Subspace Decomposition for Channel Estimation in SC-FDE Systems

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***Abstract*—Single carrier frequency division multiple access (SC-FDMA), a multiple user access scheme based on the single carrier frequency domain equalization (SC-FDE) technique, has been proposed for the uplink in fourth generation (4G) mobile communications. SC-FDE requires reliable channel estimates to maintain an acceptable bit error rate (BER) performance. Much research focuses on the use of pilot symbols which reduces the spectral efficiency of SC-FDE systems. In this paper, we propose a subspace decomposition approach for semi-blind channel estimation in SC-FDE systems as a means to reduce or eliminate the need for pilot symbols and increase their overall spectral efficiency. Simulation results show that, under the conditions presented in this paper, a BER of 10^{-3} can be achieved with a power loss of roughly 1 dB compared to a system with perfect channel estimates. This result is achieved with channel estimates that have a normalized mean square error (NMSE) less than 1%.**

I. Introduction

One of the main goals of the long term evolution (LTE) project in 3GPP is to design communications systems which provide high data rates over broadband wireless channels [1]. Spectral efficiency is improved by using higher order modulation schemes. Frequency-selective fading presents a challenge when transmitting over broadband wireless channels. Orthogonal frequency division multiplexing (OFDM) divides the signal to be transmitted over many narrowband carriers, so that the fading encountered by a particular carrier is flat [2]. However, single carrier frequency division multiple access (SC-FDMA), a multiple user version of single carrier frequency domain equalization (SC-FDE), is proposed for the uplink due to the high peak-to-average power ratio (PAR) of OFDM systems [1,3].

In OFDM systems, if the channel gain on a particular carrier is poorly estimated, the estimation error affects only the detection of the symbol that is carried over that channel. However, in SC-FDE systems, the equalizer operates on both signal and channel in the frequency domain. As a consequence, all symbols are affected by a single error in the channel's frequency response. Therefore SC-FDE systems have a heightened sensitivity to channel estimation errors compared to OFDM. Despite this, there is significantly less literature on channel estimation techniques for SC-FDMA and SC-FDE systems compared to OFDM. Existing literature focuses on using pilots in either the time or the frequency domain to estimate the frequency response of the channel [4,5]. In the uplink of 4G systems, the use of pilots for channel estimation is proposed [6]. Periodically, a frame made up entirely of pilots is transmitted for the purpose of channel estimation. However, in doing so, the spectral efficiency is reduced as well as the amount of power that is allocated to the transmission of data by the pilot frame insertion ratio.

To perform channel estimation without a corresponding loss in spectral efficiency, one can use blind techniques which allow the channel to be estimated while transmitting data that is not known to the receiver. By reducing the dependency on pilot data, the overall spectral efficiency is increased. Subspace decomposition is one of a number of second order statistical methods that can be used for blind channel estimation. When multiple receive antennas are used, the autocorrelation of the observation can be used to determine the channel impulse responses between all transmit-receive antenna pairs, up to a constant [7-10]. Determination of this constant, referred to as the ambiguity, can be achieved by the insertion of a small number of known data points, thus resulting in a semi-blind approach. However, in order for the subspace decomposition technique to yield better spectral efficiencies than typical pilot based approaches, the pilot insertion rate to correct the ambiguity must be comparatively small.

In this paper, we propose the use of subspace-based channel estimation for SC-FDE systems. Although computationally complex, it provides the advantage of requiring little pilot data which results in increased spectral efficiencies. It can also provide accurate channel estimates even in low signal-to-noise ratio (SNR) conditions, which is important for SC-FDE, due to its sensitivity to channel estimation errors. Although subspace-based channel estimation has been considered for OFDM systems (e.g. [11]), very little research seems to have been done on its use in SC-FDE or SC-FDMA systems.

The remainder of this paper is organized as follows: Section II describes the SC-FDE system model. Section III details subspace decomposition for channel estimation in an SC-FDE system. In Section IV, the accuracy of subspace-based channel estimation for SC-FDE and the corresponding BER performance are determined by Monte Carlo simulations. In Section V, we present our conclusions.

