Constrained Adaptive Echo Cancellation for Discrete Multitone Systems

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***Abstract*—In communication systems where full-duplex transmission is required, echo cancellers are deployed to cancel the interference of the transmitted signal at the collocated receiver. For systems using discrete multitone (DMT) modulation, echo cancellation is performed partially in the time and frequency domains to decrease the processing complexity. In this paper, echo cancellation for DMT systems is reformulated as a constrained optimization problem, where a cost function is minimized over an extended linear space. This extended space contains the weights of the finite-impulse-response (FIR) filter emulating the echo channel in the time and the frequency domains, while linear constraints are used to ensure the proper mapping between these two domains. Based on this proposed formulation, a new constrained adaptive echo cancellation structure for DMT-based digital subscriber lines (DSL) systems is proposed. The proposed formulation provides a unifying framework for different practical DSL systems (i.e., frame asynchronous and multirate), as well as additional flexibility in implementation by allowing the incorporation of supplementary constraints that can improve the performance of the system. As an illustrative example, we show how the robustness of the echo canceller can be improved in the presence of radio frequency interference by adding appropriate constraints on the extended linear space.**

***Index Terms*—Adaptive filtering, constrained optimization, discrete multitone, DSL systems, echo cancellation, radio frequency interference.**

I. Introduction

In high-speed digital communication systems, it is of interest to have a full-duplex, i.e., simultaneous bidirectional, transmission between the transmitter and the receiver, as it generally leads to lower latency and greater data throughput. However, full-duplex transmission results in an unwanted interference of the transmitted signal on the collocated received signal, which is known as echo. The echo can be prevented by using different methods such as, electrically shielding the transmitter and the receiver, frequency division duplexing or deploying digital echo cancellers (a combination of these methods can also be used).

In digital subscriber line (DSL) systems, which are widely used for broadband communication over the existing telephone lines, a single pair of wires is used for full-duplex transmission. Thus, a hybrid circuit (i.e., a four-wire to two-wire interface) is used to separate the transmitter and receiver circuits from the line [1]. However, because of the impedance mismatch in the hybrid, the transmitted signal is not completely blocked and leaks into the collocated receiver, causing an undesirable echo. In these systems, the frequency division duplexing, where a frequency gap between the two transmission directions is required, can be used to prevent the deteriorating effect of the echo on the received signal; however, this demands precise analog filters at the receiver which is both expensive and restrictive. Alternately, an echo canceller can be used which removes the echo by subtracting an emulated echo from the received signal at the receiver. As a result, the two directions can overlap, so the frequency band is used more efficiently and higher data rates are achieved. In a basic echo canceller, an estimation of the echo is first generated by performing the linear convolution between the transmitted signal (causing the echo) and the estimated echo channel; the estimated echo is then subtracted from the received signal. In this method, the echo channel is modeled by a finite impulse response filter which is adaptively updated.

In DSL systems based on the discrete multitone (DMT) modulation, the channel is divided into orthogonal subchannels (or tones), and data is modulated and demodulated by the use of the inverse discrete Fourier transform (IDFT) and DFT operations. The direct time domain implementation of the basic echo canceller for DMT-based systems is characterized with both high computational cost and slow convergence; however, this can be improved by exploiting the structure in the transmitted signal. In [2], Cioffi *et al.* proposed a mixed frequency and time domains echo canceller where echo emulation is performed in the time domain (including the cross-tones echo emulation) and the echo weights are updated independently in the frequency domain, resulting in an improved convergence and lower complexity of the echo canceller. Later, in [3], Ho *et al.* proposed the circular echo synthesis (CES) method, where the circular part of the echo is also emulated and cancelled efficiently in the frequency domain and only the residual echo is cancelled in the time domain.

Other methods for echo cancellation in DMT-based systems propose modifications to this latter echo canceller in order to lower its complexity and/or improve its performance. For instance in [4], Jones has shown that the proper choice of the delay between the transmitted and received symbols can help reduce the complexity of the system. In [5], Ysebaert *et al.* have added double talk cancellation to the receiver, i.e., removing the effect of the far end signal, in order to improve the convergence of the adaptive weight update. In [6], they have also proposed an asynchronous echo canceller (integrated with double talk cancellation), which is used when there is a misalignment between the transmitted symbols and received symbols. In [7], Ysebaert *et al.* also proposed a circular decomposition echo canceller,

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which improves the convergence of the canceller without requiring extra power on the unused tones, and in [8], Pisoni *et al.* extend and modify this echo canceller by using symmetric decomposition. There are other methods for echo cancellation which perform both the echo emulation and weight update in the frequency domain, such as in [9] and [10] where per-tone echo cancellers are proposed.

In this paper, we show that the mixed time and frequency domains adaptive echo canceller for DMT-based system (discussed in [3]) can be viewed as a constrained optimization problem, where a cost function is minimized over an extended linear space. This extended space contains the weights of the finite-impulse-response (FIR) filter modeling the echo channel both in the time and frequency domains. In most of the echo cancellation methods for DMT-based systems, the frequency-domain adaptive echo weights are mapped into the time domain filtering weights used for the echo emulation by means of the inverse Fourier transform. In the proposed constrained echo canceller formulation, this mapping can be regarded as a linear constraint on an extended set of weights. Based on this interpretation, we introduce a constrained adaptive echo canceller structure for DMT-based systems. We examine two implementations for this echo canceller, the first one is based on the work by Frost in [11] and the second one is based on the generalized sidelobe canceller (GSC) structure by Griffiths *et al.* in [12]. A preliminary implementation of the constrained echo canceller has been previously introduced by authors in [13].

In this paper, we also show that the proposed constrained echo canceller structure can be applied to most of the DSL systems used in practice, such as frame asynchronous and multirate systems. Therefore, the proposed structure provides a unified framework for the existing methods for echo cancellation in DMT-based DSL systems and opens the door to a realm of new opportunities. For instance, using the proposed constrained optimization framework, supplementary constraints can be applied in time or/and frequency domain to the echo canceller to improve the performance of the system. One such application is in the presence of radio frequency (RF) interference in DSL systems; these interferences have various sources, such as AM broadcasts and HAM amateur radios [1]. In a system affected by RF interference, constraints can be appended to the system to improve its robustness. We have implemented both linearly and quadratically constrained adaptive echo cancellers in the presence of RF interference and illustrate their capability to combat the deteriorating effect of such interferences, and especially targeting the drifting problem.

This paper is organized as follows. In Section II, echo cancellation for DMT-based DSL systems is reviewed, including frame asynchronous and multirate echo cancellers. In Section III, the linearly constrained adaptive echo cancellation method for DMT-based systems is introduced. In Section IV, implementation of the current echo cancellers for DSL systems based on the proposed method is discussed, and the constrained echo canceller is also examined in the presence of the RF interference, and the improvements gained from the supplementary linear and quadratic constraints are discussed. Section V contains the simulation results examining the convergence of the discussed algorithms and computational complexity comparison. Finally, Section VI concludes the research.

