

Realizing Fully Digital Precoders in Hybrid A/D Architecture With Minimum Number of RF Chains

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Abstract—In this letter, the minimum number of required radio frequency (RF) chains for realizing any MIMO fully digital precoder (FDP) with the hybrid analog/digital precoder (HADP) architecture is investigated from a new perspective. First, in order to take full advantage of the digital domain, an extended system formulation of the hybrid structure is presented in which the digital precoder is taken from an abstract transformation space. Then, a constrained optimization problem is formulated, where the aim is to minimize the number of RF chains subject to an equality constraint on the multi-stream FDP and HADP outputs. We show that non-unique optimum solutions exist for only two RF chains. Based on these results, we further develop a single RF chain design. The implementation of the latter does not require extra hardware or computational complexity. We compare the proposed HADP design with recent works that require larger number of RF chains and demonstrate that significant improvement in performance can be achieved over direct HADP design.

Index Terms—Hybrid beamforming, hybrid analog/digital precoder, massive MIMO, precoding, beamforming.

I. INTRODUCTION

MASSIVE-MIMO empowered by millimeter wave (mmWave) technology is emerging as a strong candidate for 5G and future wireless systems [1]–[7]. Precoding is undoubtedly a fundamental and powerful technique in the design of MIMO systems. In massive-MIMO, precoding plays a more critical role due to the severe path loss affecting mmWave communications. However, because of the very large number of antennas in massive-MIMO systems, the implementation of conventional fully digital precoders (FDPs) is not efficient. Indeed, FDP-based techniques require the use of one RF chain per antenna element, which increases the overall system cost and power consumption [4].

Recently, hybrid analog/digital precoders (HADP) have attracted considerable interest as a practical solution to this problem. Indeed, by cascading an analog (RF) precoder after the baseband digital precoder, it is possible to reduce the number of required RF chains. In [1], the first attempt was made to realize any FDP by means of the HADP structure with two RF chains for the case of a single data stream, i.e., one symbol per transmission. In [2] and [3] multi-stream hybrid designs were presented in which the number of RF

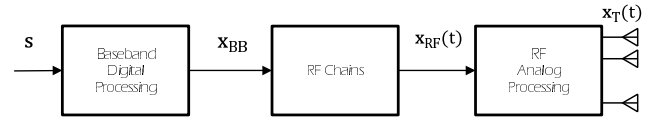


Fig. 1. Transmitter Modules.

chains must be set equal to the number of symbols per transmission. The HADP has an intricate structure where the entries of the RF precoder matrix satisfy constant modulus constraint (i.e., phase-shifters). Since the ensuing precoder optimization is non-convex, many works have alternatively focused on designing HADP directly using heuristic iterative algorithms or reconstruction algorithms [3], [4]. Additional practical power constraints for the analog precoder are discussed in [5].

In this paper, motivated by these works, we focus on realizing any given FDP with the HADP architecture while minimizing the required number of RF chains. To this end, we first introduce a generalized system model where the baseband digital precoder is viewed as an abstract transformation. Then, we formulate a constrained optimization problem to minimize the number of required RF chains such that the FDP and HADP outputs are equal. We show that feasible optimum solutions exist for only two RF chains; however, optimal solutions are not unique, which allows for a freedom in design. Based on the optimum solution so obtained, we then propose a simple yet novel HADP design for the realization of any given FDP with a single RF chain, which does not require extra hardware or computational complexity in its implementation. Finally, we compare the proposed HADP design to recent works that require a larger number of RF chains and show that significant performance improvement can be achieved by realizing FDP with the proposed HADP architecture.

II. SYSTEM MODEL

Consider a generic massive-MIMO transmitter with N_T antennas and N_{RF} RF chains. Within each transmission (time slot), there are K users to be served and for each user, D symbols are transmitted.