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II. SC-FDE SYSTEM

The block diagram of a SC-FDE transmitter is given in Figure 1. The data is modulated using M-ary quadrature amplitude modulation (M-QAM) and grouped into blocks of N symbols. A cyclic prefix is added to the beginning of each block so that convolution of the data block with the channel impulse response appears as a circular convolution. The ith message block is:

$$\mathbf{x}^{(i)} = [x^{(i)}(0)\ x^{(i)}(1)\ \cdots\ x^{(i)}(N-1)]^T \tag{1}$$

where i denotes the message block index.

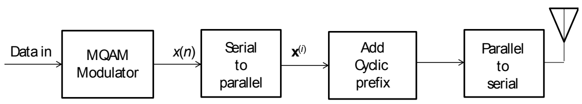

Fig. 1. Block diagram of a SC-FDE Transmitter.

We consider a system where the transmitter is equipped with a single antenna while the receiver employs $N_r > 1$ antennas along with maximal ratio combining (MRC); accordingly, there are N_r channels. The discrete time impulse response of the lth channel during the transmission of the ith message block is given by

$$h_l^{(i)}(n) = \sum_{k=0}^{M} h_{l,k}^{(i)} \delta(n-k) \tag{2}$$

where M is the channel order and $h_{l,k}^{(i)}$ is the complex gain of the kth resolvable path on the lth channel ($l = 1, 2, ..., N_r$) at time i. We assume that the fading is frequency selective Rayleigh distributed and therefore the set of channel coefficients $h_{l,k}^{(i)}$ are independent circularly symmetric complex Gaussian random variables. Furthermore, we assume that the channel responses change sufficiently slowly so that the channel gains can be considered constant for the duration of a message block.

The multiple antenna receiver for a SC-FDE system is shown in Figure 2. The ith received message block on the lth antenna after removal of the CP is given by:

$$\mathbf{y}_l^{(i)} = [y_l^{(i)}(0)\ y_l^{(i)}(1)\ \cdots\ y_l^{(i)}(N-1)]^T \tag{3}$$

Due to the insertion of the cyclic prefix at the transmitter, the received signal appears to be circularly convolved. Therefore

$$\mathbf{y}_l^{(i)} = \mathbf{x}^{(i)} \circledast \mathbf{h}_l^{(i)} + \mathbf{z}_l^{(i)} \tag{4}$$

where $\circledast$ denotes circular convolution, $\mathbf{h}_l^{(i)}$ is given by

$$\mathbf{h}_l^{(i)} = [h_{l,0}^{(i)}\ h_{l,1}^{(i)}\ \cdots\ h_{l,M}^{(i)}]^T \tag{5}$$

and the noise term

$$\mathbf{z}_l^{(i)} = [z_l^{(i)}(0)\ z_l^{(i)}(1)\ \cdots\ z_l^{(i)}(N-1)]^T \tag{6}$$

Let $\mathbf{X}^{(i)} = [X^{(i)}(0)\ X^{(i)}(1)\ \cdots\ X^{(i)}(N-1)]^T$, $H_l^{(i)} = [H_l^{(i)}(0)\ H_l^{(i)}(1)\ \cdots\ H_l^{(i)}(N-1)\ ]^T$, $\mathbf{Y}_l^{(i)} = [Y_l^{(i)}(0)\ Y_l^{(i)}(1)\ \cdots\ Y_l^{(i)}(N-1)]^T$ and $\mathbf{Z}_l^{(i)} = [Z_l^{(i)}(0)\ Z_l^{(i)}(1)\ \cdots\ Z_l^{(i)}(N-1)]^T$ be the N-point FFTs of $\mathbf{x}^{(i)}$, $\mathbf{h}_l^{(i)}$, $\mathbf{y}_l^{(i)}$, and $\mathbf{z}_l^{(i)}$ respectively. We assume that $N > M$, therefore the N-point FFT of $\mathbf{h}_l^{(i)}$ is obtained by padding it with $N - M$ zeros. It follows from (4) that ($0 \leq m \leq N-1$)