The following notations are used throughout the paper. The square identity matrix of size N is denoted by $\mathcal{I}_N$. The all zero matrix of size $N \times M$ is denoted by $\mathcal{O}_{N\times M}$. The discrete Fourier transformation and inverse discrete Fourier transformation matrices of size $N \times N$ are denoted by $\mathcal{F}_N$ and $\mathcal{F}_N^{-1}$, respectively, where $[\mathcal{F}_N]_{k,l} = \left(1/\sqrt{N}\right) e^{-j2\pi kl/N}$ and $\mathcal{F}_N^{-1} = \mathcal{F}_N{}^H$. The superscripts of $(\cdot)^H$ and $(\cdot)^T$ indicate the conjugate transpose and transpose operations, respectively. An $N \times N$ Toeplitz matrix, with the first column given by $[c_0, c_1, \ldots, c_{N-1}]^T$ and the first row given by $[c_0, r_1, \ldots, r_{N-1}]$, is denoted by

$$T\{c_{N-1}, \ldots, c_1, \underline{c_0}, r_1, \ldots, r_{N-1}\}.$$

In addition, an $N \times N$ circulant matrix, with the first column given by $[c_0, c_1, \ldots, c_{N-1}]^T$, is denoted by $C\{c_0, c_1, \ldots, c_{N-1}\}$. Finally, $\mathrm{diag}\{\mathbf{v}\}$ indicates a diagonal matrix whose diagonal elements are given by vector $\mathbf{v}$.

II. Background

In this section, we examine the echo canceller structure using CES, as introduced by Ho *et al.* for DMT-based DSL systems [3]. In this method, echo cancellation is performed by subtracting the emulated echo from the received signal using both time and frequency representations. If the echo is periodic (i.e., generated by a circular convolution in the time domain), it can be regenerated with reduced complexity in the frequency domain. Therefore, in the echo canceller with CES, the only process performed in the time domain is to make the echo periodic by reconstructing and canceling the echo generated by the tail and the head, respectively, of the transmitted signal; the remaining echo is then removed in the frequency domain with one complex multiplication per tone. A mathematical formulation of this method is presented in Section II-A, based on the frame synchronous DSL systems. Extension to the frame asynchronous and multirate systems are briefly discussed in Sections II-B and II-C, respectively.

A. Synchronous Echo Cancellation

A multicarrier structure is assumed where DMT modulation and demodulation are performed by the use of IDFT/DFT of equal length N both at the transmitter and receiver. The transmitted time domain symbol at symbol period k is represented by $\mathbf{u}(k) = \left[u_0^k, \ldots, u_{N-1}^k\right]^T$, which is obtained as the N-point IDFT of the vector $\mathbf{U}(k)$ which contains the QAM modulated data of the transmitter in the frequency domain. Later, a cyclic prefix with the length υ (which is larger than the assumed length of the far end channel) is added to each time domain symbol. This setup ensures that the channel is equivalent to a bank of parallel independent subchannels. The true echo channel, assumed to be of length $M \leq N$ samples is represented by the zero-padded vector $\mathbf{h}$ of size $N \times 1$. The latter models the effect of the hybrid circuit, the digital and analog front end filters and the time domain equalizer (TEQ).

In the synchronous case where the received and echo frames are aligned, the echo symbol $\mathbf{y}(k)$ affecting the received symbol at symbol period k can be described completely in terms of the two consecutive transmitted symbols $\mathbf{u}(k-1)$ and $\mathbf{u}(k)$ (two symbols are needed because the length of the echo channel is usually longer than the cyclic prefix). For clarity, we refer to these transmitted symbols as the echo reference symbols. The

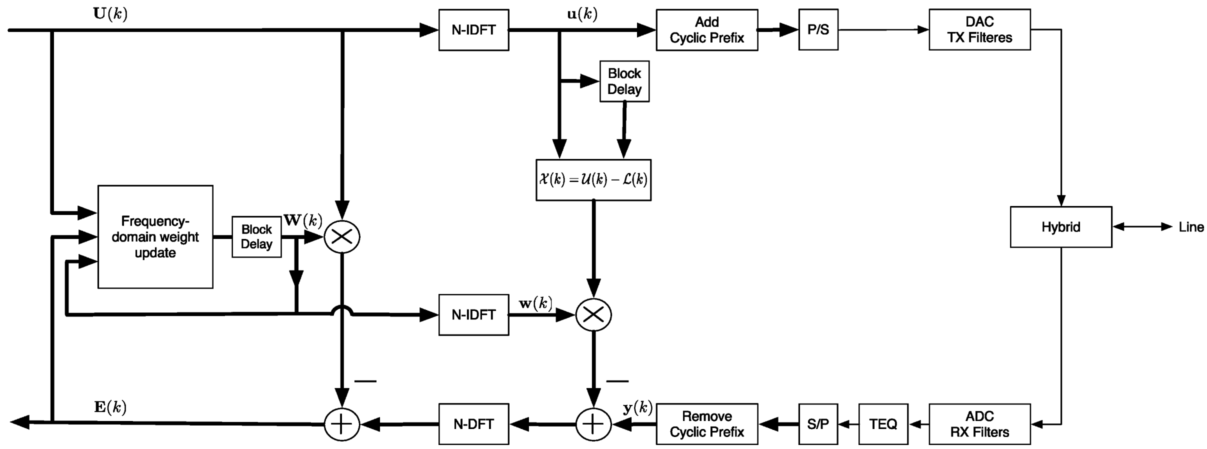

Fig. 1. Block diagram for CES echo canceller.

echo symbol is generated by the linear convolution of the echo reference symbols and the echo channel and can be expressed in the form

$$\mathbf{y}(k) = \mathcal{U}(k)\mathbf{h} \tag{1}$$

where $\mathcal{U}(k)$ is an $N \times N$ Toeplitz matrix consisting of the elements from symbols $\mathbf{u}(k)$ and $\mathbf{u}(k-1)$, defined by

$$\mathcal{U}(k) = T\left\{u_{N-1}^k, \ldots, \underline{u_0^k}, u_{N-1}^k, \ldots, u_{N-\upsilon}^k, u_{N-1}^{k-1}, \ldots, u_{\upsilon+1}^{k-1}\right\}. \tag{2}$$

The true echo channel $\mathbf{h}$ is unknown in practice, so the emulated echo is estimated by replacing $\mathbf{h}$ by $\mathbf{w}(k)$ which is the weight vector of the adaptive filter modeling the echo channel at time k. Using (1), the emulated echo is then given by

$$\mathbf{y}_e(k) = \mathcal{U}(k)\mathbf{w}(k). \tag{3}$$

The echo weights can be obtained adaptively using various methods, e.g., the least mean square (LMS) algorithm, in which an error signal is used to update the weights iteratively [14]. The error signal is the difference between the received signal and the emulated echo, i.e.,

$$\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{y}_e(k). \tag{4}$$

As proposed in [3], in order to avoid the matrix multiplication in (3), the echo emulation is performed partially in the time and frequency domains. This can be achieved by rewriting the matrix $\mathcal{U}(k)$ as a sum of a circulant matrix $\mathcal{L}(k)$ and a correction matrix $\mathcal{X}(k)$, as follows:

$$\mathcal{U}(k) = \mathcal{X}(k) + \mathcal{L}(k) \tag{5}$$

where $\mathcal{L}(k)$, defined as

$$\mathcal{L}(k) = C\left\{u_0^k, u_1^k, \ldots, u_{N-1}^k\right\} \tag{6}$$

is the periodic part of $\mathcal{U}(k)$, and contains only the elements of symbol $\mathbf{u}(k)$, and $\mathcal{X}(k) = \mathcal{U}(k) - \mathcal{L}(k)$ is an upper triangular matrix. Using (5), the emulated echo can be expanded as

$$\mathbf{y}_e(k) = \mathcal{X}(k)\mathbf{w}(k) + \mathcal{L}(k)\mathbf{w}(k). \tag{7}$$

The circular term $\mathcal{L}(k)\ \mathbf{w}(k)$ can be implemented in the frequency domain with less complexity, by diagonalizing the circulant matrix $\mathcal{L}(k)$. This can be achieved by using DFT and IDFT matrices, resulting into the decomposition

$$\mathcal{L}(k) = \mathcal{F}_N^{-1}\ \mathrm{diag}\left\{\mathbf{U}(k)\right\}\mathcal{F}_N \tag{8}$$

where $\mathbf{U}(k) = \mathcal{F}_N\ \mathbf{u}(k)$. Therefore, the error signal in the frequency domain (*i.e.*, $\mathbf{E}(k) = \mathcal{F}_N\ \mathbf{e}(k)$) can be written as

$$\mathbf{E}(k) = \mathcal{F}_N\left(\mathbf{y}(k) - \mathcal{X}(k)\mathbf{w}(k)\right) - \mathrm{diag}\left\{\mathbf{U}(k)\right\}\mathbf{W}(k) \tag{9}$$

where $\mathbf{W}(k) = \mathcal{F}_N\ \mathbf{w}(k)$ contains the echo channel weights in the frequency domain. As shown in Fig. 1, partial echo cancellation is performed in the time domain, and the residual signal is transferred into the frequency domain, where the remaining echo is emulated and cancelled on a per tone basis.