Let $\mathbf{s}_i = [s_{i,1}, s_{i,2}, \dots, s_{i,D}]^T$ be the symbol vector of the i th user, where $s_{i,j}$ is taken from a discrete constellation \mathcal{A} (such as M-QAM or M-PSK). Hence, the symbol vector $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_K^T]^T \in \mathcal{A}^{N_s}$, where $N_s = DK$ must be precoded and then transmitted. As shown in Fig. 1 which depicts the block diagram of the transmitter, vector \mathbf{s} is first processed by a baseband digital precoder, whose output vector is denoted as $\mathbf{x}_{BB} \in \mathbb{C}^{N_{RF}}$. Then, N_{RF} parallel RF chains are used to convert the baseband digital vector signal \mathbf{x}_{BB} into the bandpass modulated RF vector signal $\mathbf{x}_{RF}(t)$. The latter is next fed to the analog precoder which consists of relatively simple RF circuitry. The resulting output signal $\mathbf{x}_T(t)$ is then transmitted by means of an antenna array. In the sequel we let

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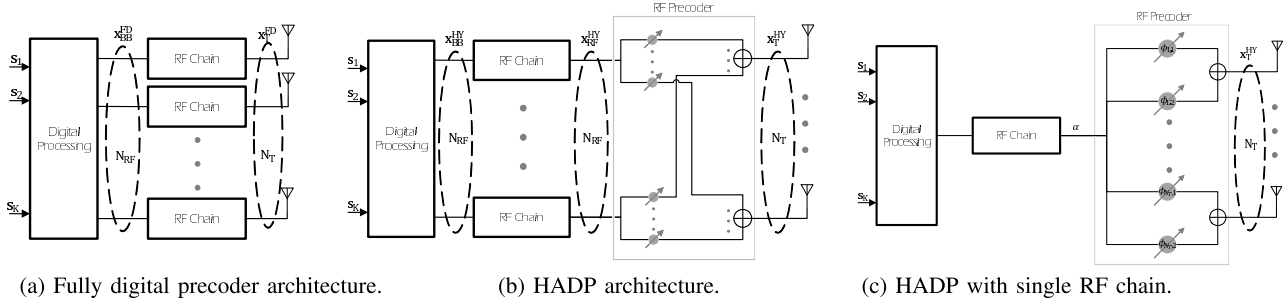


Fig. 2. Different precoder architectures.

$\mathbf{x}_{RF} \in \mathbb{C}^{N_{RF}}$ and $\mathbf{x}_T \in \mathbb{C}^{N_T}$ represent the baseband sampled version of the analog signals $\mathbf{x}_{RF}(t)$ and $\mathbf{x}_T(t)$, respectively, and drop the time dependence without loss of generality.

III. GENERALIZED PRECODER ARCHITECTURE

Fig. 2a illustrates a conventional FDP where each antenna element is directly connected to an RF chain, so that $N_{RF} = N_T$. Thus, the transmitted signal of each antenna element can be processed and controlled by the digital baseband processor. Hence, RF precoding is not required and we can write: $\mathbf{x}_T^{FD} = \mathbf{x}_{RF}^{FD} = \mathbf{x}_{BB}^{FD} \in \mathbb{C}^{N_T}$. Letting $\mathbf{P}_{FD} \in \mathbb{C}^{N_T \times N_s}$ denote the digital precoder matrix, the output of the FDP can be written as [4]:

$$\mathbf{x}_T^{FD} = \mathbf{P}_{FD}\mathbf{s}. \quad (1)$$

Fig. 2b illustrates a massive-MIMO transmitter with an HADP implementation. In this case, due to inherent feature of an HADP design, we have that $N_{RF} \ll N_T$. In this work, we consider a general framework in which the output of the digital baseband processor can be written as:

$$\mathbf{x}_{BB}^{HY} = \mathcal{P}(\mathbf{s}) \in \mathbb{C}^{N_{RF}}. \quad (2)$$

where $\mathcal{P}: \mathcal{A}^{N_s} \rightarrow \mathbb{C}^{N_{RF}}$ is a mapping from the given symbol vector \mathbf{s} to desired output \mathbf{x}_{BB}^{HY} . This is in contrast to the previous works in the literature, e.g. [1]–[5], where the digital precoder is limited to a linear transformation, i.e.:

$$\mathcal{P}(\mathbf{s}) = \mathbf{P}_D\mathbf{s}, \quad (3)$$

where $\mathbf{P}_D \in \mathbb{C}^{N_{RF} \times N_s}$ is a digital precoder with $N_{RF} \geq N_s$. It can be observed from Fig. 2b that in the adopted baseband notation, $\mathbf{x}_{RF}^{HY} = \mathbf{x}_{BB}^{HY} \in \mathbb{C}^{N_{RF}}$ since the digital and analog modules are connected via N_{RF} dedicated RF chains.