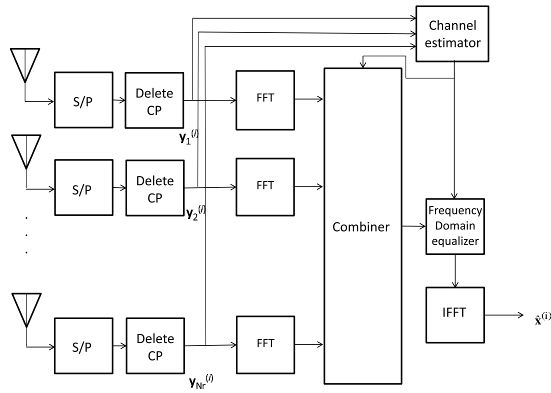

Fig. 2. Block diagram of multi-antenna SC-FDE receiver with channel estimator.

$$Y_l^{(i)}(m) = X^{(i)}(m) H_l^{(i)}(m) + Z_l^{(i)}(m), \tag{7}$$

Let $\hat{H}_l^{(i)}(m)$ be the estimate of $H_l^{(i)}(m)$ produced at the receiver. To combine the contributions from all antennas, the frequency domain data $Y_l^{(i)}(m)$ are multiplied by the complex conjugate of the channel gain estimates and then added together which is then expressed as:

$$Y^{(i)}(m) = \sum_{l=1}^{N_r} Y_l^{(i)}(m) \hat{H}_l^{(i)*}(m) \tag{8}$$

The frequency domain equalizer then produces an estimate of $X^{(i)}(m)$, $\hat{X}^{(i)}(m)$ as shown in

$$\hat{X}^{(i)}(m) = \frac{Y^{(i)}(m)}{\sum_{l=1}^{N_r} |\hat{H}_l^{(i)}(m)|^2 + \sigma_z^2} \tag{9}$$

Assuming perfect channel estimates, then (9) minimizes the mean square error (MSE) between $\hat{X}^{(i)}(m)$ and $X^{(i)}(m)$ [4].

In SC-FDE and SC-FDMA systems, the information block is then determined by taking the IFFT of the estimate found in (9). A poor channel gain estimate at frequency m will affect only the corresponding estimate $\hat{X}^{(i)}(m)$. However, when the IFFT is performed, all symbol estimates are functions of $\hat{X}^{(i)}(m)$. Therefore any channel gain estimate error affects all symbols in the data block. For this reason, channel estimates must be very accurate in SC-FDE based systems.

III. SUBSPACE DECOMPOSITION FOR CHANNEL IMPULSE RESPONSE ESTIMATION IN SC-FDE SYSTEMS

Let $L < N$ denote the size of the data vectors used at each antenna for the purpose of channel estimation. Considering the current as well as previous $L-1$ outputs of each antenna after the CP has been removed, we define the corresponding data vector as

$$\mathbf{y}_l^{(i)}(n) = [y_l^{(i)}(n)\ y_l^{(i)}(n-1)\ \cdots y_l^{(i)}(n-L+1)]^T \tag{10}$$

where $L-1 \leq n \leq N-1$ (all observations occur in the same data block) and we also define the complete $N_r L \times 1$ data vector as

$$\mathbf{y}^{(i)}(n) = [\mathbf{y}_1^{(i)}(n)^T \ \mathbf{y}_2^{(i)}(n)^T \ \cdots \mathbf{y}_{N_r}^{(i)}(n)^T]^T \quad (11)$$