Finally, the error signal is used to update the echo weights. In [3], an approximate LMS update in the frequency domain is used, as given by

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \mu\ \mathrm{diag}\left\{\mathbf{U}^*(k)\right\}\mathbf{E}(k) \tag{10}$$

where μ is the step size. The use of the approximate diagonal update (10) in place of the exact LMS update deteriorates the convergence but decreases the complexity. The inverse Fourier transform is then applied to map the updated weights into the time domain where the last $N-M$ elements are zeroed. The complete block diagram for this echo canceller is shown in Fig. 1.

B. Frame Asynchronous Echo Cancellation

In the previous section, the frame synchronous echo cancellation has been discussed. In practical systems, there is a frame misalignment between the echo reference symbols and the received echo symbols. If this misalignment is set to a predetermined value at the transceiver on one side, then the misalignment at the other side is imposed by the propagation delay of the channel. Therefore, only one transceiver can perform echo cancellation synchronously, and at the other side it should be done asynchronously. Ho *et al.* have shown, in [3], that by adding an extra pair of DFT/IDFT to the echo canceller it becomes possible to adjust the alignment between echo frames and received frames at the receiver independently of their original alignment. The extra IDFT maps the frequency domain emulated echo to the time domain where it is then subtracted serially; DFT maps the error signal back to the frequency domain where it is used in the adaptive update. In [5], Ysebaert *et al.* have shown that if the error signal does not contain the far end signal, the convergence of the echo canceller improves. This method, known as *double talk cancellation*, can be easily implemented if the echo canceller and demodulator share the DFT at the receiver side. However, adding the extra DFT/IDFT pair at the receiver side, used for asynchronous echo canceller based on [3], inhibits the low complexity implementation of the double talk cancellation. Therefore, in [6], an asynchronous echo canceller that can be implemented with just one additional DFT at the transmitter is presented, which enables the implementation of the double talk cancellation. In this section we consider the method introduced in [6].

In a synchronous echo canceller, the echo is completely determined from two consecutive echo reference symbols. However, in the asynchronous case, where there is a misalignment or delay of Δ between the echo and received frames, three consecutive echo reference symbols, *i.e.*, $\mathbf{u}(k-1)$, $\mathbf{u}(k)$ and $\mathbf{u}(k+1)$, are needed to completely determine the echo signal. Therefore, the echo is given by

$$\mathbf{y}(k) = \mathcal{U}_a(k)\mathbf{h} \tag{11}$$

where the matrix $\mathcal{U}_a(k)$ is given by

$$\begin{aligned}\mathcal{U}_a(k) = T\Big\{&u_{\Delta-\upsilon-1}^{k+1}, \ldots, u_0^{k+1}, u_{N-1}^{k+1}, \ldots, u_{N-\upsilon}^{k+1},\\ &u_{N-1}^{k}, \ldots, \underline{u_{\Delta}^{k}}, \ldots, u_0^{k}, u_{N-1}^{k}, \ldots,\\ &u_{N-\upsilon}^{k}, u_{N-1}^{k-1}, \ldots, u_{\Delta+\upsilon+1}^{k-1}\Big\}.\end{aligned} \tag{12}$$

This matrix can be decomposed as $\mathcal{U}_a(k) = \mathcal{X}_a(k) + \mathcal{L}_a(k)$. Unlike the previous section, the circulant matrix $\mathcal{L}_a(k)$ in general contains symbols from two consecutive frames k and $k-1$ and is defined as $\mathcal{L}_a(k) = C\{\mathbf{u}_a(k)\}$, where $\mathbf{u}_a(k)$ is given by

$$\begin{aligned}\mathbf{u}_a(k) = \big[&u_{\Delta}^{k}, \ldots, u_{N-N_b+\Delta-1}^{k}, u_{N-N_b+\Delta+\upsilon}^{k-1}, \ldots, u_{N-1}^{k-1},\\ &u_{N-\upsilon}^{k}, \ldots, u_{N-1}^{k}, u_0^{k}, \ldots, u_{\Delta-1}^{k}\big].\end{aligned} \tag{13}$$

N_b is a design parameter which can be used to reduce the complexity independent of the value of the delay. As shown in [6], if N_b is equal to half of the length of the echo channel, the number of the operations in the time domain part is minimized. For delay values such that the matrix $\mathcal{L}_a(k)$ contains elements from two symbols, an extra DFT is needed at the transmitter side. Similar to the previous section, the expression for the error signal $\mathbf{E}(k)$, used in the adaptive update, is obtained as follows:

$$\mathbf{E}(k) = \mathcal{F}_N\left(\mathbf{y}(k) - \mathcal{X}_a(k)\mathbf{w}(k)\right) + \mathrm{diag}\left\{\mathbf{U}_a(k)\right\}\mathbf{W}(k) \tag{14}$$

where $\mathbf{U}_a(k) = \mathcal{F}_N\,\mathbf{u}_a(k)$.

C. Multirate Echo Cancellation

In DSL systems, different data rates are used for the downstream and the upstream. Therefore, if the symbol rate is equal in both directions, more samples are received at the remote terminal (RT) where higher data rates are required than at the central office (CO). Different multirate echo cancellation schemes are available in the literature such as [3] and [15]. In the following, we discuss the interpolated echo canceller used at the RT side and decimated echo canceller used at the CO side based on [3].

1) Interpolated Echo Canceller: We assume that in the RT transceiver the transmitted signal bandwidth is κ times smaller than that of the received signal. Because of the non-ideal reconstruction filter at the DAC, the higher frequencies of the transmitted signal leak into the received signal. This aliasing is modeled by interpolation of the transmitted signal before the convolution, which in the time domain corresponds to padding $\kappa-1$ zeros between adjacent samples. Thus, the error signal is given by

$$\mathbf{E}(k) = \mathcal{F}_{\kappa N}\left(\mathbf{y}(k) - \mathcal{X}_i(k)\mathbf{w}(k)\right) - \mathrm{diag}\left\{\mathcal{M}\mathbf{U}(k)\right\}\mathbf{W}(k) \tag{15}$$

where $\mathcal{X}_i(k)$ is defined as in the previous section with the difference that here, it is constructed from the interpolated (*i.e.*, zero-padded) data; in the frequency domain, the interpolation is performed by the use of a matrix $\mathcal{M}$ which is the vertical concatenation of κ identity matrices of order N (*i.e.*, $\mathcal{M} = [\mathcal{I}_N|\cdots|\mathcal{I}_N]^T$). Note that in (15), the DFT matrix is of size κN and the weight vectors $\mathbf{w}(k)$ and $\mathbf{W}(k)$ are of length κN. The echo channel weight is subsequently updated by