In the HADP literature [1]–[5], entries of analog precoder are constrained to have equal magnitude, where in [5] further practical power constraints are discussed. For simplicity, we assume the entries of the analog precoder all have unit norm. However, for any other given magnitude constraint $\kappa > 0$, as dictated by practical RF considerations, the RF precoder matrix \mathbf{P}_A and baseband digital output vector \mathbf{x}_{BB}^{HY} designed with the proposed approach can be scaled by κ and $1/\kappa$ respectively, while maintaining the same overall analog output. Thus, without loss of generality, we model the analog precoder as a linear transformation, represented by matrix $\mathbf{P}_A \in \mathbb{U}^{N_T \times N_{RF}}$ where \mathbb{U} is the set of complex number with unit norm:

$$\mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}. \quad (4)$$

Thus, the output of the analog precoder can be expressed as:

$$\mathbf{x}_T^{HY} = \mathbf{P}_A\mathbf{x}_{RF}^{HY}. \quad (5)$$

Hence, from (2) and (5), the output of the generalized HADP can be written as:

$$\mathbf{x}_T^{HY} = \mathbf{P}_A\mathcal{P}(\mathbf{s}). \quad (6)$$

Also, from (3) and (6), we observe that the conventional hybrid structure in [1]–[5] is a special case of (6) and has the following form:

$$\mathbf{x}_T^{HY} = \mathbf{P}_A\mathbf{P}_D\mathbf{s}. \quad (7)$$

IV. REALIZING ANY FDP IN HADP ARCHITECTURE

In the recent literature, based on the model in (7), the HADP design problem is reduced to the design of the matrix factors \mathbf{P}_A and \mathbf{P}_D . Consequently, for a given symbol vector \mathbf{s} , the effective precoder determines the output of the HADP:

$$\mathbf{x}_T^{HY} = \mathbf{P}_{\text{eff}}\mathbf{s} = \mathbf{P}_A\mathbf{P}_D\mathbf{s}. \quad (8)$$

Hence, designing \mathbf{P}_A and \mathbf{P}_D such that $\mathbf{P}_{\text{eff}} = \mathbf{P}_{FD}$, where \mathbf{P}_{FD} is the given FDP, under the constraint $N_{RF} \ll N_T$ is the main goal in [1]–[3]. Extending the design of [1] to multi stream results in utilizing $N_{RF} = 2N_s$ RF chains for realizing any fully digital precoder in hybrid structure. In [2] and [3], realizations of any FDP in hybrid architecture with $N_{RF} = N_s$ RF chains are proposed.

Here, we investigate the minimum number of required RF chains for realizing any FDP in hybrid structure. To this end, we use the generalized digital baseband precoding model (2) and seek to realize the underlying given FDP by means of the proposed generalized HADP structure, that is:

$$\mathbf{P}_A\mathcal{P}(\mathbf{s}) = \mathbf{P}_{FD}\mathbf{s}. \quad (9)$$

Accordingly, we formulate the following optimization problem for minimizing the number of RF chains whilst realizing any arbitrary FDP \mathbf{P}_{FD} with HADP:

$$\min_{\mathbf{P}_A, \mathcal{P}(\cdot)} N_{RF} \quad (10a)$$

$$\text{subject to } \mathbf{P}_A\mathcal{P}(\mathbf{s}) = \mathbf{P}_{FD}\mathbf{s} \quad (10b)$$

$$\mathbf{P}_A \in \mathbb{U}^{N_T \times N_{RF}} \quad (10c)$$

This optimization is clearly not convex as \mathbb{U} is a non-convex set as shown in (4). Moreover, the generality of \mathcal{P} makes it difficult to solve this problem. To overcome this limitation, we first replace $\mathcal{P}(\mathbf{s})$ by the auxiliary signal \mathbf{x}_{BB}^{HY} in (2) and then solve the problem for \mathbf{x}_{BB}^{HY} for the given \mathbf{s} . Subsequently, we shall seek \mathcal{P} such that $\mathcal{P}(\mathbf{s}) = \mathbf{x}_{BB}^{HY}$. Hence, we have:

$$\min_{\mathbf{P}_A, \mathbf{x}_{BB}^{HY}} N_{RF} \quad (11a)$$

$$\text{subject to } \mathbf{P}_A\mathbf{x}_{BB}^{HY} = \mathbf{P}_{FD}\mathbf{s} \quad (11b)$$

$$\mathbf{P}_A \in \mathbb{U}^{N_T \times N_{RF}} \quad (11c)$$

The existence of a solution to this problem is asserted by Theorem 1 below, whose proof relies on the following lemma.