Assuming the channel variations are sufficiently slow so that the channel can be considered static over the duration of the data block, then we can express (11) as

$$\begin{aligned} \mathbf{y}^{(i)}(n) &= \mathcal{H}^{(i)}\mathbf{x}^{(i)}(n) + \mathbf{z}^{(i)}(n) \\ &= [\mathcal{H}_1^{(i)T} \ \mathcal{H}_2^{(i)T} \ \cdots \mathcal{H}_{N_r}^{(i)T}]^T \mathbf{x}^{(i)}(n) + \mathbf{z}^{(i)}(n) \end{aligned} \quad (12)$$

where

$$\mathcal{H}_l^{(i)} = \begin{bmatrix} h_{l,0}^{(i)} & h_{l,1}^{(i)} & \cdots & h_{l,M}^{(i)} & 0 & \cdots & 0 \\ 0 & h_{l,0}^{(i)} & h_{l,1}^{(i)} & \cdots & h_{l,M}^{(i)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{l,0}^{(i)} & h_{l,1}^{(i)} & \cdots & h_{l,M}^{(i)} \end{bmatrix} \quad (13)$$

is the $L \times (L+M)$ channel matrix for the lth channel,

$$\mathbf{x}^{(i)}(n) = [x^{(i)}(n) \ x^{(i)}(n-1) \ \cdots \ x^{(i)}(n-M-L+1)]^T \quad (14)$$

is the $(L+M) \times 1$ channel input vector,

$$\mathbf{z}^{(i)}(n) = [\mathbf{z}_1^{(i)}(n)^T \ \mathbf{z}_2^{(i)}(n)^T \ \cdots \ \mathbf{z}_{N_r}^{(i)}(n)^T]^T \quad (15)$$

and

$$\mathbf{z}_l^{(i)}(n) = [z_l^{(i)}(n) \ z_l^{(i)}(n-1) \ \cdots \ z_l^{(i)}(n-L+1)]^T \quad (16)$$

Assuming that the transmitted data is temporally independent with zero mean and unit variance, then $E[\mathbf{x}^{(i)}(n)\mathbf{x}^{(i)H}(n)] = \mathbf{I}_{N_r(M+L)}$, which is an identity matrix of order $N_r(M+L)$. We define the output correlation matrix of $\mathbf{y}^{(i)}(n)$ as $\mathbf{R}_y^{(i)} = E[\mathbf{y}^{(i)}(n)\mathbf{y}^{(i)}(n)^H]$, which can be expressed as:

$$\mathbf{R}_y^{(i)} = \mathcal{H}^{(i)}\mathcal{H}^{(i)H} + \sigma_Z^2 \mathbf{I}_{N_r L} \quad (17)$$

where $\mathcal{H}^{(i)}\mathcal{H}^{(i)H}$ is the signal induced part of $\mathbf{R}_y^{(i)}$ while $\sigma_Z^2 \mathbf{I}_{N_r L}$ is the noise induced part. We assume that $\mathcal{H}^{(i)}$ is full column rank, i.e. it has rank $(L+M)$. Therefore $\mathcal{H}^{(i)}\mathcal{H}^{(i)H}$ also has rank $(L+M)$ which must be less than the rank of $\mathbf{R}_y^{(i)}$, i.e. $N_r L$, to enable subspace decomposition. Therefore for subspace decomposition to be possible $L > M/(N_r - 1)$.

The $N_r L \times N_r L$ correlation matrix $\mathbf{R}_y^{(i)}$ can be expressed in terms of its eigenvalues and associated orthonormalized eigenvectors as:

$$\mathbf{R}_y^{(i)} = \sum_{k=0}^{N_r L - 1} \lambda_k \mathbf{q}_k \mathbf{q}_k^H \quad (18)$$

where the eigenvalues are arranged in decreasing order, i.e. $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{N_r L-1}$. Since $\mathbf{R}_y^{(i)}$ has rank $N_r L$ and the signal part has rank $L+M$ then we can divide these eigenvalues into two groups [12]:

1. $\lambda_k > \sigma_Z^2, \qquad 0 \leq k \leq M+L-1$
2. $\lambda_k = \sigma_Z^2, \qquad M+L \leq k \leq N_r L - 1$.