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \mu\,\mathrm{diag}\left\{\mathcal{M}\mathbf{U}^*(k)\right\}\mathbf{E}(k). \tag{16}$$

2) Decimated Echo Canceller: At the CO transceiver the transmitted signal bandwidth is κ times larger than that of the received signal and aliasing occurs because of the non-ideal anti-aliasing filter at the ADC. In order to consider the effect of the aliasing in the echo canceller the transmitted signal is decimated; in the time-domain, this corresponds to downsampling by κ and in the frequency-domain, it corresponds to a block and add operation, *i.e.*, pre-multiplication by $\mathcal{M}^T$. Thus, the error signal is given by

$$\mathbf{E}(k) = \mathcal{F}_N\left(\mathbf{y}(k) - \mathcal{X}_d(k)\mathbf{w}(k)\right) - \mathcal{M}^T\,\mathrm{diag}\left\{\mathbf{U}(k)\right\}\mathbf{W}(k) \tag{17}$$

where $\mathcal{X}_d(k)$ contains the decimated data in the time domain. Note that in (17), the weight vectors $\mathbf{w}(k)$ and $\mathbf{W}(k)$ are of length κN and are updated by

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \mu\,\mathrm{diag}\left\{\mathbf{U}^*(k)\right\}\mathcal{M}\mathbf{E}(k). \tag{18}$$

where matrix $\mathcal{M}$ is used to replicate the error vector $\mathbf{E}(k)$ (equivalent to the time domain upsampling).

III. Linearly Constrained Adaptive Echo Cancellation

As discussed in the previous section, the echo cancellation in DMT-based DSL systems relies on the dual filtering in the time and frequency domains, where the filtering weights are mapped from the frequency domain into the time domain. In this section, we propose a novel formulation of the echo cancellation problem as a constrained optimization problem on an extended linear space containing both the time and frequency domain weights. The constraints originally are used to ensure the proper mapping of the weights from the frequency domain into the time domain. Later, additional constraints can be added to improve the performance of the system, which is discussed in Section IV-C.

In this section, linearly constrained adaptive (LCA) echo cancellers which can be employed in systems with DMT modulation are developed. In our derivation, we use the approaches by Frost in [11] and the generalized sidelobe canceller (GSC) by Griffiths and Jim in [12] (with more emphasis on the later one). In this section, the frame synchronous system with equal rates at the transmitter and receiver is examined, and in Section IV, the extension to frame asynchronous and multirate systems is discussed.

A. LCA Echo Canceller Based on Constrained LMS Algorithm

Let us consider a frame synchronous system with equal rates at the transmitter and the receiver. The echo cancellation for DMT-based systems can be represented as a constrained optimization problem, where a cost function is minimized under certain constraints. The optimization is performed over an extended linear space, which is spanned by the extended weight vector, containing the weights both in the time and frequency domains, as given by

$$\boldsymbol{\omega}(k) = \left[\frac{\mathbf{w}(k)}{\mathbf{W}(k)}\right] \tag{19}$$

where $\boldsymbol{\omega}(k)$ is a vector of size $2N \times 1$. As seen in the previous section, the error signal, given in (9), is used to update the adaptive weights. This signal can also be rewritten in terms of the extended weight vector, as follows:

$$\mathbf{E}(k) = \mathcal{F}_N \mathbf{y}(k) - [\mathcal{F}_N \mathcal{X}(k) | \mathrm{diag}\,\{\mathbf{U}(k)\}]\boldsymbol{\omega}(k). \tag{20}$$

Further simplification can be achieved by defining an extended echo reference matrix $\Phi(k)$ of size $2N \times N$ as

$$\Phi(k) = \left[\frac{\mathcal{X}^H(k)\mathcal{F}_N^{-1}}{\mathrm{diag}\,\{\mathbf{U}^*(k)\}}\right] \tag{21}$$

and by considering that the received signal in the frequency domain is denoted by $\mathbf{Y}(k) = \mathcal{F}_N\ \mathbf{y}(k)$. Using the above definitions, the error signal in (20) can be rewritten as

$$\mathbf{E}(k) = \mathbf{Y}(k) - \Phi^H(k)\boldsymbol{\omega}(k). \tag{22}$$

Using this extended notation, the echo cancellation can be expressed as a constrained optimization, where the error power is minimized subject to a linear constraint on $\boldsymbol{\omega}(k)$. This can be formulated as

$$\min E[\|\mathbf{E}(k)\|^2] \quad \text{s.t.} \quad \mathcal{C}^H\boldsymbol{\omega}(k) = \mathbf{g} \tag{23}$$

where $\mathrm{E}[\cdot]$ denotes statistical expectation. In our derivation, we assume a general set of M_c linear constraints described by the constraint matrix $\mathcal{C}$ of size $2N \times M_c$ and vector $\mathbf{g}$ of size $M_c \times 1$, where $M_c < 2N$ and matrix $\mathcal{C}$ is full column rank. The constraint is primarily used to ensure the Fourier transform relation between the weights in the time and frequency domains. However, additional constraints can be added to improve the system performance, as discussed later.

Using the method of Lagrange multipliers [14], the constrained optimization can be transformed into an unconstrained one. Therefore, the constraint can be appended to the cost function, as given by

$$J = \mathrm{E}[\|\mathbf{E}(k)\|^2] + \left(\boldsymbol{\omega}^H(k)\mathcal{C} - \mathbf{g}^H\right)\boldsymbol{\lambda} + \boldsymbol{\lambda}^H\left(\mathcal{C}^H\boldsymbol{\omega}(k) - \mathbf{g}\right) \tag{24}$$

where $\boldsymbol{\lambda}$ is the Lagrange multipliers vector of size $M_c \times 1$. Complying with the method in [11], constrained gradient-descent optimization is used where the weight vector is adaptively updated by the gradient of the constrained cost function, as follows (calculated based on [16]):

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}(k) - \mu\nabla_{\boldsymbol{\omega}^H} J \tag{25}$$

where

$$\nabla_{\boldsymbol{\omega}^H} J = \mathcal{R}\boldsymbol{\omega}(k) - \mathbf{P} + \mathcal{C}\boldsymbol{\lambda}. \tag{26}$$

The matrix $\mathcal{R}$ and the vector $\mathbf{P}$ are the correlation matrix and cross correlation vector, respectively, defined by

$$\mathcal{R} = \mathrm{E}[\Phi(k)\Phi^H(k)] \tag{27}$$

$$\mathbf{P} = \mathrm{E}[\Phi(k)\mathbf{Y}(k)]. \tag{28}$$

The Lagrange multipliers ($\boldsymbol{\lambda}$) is found by forcing $\boldsymbol{\omega}(k+1)$ to satisfy the constraint. Replacing the obtained value of $\boldsymbol{\lambda}$ in (25), the iterative weight update is given by

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}_q + \left(\mathcal{I}_N - \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathcal{C}^H\right)\left(\boldsymbol{\omega}(k) - \mu\left(\mathcal{R}\boldsymbol{\omega}(k) - \mathbf{P}\right)\right) \tag{29}$$

where $\boldsymbol{\omega}_q = \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathbf{g}$ is the quiescent term and depends only on the constraint. The term $\mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathcal{C}^H$ represents the projection onto the subspace spanned by the constraint matrix $\mathcal{C}$ (denoted by $\mathcal{P}_c$), and the matrix

$$\mathcal{P}_c^{\perp} = \mathcal{I}_N - \mathcal{P}_c \tag{30}$$

is the projection onto its orthogonal complement. Hence, (29) can be rewritten as