Lemma 1: (See [1, Th. 1]): For positive real numbers β_1 and β_2 , any complex number z where $|\beta_1 - \beta_2| \leq |z| \leq |\beta_1 + \beta_2|$ can be written as: $z = \beta_1 e^{j\theta_1} + \beta_2 e^{j\theta_2}$ where $\theta_1, \theta_2 \in [0, 2\pi]$ and θ_1, θ_2 might be non-unique.

Theorem 1: There exist non-unique solutions for \mathbf{P}_A and \mathbf{x}_{BB}^{HY} which minimize (11) with optimum objective function value $N_{RF} = 2$.

Proof: Since $N_{RF} \in \{1, 2, 3, \dots\}$ is a natural number, in order to solve (11), we can take the simple approach of starting from the least possible value for N_{RF} and show whether the constraints can be satisfied or not; if not, we move on to the next number and so on until the constraints are satisfied. As \mathbf{P}_{FD} and \mathbf{s} are given, the right hand side of (11b), i.e., $\mathbf{x}_T^{FD} = \mathbf{P}_{FD}\mathbf{s}$ in (1), is an arbitrary complex vector of size N_T .

1) $N_{RF} = 1$: In this case, \mathbf{P}_A is a vector of size N_T in $\mathbb{U}^{N_T \times 1}$ and \mathbf{x}_{BB}^{HY} is a complex scalar. Since \mathbf{P}_A has unit modulus entries, $\mathbf{P}_A \mathbf{x}_{BB}^{HY}$ would be a vector of size N_T where all the entries have the same magnitude. Therefore, because \mathbf{x}_T^{FD} is an arbitrary complex vector, $\mathbf{P}_A \mathbf{x}_{BB}^{HY} = \mathbf{x}_T^{FD}$ is not generally satisfied. Thus, $N_{RF} = 1$ is not the minimum of (11) under the given constraints.

2) $N_{RF} = 2$: Let $x_i = |x_i|e^{j\vartheta_i}$ denote the polar representation of the i th entry of the vector \mathbf{x}_T^{FD} . Define $|x|_{min}$ and $|x|_{max}$ as the minimum and maximum values of $|x_i|$ for $i = 1, \dots, N_T$, respectively. In the case $N_{RF} = 2$, \mathbf{P}_A and \mathbf{x}_{BB}^{HY} can be explicitly written as:

$$\mathbf{P}_A = \begin{bmatrix} e^{j\phi_{1,1}} & e^{j\phi_{2,1}} & \dots & e^{j\phi_{N_T,1}} \\ e^{j\phi_{1,2}} & e^{j\phi_{2,2}} & \dots & e^{j\phi_{N_T,2}} \end{bmatrix}^T \quad (12a)$$

$$\mathbf{x}_{BB}^{HY} = [\alpha_1 \quad \alpha_2]^T. \quad (12b)$$

From Lemma 1, there exist non-unique angles $\phi_{i,1}$ and $\phi_{i,2}$ and positive real numbers α_1 and α_2 such that:

$$x_i = \alpha_1 e^{j\phi_{i,1}} + \alpha_2 e^{j\phi_{i,2}}, \quad (13a)$$

$$|\alpha_1 - \alpha_2| \leq |x|_{min}, \quad |x|_{max} \leq |\alpha_1 + \alpha_2|. \quad (13b)$$

Thus, we have shown that for any given vector \mathbf{s} and FDP matrix \mathbf{P}_{FD} , there exist optimal solutions for \mathbf{P}_A (12a) and \mathbf{x}_{BB}^{HY} (12b) such that the constraint $\mathbf{P}_A \mathbf{x}_{BB}^{HY} = \mathbf{P}_{FD} \mathbf{s}$ is satisfied. ■

Theorem 1 indicates that in order to realize an arbitrary FDP with an HADP architecture, each antenna requires only two phase shifters and therefore only two RF chains are needed to provide $\mathbf{x}_{BB}^{HY} = [\alpha_1, \alpha_2]^T$ to the two sets of phase shifters. (i.e., α_1 is connected to the phase shifters of \mathbf{P}_A 's first column and α_2 to the second column). From Lemma 1, however, we can see that the optimal solutions for \mathbf{P}_A and \mathbf{x}_{BB}^{HY} are not unique and there exists degrees of freedom for designing \mathbf{P}_A and \mathbf{x}_{BB}^{HY} . In particular, if there exists an optimal solution where $\alpha_1 = \alpha_2 = \alpha$, i.e., $\mathbf{x}_{BB}^{HY} = [\alpha, \alpha]^T$, then, only one RF chain is sufficient to realize any FDP in the HADP system, as depicted in Fig. 2c. In the following theorem, an optimal solution of (11) is presented under the constraint $\alpha_1 = \alpha_2$ which leads to a novel HADP design with a single RF chain.