The space spanned by the eigenvectors of $\mathbf{R}_y^{(i)}$ can then be separated into two subspaces: the signal subspace is spanned by the eigenvectors associated to the first group while the noise subspace is spanned by those associated to the second group. The ability to separate the eigenvectors into two subspaces implies that M is known. A method for determining M is discussed in [13].

Let $\mathbf{g}_k = \mathbf{q}_{M+L+k}$ for $0 \leq k \leq N_r L - M - L - 1$ be the set of eigenvectors of $\mathbf{R}_y^{(i)}$ which span the noise subspace. It is shown in [12] that:

$$\mathcal{H}^{(i)H}\mathbf{g}_k = \mathbf{0} \quad (19)$$

which means that the columns of the unknown multichannel matrix are orthogonal to the noise subspace. Equivalently, we can rewrite (19) as:

$$\mathbf{g}_k^H \mathcal{H}^{(i)}\mathcal{H}^{(i)H}\mathbf{g}_k = 0 \quad (20)$$

Since $\mathcal{H}^{(i)}$ has a partitioned structure as shown in (12), it is natural to assume a corresponding structure for the $N_r \times 1$ eigenvector $\mathbf{g}_k$. Therefore we rewrite $\mathbf{g}_k$ as:

$$\mathbf{g}_k = [\mathbf{g}_k^{(1)T} \ \mathbf{g}_k^{(2)T} \ \cdots \ \mathbf{g}_k^{(N_r)T}]^T \quad (21)$$

where $\mathbf{g}_k^{(l)} = [g_{k,0}^{(l)} \ g_{k,1}^{(l)} \ \cdots \ g_{k,L-1}^{(l)}]^T$.

It is shown in [12] that:

$$\mathbf{g}_k^H \mathcal{H}^{(i)}\mathcal{H}^{(i)H}\mathbf{g}_k = \mathbf{h}^{(i)H}\mathcal{G}_k\mathcal{G}_k^H\mathbf{h}^{(i)} = 0 \quad (22)$$

where

$$\mathcal{G}_{\mathbf{k}} = \left[\mathbf{G}_k^{(1)T} \ \mathbf{G}_k^{(2)T} \ \cdots \ \mathbf{G}_k^{(N_r)T}\right]^T \quad (23)$$

$$\mathbf{G}_k^{(l)} = \begin{bmatrix} g_{k,0}^{(l)} & \cdots & g_{k,L-1}^{(l)} & 0 & \cdots & 0 \\ 0 & g_{k,0}^{(l)} & \cdots & g_{k,L-1}^{(l)} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{k,0}^{(l)} & \cdots & g_{k,L-1}^{(l)} \end{bmatrix} \quad (24)$$

is an $(M+1) \times (M+1)$ matrix and

$$\mathbf{h}^{(i)} = [\mathbf{h}_1^{(i)T} \ \mathbf{h}_2^{(i)T} \ \cdots \ \mathbf{h}_{N_r}^{(i)T}]^T \quad (25)$$

is an $N_r(M+1) \times 1$ channel gain vector, $\mathbf{h}_l^{(i)}$ being the desired channel gain vector in (5).

In practice, we need to estimate $\mathbf{R}_y^{(i)}$ and its corresponding eigenvectors $\mathbf{g}_k^{(i)}$ by averaging over time. This is done by creating an $(N-L+1) \times L$ data matrix $\mathbf{A}_l^{(i)}$ whose Hermitian transpose is given by:

$$\mathbf{A}_l^{(i)H} = [\ \mathbf{y}_l^{(i)}(L-1) \quad \mathbf{y}_l^{(i)}(L) \quad \cdots \quad \mathbf{y}_l^{(i)}(N-1)\ ] \quad (26)$$

where $\mathbf{y}_l^{(i)}(n)$ is given by (10). We then define the $(N-L+1) \times N_r L$ data matrix $\mathbf{A}^{(i)}$ as

$$\mathbf{A}^{(i)} = [\mathbf{A}_1^{(i)} \ \mathbf{A}_2^{(i)} \ \cdots \ \mathbf{A}_{N_r}^{(i)}] \quad (27)$$