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}_q + \mathcal{P}_c^{\perp}\left(\boldsymbol{\omega}(k) - \mu\left(\mathcal{R}\boldsymbol{\omega}(k) - \mathbf{P}\right)\right). \tag{31}$$

The adaptive update in (31) requires the knowledge of the correlation matrix $\mathcal{R}$ and the vector $\mathbf{P}$, which is unavailable *a priori*. However, for a LMS style approach, these quantities can be approximated by their instantaneous values given by

$$\mathcal{R}(k) = \Phi(k)\Phi^H(k) \tag{32}$$

$$\mathbf{P}(k) = \Phi(k)\mathbf{Y}(k). \tag{33}$$

Using the instantaneous approximations, the iterative update of the extended weight vector $\boldsymbol{\omega}(k+1)$ based on the constrained LMS algorithm is given by

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}_q + \mathcal{P}_c^{\perp}\left(\boldsymbol{\omega}(k) + \mu\Phi(k)\mathbf{E}(k)\right) \tag{34}$$

where $\mathbf{E}(k)$ is given in (22). As shown in [11], this algorithm converges if

$$0 < \mu < \frac{1}{\lambda_{\max}} \tag{35}$$

where $\lambda_{\max}$ is the largest eigenvalue of the matrix $\mathcal{P}_c^{\perp}\ \mathcal{R}\ \mathcal{P}_c^{\perp}$. In Section III-B, a more efficient implementation of the above LCA echo canceller is introduced based on the approach in [12].

B. LCA Echo Canceller Based on Generalized Sidelobe Canceller

One of the efficient implementations for adaptive constrained optimization is proposed by Griffiths and Jim in [12]. In this method, known as generalized sidelobe canceller (GSC), the weight vector space is decomposed into two orthogonal subspaces, the constraint subspace spanned by the columns of the constraint matrix $\mathcal{C}$ and its orthogonal complement subspace. Originally, this adaptive processor was introduced in the context of beamforming applications, where it is partitioned into two processing paths. One path processes the signal from the desired direction, often subject to some constraints, and the other one filters out (blocks) the interference and the noise.

Employing this method, the $2N$-dimensional extended weight vector $\boldsymbol{\omega}(k)$ can be written as the sum of two components resulting from the projection onto the two orthogonal subspaces. That is

$$\boldsymbol{\omega}(k) = \boldsymbol{\omega}_c(k) + \boldsymbol{\omega}_b(k) \tag{36}$$

where $\boldsymbol{\omega}_c(k)$ is the projection of $\boldsymbol{\omega}(k)$ onto the constraint subspace and $\boldsymbol{\omega}_b(k)$ is the projection onto the orthogonal subspace. If the constraint subspace is spanned by the columns of matrix $\mathcal{C}$ of size $2N \times M_c$, the orthogonal subspace is spanned by the columns of a so-called blocking matrix $\mathcal{B}$ of size $2N \times (2N - M_c)$. The blocking matrix can be any full column rank matrix with columns orthogonal to the columns of $\mathcal{C}$, i.e.,

$$\mathcal{B}^H\mathcal{C} = \mathcal{O}_{(2N-M_c)\times M_c}. \tag{37}$$

It is often convenient but not necessary to choose matrix $\mathcal{B}$ such that $\mathcal{B}^H\ \mathcal{B} = \mathcal{I}_{(2N-M_c)}$.

The projection of $\boldsymbol{\omega}(k)$ onto the constraint subspace is performed by its left multiplication with the projection matrix $\mathcal{P}_c\ \left(\mathcal{P}_c = \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathcal{C}^H\right)$. Thus, from the constraint in (23), it can be seen that

$$\boldsymbol{\omega}_c(k) = \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathbf{g} = \boldsymbol{\omega}_q. \tag{38}$$

As noted in the previous section, $\boldsymbol{\omega}_q$ depends only on the constraint and is not adaptive. Similarly, the projection of $\boldsymbol{\omega}(k)$ onto the orthogonal subspace can be derived via left multiplication with the projection matrix onto the orthogonal subspace $\left(\mathcal{P}_c^{\perp}\right)$, which in terms of the blocking matrix is given as follows:

$$\mathcal{P}_c^{\perp} = \mathcal{B}(\mathcal{B}^H\mathcal{B})^{-1}\mathcal{B}^H. \tag{39}$$

Using some arithmetic, it can be shown that $\boldsymbol{\omega}_b(k)$ is given by

$$\boldsymbol{\omega}_b(k) = \mathcal{B}\boldsymbol{\omega}_a(k) \tag{40}$$

where $\boldsymbol{\omega}_a(k)$ is the unconstrained solution of

$$\min E\left[\left\|\mathbf{Y}(k) - \Phi^H(k)\boldsymbol{\omega}_q - \Phi^H(k)\mathcal{B}\boldsymbol{\omega}_a(k)\right\|^2\right]. \tag{41}$$

Thus, $\boldsymbol{\omega}_a(k)$ can be adaptively updated by the gradient of (41), as

$$\boldsymbol{\omega}_a(k+1) = \boldsymbol{\omega}_a(k) + \mu\mathcal{B}^H\left(\mathbf{P} - \mathcal{R}\boldsymbol{\omega}_q - \mathcal{R}\mathcal{B}\boldsymbol{\omega}_a(k)\right) \tag{42}$$

where the matrix $\mathcal{R}$ and the vector $\mathbf{P}$ are defined in (27) and (28), respectively. The decomposition of $\boldsymbol{\omega}(k)$ as in (36) makes it possible to transform the constrained optimization over $\boldsymbol{\omega}(k)$ into an equivalent unconstrained one over $\boldsymbol{\omega}_a(k)$. In addition, the term $\boldsymbol{\omega}_c(k) = \boldsymbol{\omega}_q$ ensures that the constraint is satisfied at all times, independent of the choice of $\boldsymbol{\omega}_a(k)$.

The iterative update for $\boldsymbol{\omega}_a(k)$ in (42) requires the knowledge of the correlation matrix $\mathcal{R}$ and the vector $\mathbf{P}$. Similar to the previous section, these values can be approximated by their instantaneous values given by (32) and (33), respectively. Consequently, the iterative update for the extended weight vector based on the GSC algorithm in [12] is given by

$$\begin{aligned}\boldsymbol{\omega}_a(k+1) &= \boldsymbol{\omega}_a(k)\\ &\quad+\mu\Phi_b(k)\left(\mathbf{Y}(k) - \Phi^H(k)\boldsymbol{\omega}_q - \Phi_b^H(k)\boldsymbol{\omega}_a(k)\right)\end{aligned} \tag{43}$$

where $\Phi_b(k) = \mathcal{B}^H\ \Phi(k)$ is the projection of the extended echo reference matrix onto the subspace spanned by the matrix $\mathcal{B}$. Using (43), a compact implementation of the GSC-based LCA echo canceller is shown in Fig. 2. It can also be seen that the adaptive update is equivalent to

$$\boldsymbol{\omega}_a(k+1) = \boldsymbol{\omega}_a(k) + \mu\Phi_b(k)\mathbf{E}(k) \tag{44}$$

where $\mathbf{E}(k)$ is the error signal in (22). It is notable that this algorithm converges if

$$0 < \mu < \frac{1}{\lambda_{\max}} \tag{45}$$

where $\lambda_{\max}$ is the largest eigenvalue of the matrix $\mathcal{R}$.

