^2 \to 0$) the SINR for BF-STMUD achieves a plateau, whereas the full ST-MUD does not (see (22)). This can be observed directly

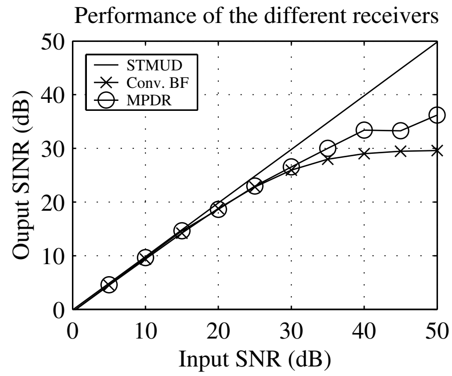


Fig. 4. SINR for the different algorithms

from Fig. 4 and is explained by the inability of the group-based BF-STMUD to completely eliminate IGI, the main source of signal degradation at high SNR.

Since MPDR has a better ability to null interferers, in general it provides higher output SINR than conventional beamforming. Table III shows the value of the SINR for the different algorithms at the operating SNR of 10dB. It can be observed that ST-MUD is closely followed by the group-based BF-STMUD of section III with MPDR and Conv. BF. The difference of 0.2dB between STMUD and BF-STMUD

Method	SINR (dB)
ST-MUD	9.834
BF-STMUD MPDR	9.627
BF-STMUD Conv. BF	9.388

TABLE III

SINR AT 10DB

with MPDR is negligible in practice. Yet, the implication in terms of complexity reduction is significant: by carefully choosing the grouping, and by applying proper spatial filters, it is possible to reduce the complexity considerably with a minimal cost on performance. This is further confirmed in the bit error rate (BER) curves for practical operation range shown in Fig. 5. It can be easily observed that the difference between the three algorithms is really small. Indeed, at BER of 10^{-4}, the corresponding differences in terms of input SNR between the full STMUD and BF-STMUD with MPDR and Conv. BF are approximately 0.2dB and 0.5dB, respectively.

We also considered the traditional decorrelator or zero-forcing (ZF) approach for the weight design. Using this method, the IGI and noise are ignored in the weight design, i.e. the terms $\mathbf{C}\mathbf{C}^H$ and $\mathbf{T}_{jj}^H\mathbf{R}_n\mathbf{T}_{jj}$ in (15) are dropped. The results obtained showed a negligible degradation in the order of a few hundredth of a dB when compared to the MMSE approach. As expected, the degradation was more important for groups that were closer together since the IGI is more prominent.

VI. CONCLUSION

In this work, we have studied the optimal linear group-based beamforming ST-MUD receiver and compared it to the full

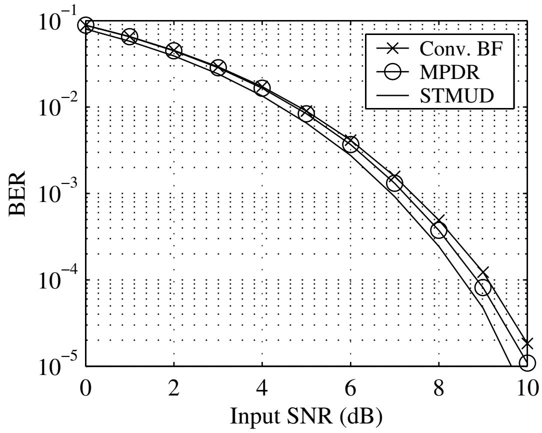


Fig. 5. BER for the different algorithms

ST-MUD receiver. Computer simulations show that the loss in performance in BER due to grouping is negligible when using traditional beamforming algorithms. The studied structure may therefore represent an approach of choice to future multiuser receiver design. In a subsequent work, we will improve the structure by removing the beamformers, since it represents an extra step in the design of the group-optimal ST-MUD weights.

APPENDIX

A. SINR derivation

Let $\hat{\mathbf{d}}'_j$ be the soft data symbol vector estimate for users of group j. Then according to (8) and (10), for any group linear data estimator $\mathbf{M}$, it can be expressed as

$$\hat{\mathbf{d}}'_j = \Re(\mathbf{M}^H \mathbf{W}_j^H \mathbf{x}). \tag{19}$$

The statistical mean power of $\hat{\mathbf{d}}'_j$ can be expressed as a sum of three terms:

$$\begin{aligned} E||\hat{\mathbf{d}}'_j||^2 &= E(\mathrm{tr}(\hat{\mathbf{d}}'_{\mathrm{j}} \hat{\mathbf{d}}'^{\mathrm{H}}_{\mathrm{j}})) \\ &= \mathrm{tr}(\mathbf{M}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}}^{\mathrm{H}} \mathbf{T}_{\mathrm{jj}} \mathbf{T}_{\mathrm{jj}}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}} \mathbf{M} \\ &\quad + \mathbf{M}^H \mathbf{W}_j^H \mathbf{T}_{j\bar{j}} \mathbf{T}_{j\bar{j}}^H \mathbf{W}_j \mathbf{M} \\ &\quad + \sigma_w^2 \mathbf{M}^H \mathbf{W}_j^H \mathbf{W}_j \mathbf{M}) \end{aligned} \tag{20}$$

where tr is the matrix trace operator. Equation (20) is obtained assuming independence between noise and data symbols from different users, i.e. $E(\mathbf{d}\mathbf{d}^H) = \mathbf{I}_N$ and $E(\mathbf{d}\mathbf{n}^H) = \mathbf{0}$. The first term in (20) represents the signal part from the group of interest, the second and third terms represent the interference and noise components, respectively. Thus, the signal to interference plus noise ratio (SINR) for group j, may be expressed as

$$\Gamma_{\mathrm{BF}}^{(j)}(\mathbf{M}) = \frac{\mathrm{tr}(\mathbf{M}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}}^{\mathrm{H}} \mathbf{T}_{\mathrm{jj}} \mathbf{T}_{\mathrm{jj}}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}} \mathbf{M})}{\mathrm{tr}(\mathbf{M}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}}^{\mathrm{H}} \mathbf{T}_{\mathrm{j}\bar{\mathrm{j}}} \mathbf{T}_{\mathrm{j}\bar{\mathrm{j}}}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}} \mathbf{M} + \sigma_{\mathrm{w}}^2 \mathbf{M}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}}^{\mathrm{H}} \mathbf{W}_{\mathrm{j}} \mathbf{M})}. \tag{21}$$

For the particular case of the full ST-MUD in (9), the beamforming is absent and since no grouping occurs, there is no IGI present. Therefore, the SINR simply becomes:

$$\Gamma_{\mathrm{STMUD}}(\mathbf{M}) = \frac{\mathrm{tr}(\mathbf{M}^{\mathrm{H}} \mathbf{T} \mathbf{T}^{\mathrm{H}} \mathbf{M})}{\mathrm{tr}(\sigma_{\mathrm{w}}^2 \mathbf{M}^{\mathrm{H}} \mathbf{M})}. \tag{22}$$

ACKNOWLEDGMENT

We would like to thank InterDigital Canada Ltée and the Natural Sciences and Engineering Research Council of Canada (NSERC) for supporting this work.

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