The sample correlation matrix of data block i, $\mathbf{\Phi}_y^{(i)}$ is then obtained as:

$$\mathbf{\Phi}_y^{(i)} = \frac{\mathbf{A}^{(i)H}\mathbf{A}^{(i)}}{N-L+1} \quad (28)$$

The expected value of (28) is $E[\mathbf{\Phi}_y^{(i)}] = \mathbf{R}_y^{(i)}$ therefore, the eigenvalues and eigenvectors of $\mathbf{\Phi}_y^{(i)}$ should approximate those of $\mathbf{R}_y^{(i)}$.

For short data frames N, $\mathbf{\Phi}_y^{(i)}$ may not approximate $\mathbf{R}_y^{(i)}$ well enough due to insufficient averaging. Therefore, we estimate $R_y^{(i)}$ by $\hat{\mathbf{R}}_y^{(i)}$ using time-averaging over successive data frames. We consider two types of averaging based on rectangular and exponential windows. In rectangular window averaging, the correlation matrix is obtained by averaging the W most recent sample correlation matrices using equal weight, i.e.

$$\hat{\mathbf{R}}_y^{(i)} = \frac{1}{W} \sum_{w=0}^{W-1} \mathbf{\Phi}_y^{(i-w)} \tag{29}$$

In exponential window averaging, the estimated correlation matrix is obtained recursively. Let $\hat{\mathbf{R}}_y^{(0)} = \mathbf{\Phi}_y^{(0)}$. Then

$$\hat{\mathbf{R}}_y^{(i)} = \mu \mathbf{\Phi}_y^{(i)} + (1-\mu)\hat{\mathbf{R}}_y^{(i-1)} \tag{30}$$

In practice, wireless channels are time varying. Channel estimates produced by time averaging are subject to two types of errors: noise errors and lag errors. Short averaging windows suffer from higher noise errors while longer averaging windows experience higher lag errors. Therefore, selection of a particular equivalent window length (W in rectangular windows and $1/\mu$ in exponential windows) results in a trade-off between noise and lag error.

Performing the eigenvector decomposition of $\hat{\mathbf{R}}_y^{(i)}$ yields the set of eigenvectors $\{\mathbf{v}_k\}$ and associated eigenvalues $\{\rho_k\}$ for $k = 0, 1, \cdots, N_r L - 1$. We approximate the eigenvectors of the noise subspace, $\mathbf{g}_k$ by selecting the eigenvectors that correspond to the $(N_r - 1)L - M$ smallest eigenvalues of $\hat{\mathbf{R}}_y^{(i)}$. Assuming that $\rho_0 \geq \rho_1 \geq \cdots \geq \rho_{N_r L-1}$ then

$$\hat{\mathbf{g}}_k = \mathbf{v}_{M+L+k}, \quad 0 \leq k \leq (N_r - 1)L - M - 1 \tag{31}$$

By replacing $\mathbf{g}_k$ by $\hat{\mathbf{g}}_k$ in (21), (23) and (24), we obtain an estimate for $\mathcal{G}_k$ which we denote as $\hat{\mathcal{G}}_k$. Then, based on (22), we define:

$$\mathcal{Q} = \sum_{k=0}^{(N_r-1)L-M-1} \hat{\mathcal{G}}_k \hat{\mathcal{G}}_k^H \tag{32}$$

and consider a cost function $\mathcal{E}(\mathbf{h}) = \mathbf{h}^H \mathcal{Q} \mathbf{h}$. Let $\mathbf{h} = \hat{\mathbf{h}}^{(i)}$ be the vector that minimizes this cost function subject to $\|\hat{\mathbf{h}}^{(i)}\| = 1$. Then the estimate of the N_r channel impulse responses is $\hat{\mathbf{h}}^{(i)}$ and is given by

