Generalized Bayesian Estimators of the Spectral Amplitude for Speech Enhancement

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***Abstract*—In this letter, we show that many existing short-time spectral amplitude (STSA) Bayesian estimators for speech enhancement all have a similarly structured cost function. On this basis, we propose a new cost function that generalizes those of existent Bayesian STSA estimators and then obtain the corresponding closed-form solution for the optimal clean speech STSA. The resulting family of estimators, which we will term the Generalized Weighted family of STSA estimators (GWSA), features a new parameter that acts only on the estimated clean speech STSA. It is found that this new parameter yields an added flexibility in terms of achievable gain curves when compared to those of existing estimators. Moreover, we show that the new estimator family tends to a Wiener filter for high instantaneous signal-to-noise ratios.**

***Index Terms*—Bayesian estimators, short-time spectral amplitude, speech enhancement.**

I. Introduction

BAYESIAN estimators of the spectral amplitude have been widely used to perform single-channel speech enhancement (see [1]–[5] and references therein). In that approach, an estimate of the clean speech is derived by minimizing the expectation of a cost function that penalizes errors in the clean speech estimate.

One well-known Bayesian estimator for speech enhancement, the minimum mean square error (MMSE) of the short-time spectral amplitude (STSA), i.e., MMSE STSA, is obtained when the chosen cost function is the squared error between the estimated and actual clean speech STSA [1]. A generalization of the MMSE STSA cost function was proposed in [2] in the β-Order STSA MMSE estimator ($\beta -$ SA). This estimator applies a power law (i.e., an exponent β) to the estimated and actual clean speech STSA in the squared error of the cost function. Another generalization of MMSE STSA was proposed in [3] where the error between the estimated and actual clean speech STSA is weighted by the STSA of the clean speech raised to an exponent (p); the resulting estimator is termed Weighted Euclidien (WE). A cost function incorporating both aspects of the WE and $\beta -$ SA estimators was proposed in [4] and lead to the W$\beta -$ SA estimator. A variant of the Itakura-Saito distortion measure, the COSH measure, and its generalization, the Weighted COSH (WCOSH), were also proposed as cost functions for Bayesian STSA estimators in [3].

In this letter, we first expose the similarities between the cost functions used in existing Bayesian STSA estimators for speech enhancement. In particular, we show that they all have a structure involving a weighted squared difference between a monotonic function of the estimated and actual clean speech STSA. Based on the highlighted structure, we propose a new cost function that generalizes those of existent Bayesian STSA estimators and then obtain the corresponding closed-form solution for the optimal clean speech STSA. The resulting family of estimators, which we will refer to as the generalized weighted family of STSA estimators (GWSA), incorporates the parameters present in other existing estimators but also features a new parameter: η. The parameters present in the estimators control the shape of its gain curve as a function of the instantaneous SNR. In contrast to the other parameters, η acts only on the estimated clean speech STSA. It is found that η yields an added flexibility in terms of achievable gain curves when compared to those of existing estimators. Finally, we also show that all the estimators belonging to the new estimator family tend to a Wiener filter for high instantaneous SNR.

The paper is organized as follows. In Section II, we briefly review Bayesian STSA estimation. In Section III we expose the similarities between various existing cost functions. In Section IV, we introduce the GWSA family of estimators and study their properties. A conclusion is presented in Section V.

II. Bayesian STSA Estimation

Let the observed noisy speech of a particular frame i be

$$y_i[n] = x_i[n] + w_i[n], \qquad 0 \leq n < N-1 \tag{1}$$

where $x_i[n]$ is the clean speech, $w_i[n]$ is the additive noise and $\{0, \cdots, N-1\}$ is the observation interval. Let $Y_{i,k}$, $X_{i,k}$ and $W_{i,k}$ denote the k^{th} complex short-time spectral components of the noisy speech, clean speech and noise, respectively. To simplify the notation, we will omit the subscript i in the sequel.