Theorem 2: Any FDP can be realized by a single RF chain HADP as shown in Fig. 2c with the following parameters:

$$\alpha \geq \frac{1}{2}|x|_{max} \quad (14a)$$

$$\phi_{i,j} = \vartheta_i + (-1)^j \cos^{-1} \left(\frac{|x_i|}{2\alpha} \right), \quad j = 1, 2. \quad (14b)$$

Proof: For $\alpha \geq \frac{1}{2}|x|_{max}$, setting $\alpha_1 = \alpha_2 = \alpha$ satisfies the necessary condition (13b) because $|x|_{min} \geq |\alpha_1 - \alpha_2| = 0$ and $|x|_{max} \leq |\alpha_1 + \alpha_2| = 2\alpha$. Also, since $0 \leq |x|_{max} \leq 2\alpha$, it follows that the argument of the \cos^{-1} functions in (14b) satisfies $0 \leq \frac{|x_i|}{2\alpha} \leq 1$. Under this condition, using the principal value of \cos^{-1} in $[0, \pi/2]$, it can be verified through algebraic manipulations that the representation (13a) is satisfied for every x_i , with $i = 1, \dots, N_T$. ■

Algorithm 1 Realizing Any FDP in HADP

Given: $\mathbf{s}, \mathbf{P}_{FD}$

Output: \mathbf{x}_T^{HY}

1. Calculate the desired transmit signal by: $\mathbf{x}_T^{FD} = \mathbf{P}_{FD} \mathbf{s}$.
2. Choose α as (14a).
3. Set \mathbf{P}_A in (12a) using (14b).
4. Feed α to the RF chain.

Note: The output of the HADP, i.e., \mathbf{x}_T^{HY} , is identical to \mathbf{x}_T^{FD} .

The only problem left for realizing any FDP in HADP architecture with minimum N_{RF} is to find a mapping \mathcal{P} such that $\mathcal{P}(\mathbf{s}) = \mathbf{x}_{BB}^{HY}$. Since our design requires only one RF chain and, more importantly, the output of the baseband digital processing module is already determined, i.e., the positive number α , this problem can be solved by taking advantage of the digital processing. In other words, instead of going through the difficult problem of finding a closed-form representation for \mathcal{P} , we can hard code the desired output to $\mathcal{P}(\mathbf{s})$. In essence, since it is the output of the digital precoder that really matters, and not the function which produces it, instead of finding \mathcal{P} , we focus on producing the output of $\mathcal{P}(\mathbf{s}) = \mathbf{x}_{BB}^{HY} = [\alpha, \alpha]^T$. Algorithm 1 summarizes our proposed design for realizing any FDP in HADP with a single RF chain.

V. IMPLEMENTATION ASPECTS

Except for its reduced number of RF chains, the proposed scheme is characterized by an HADP architecture similar to [1]–[7]. However, certain important considerations need to be taken into account for its practical implementation:

A. RF precoder Update Rate

In a conventional HADP system, to allow for accurate tracking of the wireless channel conditions, the RF precoder coefficients are updated according to the channel coherence time, T_c . In the proposed scheme however, the RF precoder coefficients are also influenced by the transmit symbol vector \mathbf{s} and as such, they must be updated according to the symbol duration T_s . Since $T_s < T_c$ in slowly-varying channels, this means that in this case, the RF precoder needs be updated at a higher rate, i.e., by a factor T_c/T_s . However, recent advances in RF semiconductor and mmWave hardware [6], [7] suggest that it will be possible to accommodate such requirements in near future. Alternatively, at the price of increasing latency,

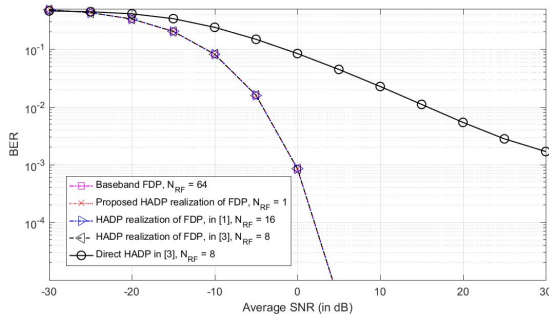


Fig. 3. BER versus SNR for different methods.

the update rate of the RF precoder can be reduced by increasing the symbol duration T_s , i.e. the length of the symbol vector.