$$\hat{\mathbf{h}}^{(i)} = c(\mathbf{h}^{(i)} + \mathbf{h}_e^{(i)}) \tag{33}$$

where $\mathbf{h}^{(i)}$ is given by (25), $\mathbf{h}_e^{(i)}$ is the estimation error and c is an unknown complex multiplicative constant defined as the ambiguity [12]. To obtain the channel estimate, we first need to determine the ambiguity. Determination of the ambiguity requires the use of a small number of pilot symbols interleaved in a data block. Techniques for scalar ambiguity determination are discussed in [14, 15].

IV. Simulation Results

In this section we evaluate the performance of the proposed channel estimation scheme in terms of normalized mean square error (NMSE) between the FFT of the actual channel gains and those of the estimated ones. We define NMSE as:

$$\text{NMSE} = \frac{E[|\hat{H}_l^{(i)}(m) - H_l^{(i)}(m)|^2]}{E[|H_l^{(i)}(m)|^2]} \tag{34}$$

We also present the BER of the SC-FDE system which employs the proposed channel estimation scheme. We consider frequency selective Rayleigh fading channels with 5 paths of equal strength. The parameters of the system that is simulated are as follows: 16QAM modulation is used, $N_r = 4$, a block size $N = 64$ is used, the channel order $M = 4$, a CP of length 8 is used and the fading rate is $B_d T = 0.025$, where B_d is the Doppler spread and T is the data block interval. We also define E_b as the energy per bit, N_o as the single sided noise spectral density and E_b/N_o as the energy per bit to noise spectral density ratio.

The channel model used in the simulation is the quasi-static model. The channel is assumed to be constant for the duration of one FFT frame. On the following FFT frame, $\mathbf{h}^{(i+1)}$ is produced using a first order autoregressive model so that $E[\mathbf{h}^{(i)H}\mathbf{h}^{(i+1)}]/(\|\mathbf{h}^{(i)}\|\|\mathbf{h}^{(i+1)}\|) = J_0(2\pi B_d T)$. We assume that ambiguity and channel order determination are performed perfectly.

Fig. 3 shows the NMSE of the channel estimate and Fig. 4 shows the corresponding BER when rectangular averaging windows are used. We note in Fig. 3 that the NMSE is lower for longer window lengths when E_b/N_o is low but the opposite is true when E_b/N_o is high, an effect of the previously discussed trade-off. At E_b/N_o around 13.5 dB (which corresponds to BERs in the 10^{-3} range in Fig. 4), $W = 2$ appears to provide the lowest NMSE. We note in Fig. 4 that $W = 3$ achieves a BER of 10^{-3} at the lowest E_b/N_o. This BER is achieved at a loss in E_b/N_o 1.5 dB when compared to an SC-FDE system with perfect channel estimates.

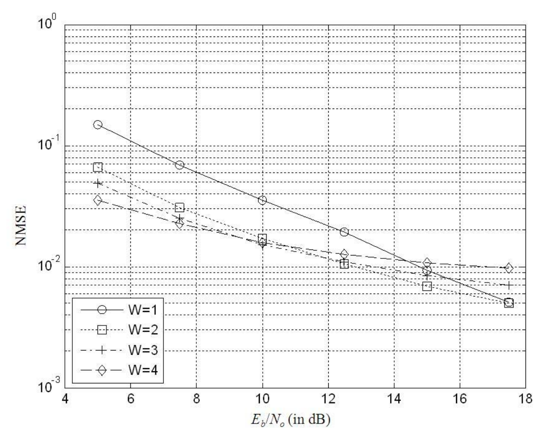

Fig. 3. NMSE of subspace-based channel estimates as a function of E_b/N_o when rectangular windows of length W are used.