In Bayesian STSA estimation for speech enhancement, the goal is to obtain the estimator $\hat{\mathcal{X}}_k^o$ of $\mathcal{X}_k \triangleq |X_k|$, i.e. the STSA of X_k, which minimizes the expectation of a given cost function $C(\mathcal{X}_k, \hat{\mathcal{X}}_k)$

$$\hat{\mathcal{X}}_k^o = \arg\min_{\hat{\mathcal{X}}_k} E\left\{C(\mathcal{X}_k, \hat{\mathcal{X}}_k)\right\} \tag{2}$$

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where E denotes statistical expectation. This estimator is then combined with the phase of the noisy speech, $\measuredangle Y_k$, to yield the estimator of the complex spectrum of the noisy speech $\hat{X}_k$

$$\hat{X}_k = \hat{\mathcal{X}}_k^o e^{j\measuredangle Y_k}. \tag{3}$$

The time-domain estimate $\hat{x}[n]$ is obtained by performing an inverse Fourier transform of $\hat{X}_k$ for each frame, which are then combined using the overlap-add method as in [1].

One well-known estimator of this type is the MMSE STSA estimator [1]. In the lateer, X_k and W_k are modeled as independent, identically distributed Gaussian random variables with zero means and known variances and a squared error cost function is used

$$C(\mathcal{X}_k, \hat{\mathcal{X}}_k) = (\mathcal{X}_k - \hat{\mathcal{X}}_k)^2. \tag{4}$$

The MMSE STSA estimator can be expressed in the following form:

$$\hat{\mathcal{X}}_k^o = G_k |Y_k| \tag{5}$$

$$G_k = \frac{\sqrt{\upsilon_k}}{\gamma_k}\Gamma(1.5)M(-0.5, 1; -\upsilon_k) \tag{6}$$

where G_k is the gain applied to the STSA of the noisy speech, $\Gamma(x)$ is the gamma function, $M(a, b; z)$ is the confluent hypergeometric function [6] and

$$\upsilon_k = \frac{\xi_k}{1+\xi_k}\gamma_k, \quad \xi_k = \frac{E\left\{\mathcal{X}_k^2\right\}}{E\left\{|W_k|^2\right\}}, \quad \gamma_k = \frac{|Y_k|^2}{E\left\{|W_k|^2\right\}}.$$

In these expressions, ξ_k acts as a long term estimator of the SNR while $\gamma_k - 1$ can be interpreted as an instantaneous SNR.

III. Similarities Between Existing Cost Functions

In this section, we examine different cost functions that have been proposed recently in order to reveal their similar structure.

The $\beta - \mathrm{SA}$ estimator was proposed in [2] as a generalization of the MMSE STSA estimator. Its cost function has the following form:

$$C_{\beta-\mathrm{SA}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) \triangleq \left(\mathcal{X}_k^\beta - \hat{\mathcal{X}}_k^\beta\right)^2 \tag{7}$$

where $\beta > -2$. We note that for the above statistical model on X_k and W_k, the estimator corresponding to the case $\beta \to 0$ is identical to the MMSE log-STSA (LSA) estimator proposed in [7] (see [4, Appendix]). The LSA cost function is expressed as

$$C_{\mathrm{LSA}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) \triangleq \left(\log(\mathcal{X}_k) - \log(\hat{\mathcal{X}}_k)\right)^2. \tag{8}$$

In [3], another generalization of the MMSE STSA estimator was proposed in the WE estimator. The WE cost function has the following form:

$$C_{\mathrm{WE}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) \triangleq \mathcal{X}_k^p(\mathcal{X}_k - \hat{\mathcal{X}}_k)^2 \tag{9}$$

where $p > -2$. The WE and $\beta - \mathrm{SA}$ estimators were combined in the $\mathrm{W}\beta - \mathrm{SA}$ estimator [4] for which the cost function is

$$C_{W\beta-\mathrm{SA}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) \triangleq \left(\frac{\mathcal{X}_k^\beta - \hat{\mathcal{X}}_k^\beta}{\mathcal{X}_k^\alpha}\right)^2 \tag{10}$$

where $\beta > 2(\alpha - 1)$, $\alpha < 1$.

A variant of the well-known Itakura-Saito distortion measure, the COSH measure, was proposed in [3] as a cost function for Bayesian STSA estimation. This cost function can be shown to have a similar structure as the cost functions enumerated above. In fact

$$C_{\mathrm{COSH}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) \triangleq \frac{1}{2}\left(\frac{\mathcal{X}_k}{\hat{\mathcal{X}}_k} + \frac{\hat{\mathcal{X}}_k}{\mathcal{X}_k}\right) - 1 = \frac{(\mathcal{X}_k - \hat{\mathcal{X}}_k)^2}{2\mathcal{X}_k\hat{\mathcal{X}}_k}. \tag{11}$$