IV. Constrained Adaptive Echo Cancellation for DMT-Based DSL Systems

In the previous section, the LCA echo canceller for a general system with DMT modulation is presented. In order to use this echo canceller in the DSL transceivers, the proper constraint matrix must be defined. In this section, first we show how the current methods for echo cancellation for DSL (as discussed in Section II) can be formulated as a LCA echo canceller, by defining the constraint matrix and expanding this method for the frame asynchronous and multirate echo cancellers. Later, we take this a step forward and show that for LCA echo canceller the performance of the system can be improved by choosing

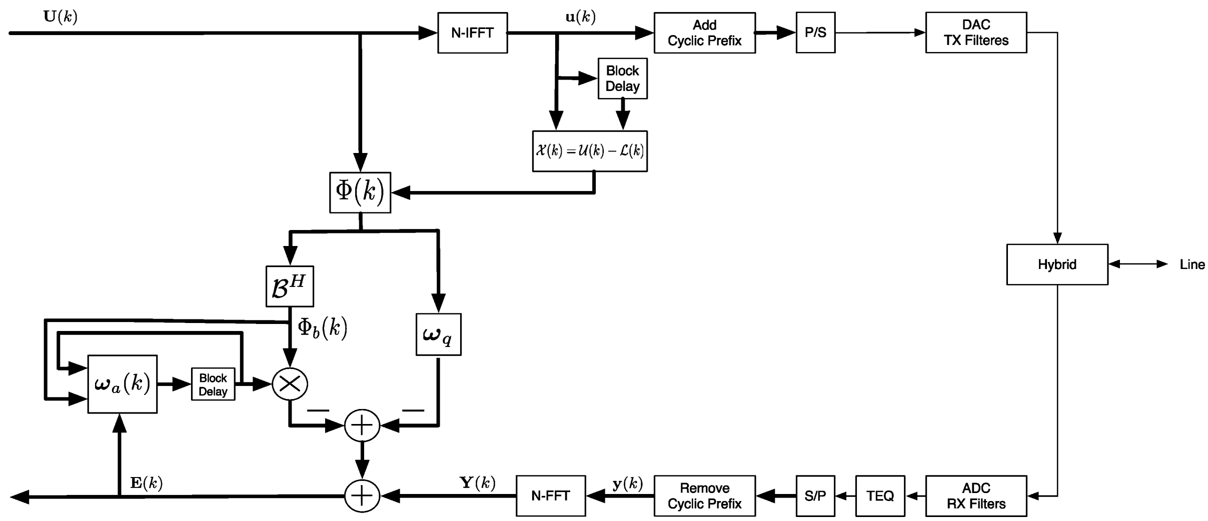

Fig. 2. Block diagram for GSC-based LCA echo canceller.

proper constraints. For instance, when the transmission is deteriorated by the radio frequency interference, the constrained echo canceller can improve the robustness of the system.

A. Formation of the Constraint Matrix

As discussed in Section II, in most of the echo cancellers used in DSL systems the weight update is performed in the frequency domain and later these weights are transformed to the time domain by means of the inverse Fourier transform. Therefore, the weights in the time and frequency domains are related by $\mathbf{w}(k) = \mathcal{F}_N^{-1}\, \mathbf{W}(k)$, which can be represented in matrix form as

$$\mathcal{C}^H\boldsymbol{\omega}(k) = \mathbf{0} \tag{46}$$

where the constraint matrix is given by

$$\mathcal{C} = [\mathcal{F}_N| - \mathcal{I}_N]^H \tag{47}$$

denoting N linear constraints $(M_c = N)$. Hence, these methods can be represented as a special case of the linearly constrained echo canceller.

In our derivation, we have assumed that the FIR adaptive filter modeling the echo channel has the same length as the transmitting symbol (N); however, usually a filter with shorter length can be used. Thus, the vector $\mathbf{w}(k)$ in the time domain can be of length M $(M < N)$ and the corresponding extended weight vector $\boldsymbol{\omega}(k)$ is of length $(N + M)$, which results in a reduced computational complexity, as discussed in Section IV-B. Consequently, this can be expressed in the form (46), where the constraint matrix in this case is modified to

$$\mathcal{C} = [\mathcal{F}_N\mathcal{S}| - \mathcal{I}_N]^H \tag{48}$$

where matrix $\mathcal{S} = [\mathcal{I}_M \,|\, \mathcal{O}_{M\times(N-M)}]^T$ is used for zero-padding before the DFT.

The projection matrix $\left(\mathcal{P}_c^\perp\right)$ and the blocking matrix $(\mathcal{B})$ required for the LCA echo cancellation can be derived based on the constraint matrix. For example, if the constraint matrix is described as in (48), the projection matrix $\mathcal{P}_c = \mathcal{C}(\mathcal{C}^H\mathcal{C})^{-1}\mathcal{C}^H$ and the corresponding orthogonal projection matrix $\mathcal{P}_c^\perp$ is given by (30). Additionally, for simple constraint matrices, the blocking matrix of size $(N + M) \times M$ can be found using (37). For instance, for the constraint matrix given in (48), the blocking matrix $(\mathcal{B})$ is as follows:

$$\mathcal{B} = \frac{1}{\sqrt{2}}\left[\mathcal{I}_M|\mathcal{S}^T\mathcal{F}_N^{-1}\right]^H. \tag{49}$$

For a more complicated constraint matrix, methods such as Gram–Schmidt or QR-factorization can be used to find the blocking matrix [17].

The extended echo reference matrix $\Phi(k)$ also needs to be adjusted in accordance with the modification in the extended weight vector $\boldsymbol{\omega}(k)$ of length $(N + M)$. In order for the error signal to satisfy (22), the matrix $\Phi(k)$ of size $(N + M) \times N$ should be redefined as

$$\Phi(k) = \left[\frac{\mathcal{S}^T\mathcal{X}^H(k)\mathcal{F}_N^{-1}}{\operatorname{diag}\{\mathbf{U}^*(k)\}}\right] \tag{50}$$

where matrix $\mathcal{S}$ is used to select the proper time-domain samples.

B. LCA Echo Cancellers for Frame Asynchronous and Multirate Systems

The LCA echo cancellers can be easily applied to the frame asynchronous and multirate systems. This extension is achieved by changing the definition of the extended echo reference matrix $\Phi(k)$ in (21). However, the constraint matrix and the corresponding projection and blocking matrices are not affected and can be applied as discussed in the previous section.

1) Frame Asynchronous LCA Echo Canceller: As seen in Section II-B, the frame synchronous echo cancellation is possible only at one of the transmission sides, and at the other side the echo cancellation must be done asynchronously. Considering the approach in [6], where the extra DFT is placed at the receiver and the double talk cancellation is easily achieved, the constrained echo cancellation can be used by redefining the extended reference matrix as

$$\Phi(k)=\left[\frac{\mathcal{X}_a^H(k)\mathcal{F}_N^{-1}}{\operatorname{diag}\{\mathbf{U}_a^*(k)\}}\right] \tag{51}$$

where $\mathcal{X}_a(k)$ and $\operatorname{diag}\{\mathbf{U}_a(k)\}$ are defined in Section II-B. The redefined $\Phi(k)$ can then be used to calculate the correlation matrices in (27) and (28).

2) Multirate LCA Echo Canceller: In practical systems different data rates are used in the upstream and downstream directions. Hence, as discussed in Section II-C, the relating emulated echo signals is interpolated or decimated in order to consider the effect of the non-ideal filters at DAC or ADC, respectively.

For the interpolated echo cancellation the same methods for constrained echo cancellation can be used by modifying $\Phi(k)$ as

$$\Phi(k)=\left[\frac{\mathcal{X}_i^H(k)\mathcal{F}_{\kappa N}^{-1}}{\operatorname{diag}\{\mathcal{M}\mathbf{U}^*(k)\}}\right] \tag{52}$$

and $\mathbf{Y}(k)$ as

$$\mathbf{Y}(k)=\mathcal{F}_{\kappa N}\mathbf{y}(k) \tag{53}$$

where $\mathcal{X}_i(k)$ contains the interpolated data as discussed in Section II-C.