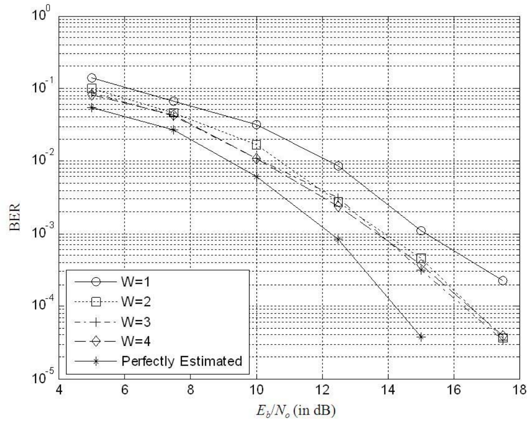

Fig. 4. BER of SC-FDE system employing subspace-based channel estimation when rectangular windows of length W are used.

The NMSE of the channel estimate for exponentially decaying windows is shown in Fig. 5 while Fig. 6 provides the corresponding BER. In Fig. 5 we note that the NMSE of the channels estimates improves as we decrease μ at low E_b/N_o but that higher values of μ become optimum as we increase E_b/N_o. At E_b/N_o around 13 dB, $\mu = 0.6$ provides the lowest NMSE. However, from Fig. 6, we obtain a BER of 10^{-3} with $\mu = 0.4$ at the lowest E_b/N_o. By using exponential window averaging with $\mu = 0.4$, we achieve a BER of 10^{-3} at a loss in E_b/N_o of 1 dB compared to the system with perfect channel estimates.

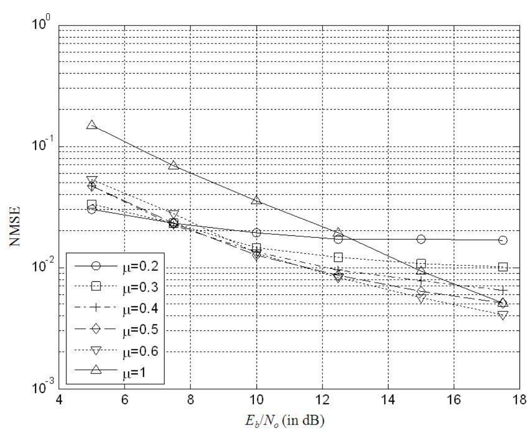

Fig. 5. NMSE of subspace-based channel estimates as a function of E_b/N_o when exponentially decaying windows are used.

The fact that the window with the lowest NMSE does not necessarily provide the lowest BER is due to the time-varying nature of the channel. As previously discussed, as the window length increases, the variance of the noise in the time estimates decreases, but the lag error increases.

V. Conclusion

Subspace-based channel estimation permits the transfer of information-bearing data while channel parameters are being estimated, thus improving spectral efficiency. In this paper, we presented an SC-FDE system that employs subspace decomposition for channel estimation purposes. For a system employing 4 receive antennas, 16QAM modulation and 64 symbol FFT blocks which operates in a slowly fading 5 tap channel, we show that the proposed technique can achieve a BER of 10^{-3} at a loss of only 1 dB compared to a system which estimates the channel gains perfectly.

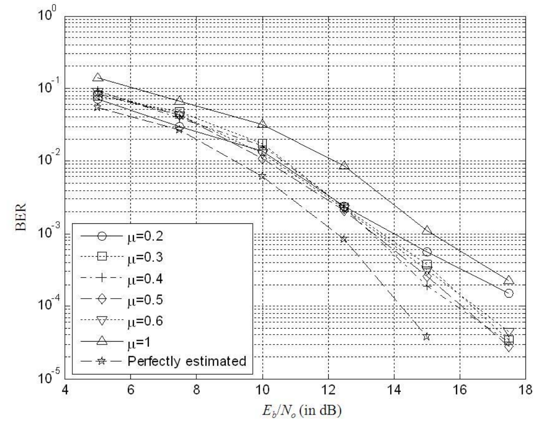

Fig. 6. BER of SC-FDE system employing subspace-based channel estimation when exponentially decaying windows are used.

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