B. Computational Complexity

The proposed HADP design does not introduce any major additional complexity. Compared to conventional HADP methods, only the digital baseband processing is modified and extended, while the RF processing has been much simplified through reduction of N_{RF} . For a chosen FDP, our approach provides its HADP realization in two phases. Every T_c , the desired FDP matrix must be calculated according to the selected algorithm, as is the case for other HADP methods [2], [3]. Since FDP techniques usually admit closed form solutions, less computational complexity is required compared with iterative HADP designs such as those in [3] and [4], where complex calculations are performed in each iteration, while the required number of iterations itself varies. Having designed the FDP matrix, within each T_s , a new symbol vector must be precoded. To this end, the HADP designs in [2]–[4] require calculations with complexity order of $O(N_{RF}N_s)$. In our design, step 1 of the algorithm is a simple matrix multiplication with complexity order $O(N_TN_s)$ while from (12) and (14), the second and thirds steps only require $O(N_T)$ complex operations or function evaluations. Thus the overall complexity order is dominated by the first step. However, this only represents a small fraction of the total computations needed for the FDP calculation or HADP design in iterative approaches [3], [4].

C. Digital Mapping \mathcal{P}

Although the digital processing is extended to the mapping $\mathcal{P}(\mathbf{s})$ in (2), as opposed to the linear transformation \mathbf{P}_{DS} , the realization of the proposed HADP scheme is similar to the conventional HADP ones because we neither need to implement the mapping nor calculate it. As shown in Algorithm 1, first the desired precoded signal \mathbf{x}_T^{FD} is computed, and then α is determined and assigned as the output of the digital domain processor to drive the RF precoder. In fact, since the desired output of $\mathcal{P}(\mathbf{s})$ is already calculated, i.e., $\mathbf{x}_{\text{BB}}^{\text{HY}}$, we do not need to implement the nonlinear mapping \mathcal{P} nor apply it on \mathbf{s} to obtain $\mathbf{x}_{\text{BB}}^{\text{HY}}$.

D. Number of Served Users

Since increasing the number of RF chains may not be feasible in an already deployed system, the number of precoded symbols per transmission in conventional HADP-based systems [1]–[5] is limited by N_{RF} (i.e., $N_s \leq N_{RF}$), which makes

a direct migration of such system to massive-MIMO unlikely. In contrast, migration of a system based on the proposed scheme does not require any change on the RF chain; only the RF precoder needs to be expanded according to N_T since the proposed HADP does not limit the number of precoded symbols per transmission.

VI. SIMULATION RESULTS

We consider a similar set up as [3], that is: 64×8 massive-MIMO system with an $L = 15$ paths directional vector channel model. At the receiver, fully digital decoding (combining) is used for all the methods. Fig. 3 shows the bit error rate (BER) performance versus signal-to-noise ratio (SNR) of the different methods for $N_s = 8$ symbols and QPSK modulation. It is observed that the optimal baseband FDP and consequently, the corresponding HADP realizations (including the proposed design) outperform the direct HADP design in [3] by more than 20 dB at a BER= 10^{-2} . Fig. 3 also illustrates that the proposed HADP design perfectly matches the performance of the other HADP realizations of FDP as well as the baseband FDP. Furthermore while the conventional baseband FDP and the corresponding HADP realizations in [1] and [3] require 64, 16 and 8 RF chains, respectively, the presented solution requires only a single RF chain.

VII. CONCLUSION

We provided a solution for an efficient implementation of HADP for massive-MIMO systems. The proposed design requires a minimum number of RF chains while matching the performance of optimal baseband FDP. First, we presented the generalized system formulation of the hybrid architecture. Then, we minimized the number of RF chains such that any given FDP can be realized in the extended HADP architecture. Based on the obtained optimum solutions, we then presented our hybrid design for realizing any FDP in the HADP architecture with a single RF chain. The presented simulation results verify that the proposed single RF chain hybrid design outperforms a direct HADP design scheme and can actually achieve the same performance as the optimal baseband FDP.

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