For the decimated echo cancellation it is only required to modify the extended echo reference matrix as follows:

$$\Phi(k)=\left[\frac{\mathcal{X}_d^H(k)\mathcal{F}_N^{-1}}{\operatorname{diag}\{\mathbf{U}^*(k)\}\mathcal{M}}\right] \tag{54}$$

where the matrix $\mathcal{X}_d(k)$ is defined in Section II-C. Using the above definitions the correlation matrices, for the constrained echo canceller, can again be calculated using (27) and (28). As shown above, the LCA echo canceller methods can be applied to all DSL systems used in practice. Although not shown here, other methods for echo cancellation such as in [7] and [8] can also be represented using the LCA echo canceller structure. Therefore, the proposed LCA formulation provides a unifying framework for examining constrained echo cancellation for all DSL systems.

C. Constrained Adaptive Echo Canceller in the Presence of the Radio Frequency Interference

In DSL systems, the twisted-pair line, specially the aerial lines, act as an antenna for the radio frequency (RF) signals present in the environment, resulting in RF interference. Predominantly, this RF interference consists of the ingress from the AM radio broadcasts and amateur HAM operator transmissions. AM radio ingress is caused by the continuous operation of the radio stations on narrow bands spreading from 560 kHz up to 1.6 MHz. Amateur radio's transmission also occupies narrow frequency bands on the range from 1.8 MHz up to 29 MHz; however, the carrier frequency for these transmissions are changed constantly every few minutes. The level of the ingress from both of these sources at the receiver is higher than the level of the crosstalk and the background noise.

The RF ingress is usually modeled as a narrowband (NB) noise, which is a signal with narrowband and high amplitude over a short period of time. In the presence of NB noise, the weights of the echo canceller corresponding to the tones affected by the noise, increase substantially. Therefore, even after the removal of the noise the echo canceller needs to compensate for the large error on these tones. This phenomenon is known as the drifting problem.

Below, two approaches are proposed where the constrained echo canceller is used in the presence of RF interference and the constraint is modified to restrain the drifting problem.

1) Linearly Constrained Adaptive Echo Canceller in the Presence of the NB Noise: So far, we have derived a framework for echo cancellation in DSL systems which makes it possible to apply desired constraints to the weights of the echo canceller. Therefore, in the presence of the NB noise, which cause the deteriorating error on the affected tones, the proper constraint can be used to prevent this problem. For the case of linear constraints, the weights of the affected tones, i.e., specific components of the frequency domain weight vector $\mathbf{W}(k)$, can be forced to zero during the presence of the noise. This can be obtained by extending the constraint matrix $\mathcal{C}$ as follows:

$$\mathcal{C}=\left[\begin{array}{c|c}\mathcal{F}_N\mathcal{S} & -\mathcal{I}_N\\ \hline \mathcal{O}_{M_l\times M} & \mathcal{T}_{M_l\times N}\end{array}\right]^H \tag{55}$$

where M_l is the number of affected tones, and matrix $\mathcal{T}$ is a submatrix of the identity matrix used to incorporate the affected tones into the constraint (corresponding entry for each affected tone is equal to 1). It should be noted that due to the extended formulation the effect of the constraint in the frequency domain will correctly be mapped onto the weights in the time domain by the modified algorithm.

This approach requires that the existence of the noise can be detected on the reserved tones [18]. As a result, it is best suited for the cases where the RF ingress is continuous e.g., AM ingress. The simulation results using these constraints, given in Section V, show a major improvement in terms of the error on the affected tones.

2) Constrained Adaptive Echo Canceller in the Presence of the NB Noise With Quadratic Constraint: Quadratic inequality constraints are mostly used to improve the robustness of constrained adaptive systems in the presence of interferences [19]. Thus, inequality constraints can be used to ensure that the energy of the desired tones is limited within a certain threshold. This can be formulated as

$$\|[\,\mathcal{O}_{M_l\times M}|\mathcal{T}_{M_l\times N}\,]\boldsymbol{\omega}(k)\|^2\leq\Upsilon \tag{56}$$

where matrix $\mathcal{T}$ is again used to include the affected tones in the constraint and leave the weights of the unaffected tones unchanged, and Υ is the limit on the energy of the affected tones. Additional step is added in the LCA echo cancellers to ensure that the above constraint is satisfied [for GSC-based echo canceller, (36) and (40) is used to map the constraint into the orthogonal subspace]. This method does not require the detection of the noise, and it can be used in cases where the frequency band of the RF interference changes quickly such as in HAM radio ingress. The inequality constraint can be applied to the weights at all times and the value of the threshold can be adjusted during the initialization period.

Finally, we note that it is possible to extend this approach so as to constrain certain components of the time domain weight vector $\mathbf{w}(k)$. This might be desirable if *a priori* information about the true echo channel is available, and it is known that some tail coefficients must be null or very small.

V. Performance of the Constrained Echo Canceller

In this section, we compare the performance of the proposed LCA echo cancellers with the current echo cancellers. We use simulation experiments to evaluate the convergence behavior of the algorithms, and we also compare their computational complexity. In addition, we investigate the constrained echo canceller with supplementary constraints in the presence of the RF interference.

In the simulations, an ADSL system over the carrier serving area (CSA) loop #1 setup is used [1]. In addition, DMT modulation is employed where, tones 7–31 and 33–255 are allocated for upstream and downstream, receptively. Each tone transmits a 4-QAM signal constellation. The downstream and upstream signal transmit with -40 dbm/Hz, and the echo reference signal contains 20 dB lower power on the unused tones to ensure convergence. The external additive noise is white Gaussian noise at -140 dBm/Hz. The transmit block length N at the upstream and downstream is 64 and 512, respectively and the corresponding cyclic prefix length υ is 5 and 40, respectively. The true echo channel contains 512 samples at 2.2 MHz, while the number of echo canceller taps used is $M = 220$. The taps are initialized with all zero, and the weights are updated after each frame is received. The echo channel transfer function includes the effect of the hybrid and the transmitter and receiver filters [1]. This setup is widely used in literature for studying the convergence of echo cancellers such as in [3], [5], [6], and [15].

A. Convergence Comparison

In the first set of simulation experiments, the convergence behavior of different algorithms for echo cancellation in DSL systems is compared. As discussed in Section II, the weights can be updated either by the exact formula using the LMS algorithm in the time domain or by the approximate per tone update in the frequency domain given by (10) (denoted as DMT method [3]). The performance of these two methods is compared with the linearly constrained echo cancellers discussed in Section III. For the latter echo cancellers, the constraint represents the Fourier transform relation between the weights in the time and frequency domains, where the constraint and the blocking matrices are given by (48) and (49), respectively.

The results for the frame asynchronous system at the RT receiver, where the interpolated echo canceller is used is depicted in Fig. 3, same results can be achieved for the echo canceller at

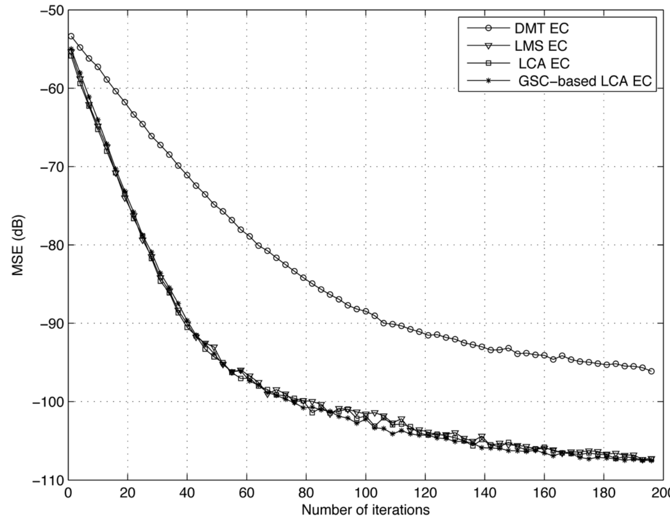

Fig. 3. Convergence behavior of different EC methods for asynchronous ADSL-RT.

the CO receiver (not shown here). As it can be seen, the DMT method has slower convergence than the other methods, because of the approximation used but it has less complexity. The linearly constrained echo cancellers both offer the same performance as the exact LMS algorithm, as expected, but provide additional capability to implement supplementary constraints.

It should be noted that unlike the DMT-CES echo canceller, the LCA echo cancellers do not require the extra power on the unused tones to achieve convergence.

B. Computational Complexity

The computational complexity of algorithms are usually approximated by the total number of required real multiplications for each iteration. In Table I, the complexity of LMS-CES and DMT-CES echo canceller (EC) are compared with the GSC-based constrained echo canceller. We assumed that both transmitter and receiver use the same data rate and are frame synchronous. In the table, M is the number of the time domain echo canceller taps, N and υ are the symbol length and cyclic prefix length, respectively. For the constrained echo canceller, the constraint and the blocking matrices are given by (48) and (49), respectively. It can be seen that the complexity of the constrained echo canceller is equal to the complexity of the LMS-CES echo canceller.[1] For a general linear constraint matrix, the complexity of the constrained echo canceller increases, which is mainly caused by the additional projection operation involved in these algorithms. The first implementation, given in (34), requires the orthogonal projection by matrix $\mathcal{P}_c^{\perp}$, which is uniquely defined. In the second implementation based on the GSC algorithm, the input signal is divided using the blocking matrix $\mathcal{B}$, which is only required to be orthogonal to the constraint matrix $\mathcal{C}$ and full rank. Therefore, the complexity of the GSC-based constrained echo canceller can be further reduced by the proper formation of matrix $\mathcal{B}$ (e.g., using the method proposed in [20]).

[1]It is assumed that the adaptive update for the constrained echo canceller is performed in the time domain, where $\mathbf{E}(k)$ is transformed into the time domain using IDFT.

TABLE I
COMPLEXITY COMPARISON

Application	**DMT-CES**	**LMS-CES**	**CLMS-GSC**
Echo Emulation	$2N + \frac{(M-\upsilon)^2}{2}$	$2N + \frac{(M-\upsilon)^2}{2}$	$2N + \frac{(M-\upsilon)^2}{2} + 2N\log N$
Adaptive Update	$2N + 2N\log N$	$MN + 4N\log N$	$MN + 2N\log N$

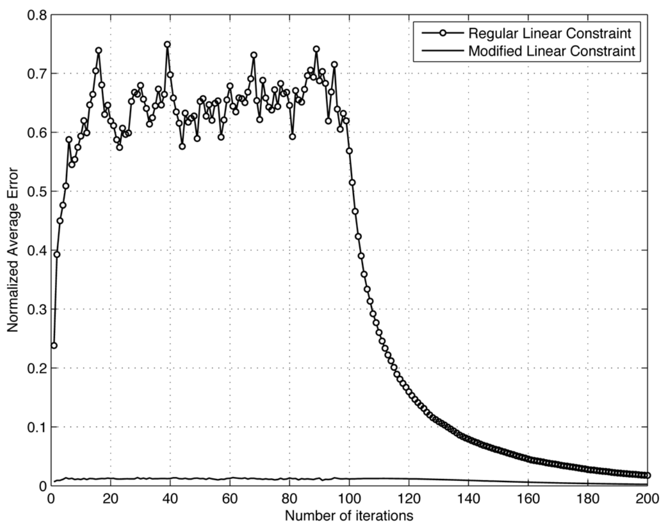

Fig. 4. Normalized average error on the weights of the affected tones with regular and modified linear constraints.

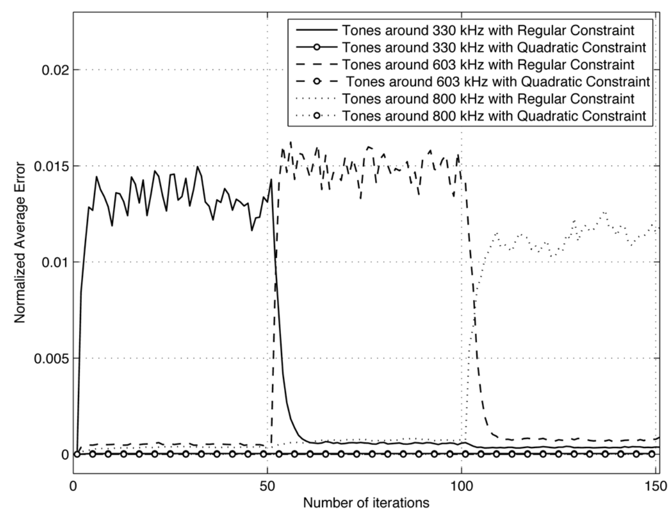

Fig. 5. Normalized average error on the weights of the affected tones with regular linear constraints and additional quadratic constraint.

C. Constrained Adaptive Echo Canceller in the Presence of the Radio Frequency Interference

In this section, we examine the effect of the supplementary constraints to improve the robustness of the system in the presence of RF interference. The interference is modeled by NB noise on the known frequencies affected by AM transmissions. In the first approach, linear constraints are added to force the weights of the affected tones to zero in the presence of the noise. The echo canceller weights are initialized with the true echo channel weights and the NB noise is added at frequency 560 kHz for the next 100 frames (iterations) and then removed, while the adaptive update is continued for next 100 iterations. The normalized average error on the echo canceller weights corresponding to the affected tones is depicted in Fig. 4. As it can be seen, the echo canceller with the regular constraint encounters a large increase on the affected tones, while the linearly constrained echo canceller with additional zero-forcing constraints is protected, in the presence of the NB noise. In addition, after the removal of the NB noise source, the regularly constrained echo canceller needs more than 50 iterations to compensate the error on the affected tones by the noise, while the weights of the later one are adjusted in few iterations.

In the second approach, a quadratic constraint is added to limit the weights on the tones suspected to be affected by the NB noise. In this simulation experiment, NB noise changes its frequency from 330 to 603 kHz and finally to 800 kHz, in iterations 1, 50, and 100. The normalized average error on the weights for the echo canceller corresponding to the affected tones with the regular constraints and the one with additional quadratic constraint is depicted in Fig. 5. As it can be seen, the regularly constrained echo canceller, not only experiences large error on the affected tones but also requires additional iterations to compensate the error on the previously affected tones even after the change in the frequency of the noise. On the other hand, the echo canceller with additional quadratic constraints shows an improved performance in the presence of the NB noise with changing frequencies.

VI. CONCLUSION

In this paper, we have introduced a constrained adaptive echo canceller for DMT-based DSL systems where echo is cancelled partially in the frequency and time domain. Two implementations for the linearly constrained echo canceller have been discussed based on [11] and [12]. LCA echo canceller can be applied to most of the practical DSL systems such as frame asynchronous and multirate systems. Thus, this formulation provides a unifying framework for studying these systems. In addition, supplementary constraints can be added to the constrained echo canceller to improve the performance of the system. For instance, it was demonstrated that the introduction of additional linear or quadratic constraints can significantly improve the robustness of the system in the presence of RF interference.

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