Moreover, a generalization of the COSH cost function, the WCOSH, was also proposed in [3]

$$C_{\mathrm{WCOSH}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) \triangleq \left(\frac{\mathcal{X}_k}{\hat{\mathcal{X}}_k} + \frac{\hat{\mathcal{X}}_k}{\mathcal{X}_k} - 1\right)\mathcal{X}_k^q \tag{12}$$

where $q > -1$. It can also be expressed in a similar form as the previous cost functions. In fact, we can modify the WCOSH cost function in the following form:

$$C'_{\mathrm{WCOSH}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) = C_{\mathrm{WCOSH}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) - \mathcal{X}_k^q = \frac{(\mathcal{X}_k - \hat{\mathcal{X}}_k)^2}{\mathcal{X}_k^{1-q}\hat{\mathcal{X}}_k} \tag{13}$$

without any modification on the final estimator since the cost function will be minimized with respect to $\hat{\mathcal{X}}_k$ in (2) to obtain the Bayesian estimator.

In all the cost functions presented above, a similar structure can be highlighted that involves a squared difference between a monotonic power function of $\mathcal{X}_k$ and $\hat{\mathcal{X}}_k$. Moreover, that squared difference can be weighted by a function of either $\mathcal{X}_k$ or $\hat{\mathcal{X}}_k$ or both.

Table I summarizes the above Bayesian STSA cost functions along with their corresponding gains G_k obtained when using the same statistical model as for the MMSE STSA estimator in [1]. The first three lines of Table I present estimators without parameters in the cost function while the remaining ones present those with parameters. The columns β, α and η of Table I will be explained in the next section.

IV. GWSA Family of Estimators

In this section, we will generalize the common structure of the cost functions highlighted above, derive the corresponding closed-form solution and perform an analysis of the resulting family of estimators.

A. *The GWSA Family of Estimators*

We propose the following cost function:

$$C_{\mathrm{GWSA}}(\mathcal{X}_k, \hat{\mathcal{X}}_k) = \left(\frac{\mathcal{X}_k^\beta - \hat{\mathcal{X}}_k^\beta}{\mathcal{X}_k^\alpha\hat{\mathcal{X}}_k^\eta}\right)^2 \tag{14}$$

where $\left(\mathcal{X}_k^\beta - \hat{\mathcal{X}}_k^\beta\right)^2$ is now weighted by both $\mathcal{X}_k^{-2\alpha}$ and $\hat{\mathcal{X}}_k^{-2\eta}$. The cost functions associated with the MMSE STSA, LSA, COSH, WE, $\beta - \mathrm{SA}$, $\mathrm{W}\beta - \mathrm{SA}$ and WCOSH estimators are then all particular cases of the proposed cost function with parameter values α, β and η as given in the corresponding columns of Table I. We note that, in contrast to the existing cost functions,

TABLE I
BAYESIAN STSA COST FUNCTIONS WITH CORRESPONDING GAINS G_k AND EQUIVALENT GWSA PARAMETER VALUES (β, α AND η)

	$C(\mathcal{X}_k, \hat{\mathcal{X}}_k)$	G_k	β	α	η
MMSE STSA [1]	$(\mathcal{X}_k - \hat{\mathcal{X}}_k)^2$	$\frac{\sqrt{\upsilon_k}}{\gamma_k}\Gamma(1.5)M(-0.5, 1; -\upsilon_k)$	1	0	0
LSA [7]	$(\log \mathcal{X}_k - \log \hat{\mathcal{X}}_k)^2$	$\frac{\upsilon_k}{\gamma_k} \exp\left\{\frac{1}{2}\int_{\upsilon_k}^{\infty} \frac{e^{-t}}{t} dt\right\}$	$\to 0$	0	0
COSH [3]	$\frac{1}{2}\left(\frac{\mathcal{X}_k}{\hat{\mathcal{X}}_k} + \frac{\hat{\mathcal{X}}_k}{\mathcal{X}_k}\right) - 1$	$\frac{\sqrt{\upsilon_k}}{\gamma_k}\sqrt{\frac{1}{2}\frac{M(-0.5,1;-\upsilon_k)}{M(0.5,1;-\upsilon_k)}}$	1	0.5	0.5
β-SA [2]	$(\mathcal{X}_k^\beta - \hat{\mathcal{X}}_k^\beta)^2$	$\frac{\sqrt{\upsilon_k}}{\gamma_k}\left[\Gamma(\frac{\beta}{2}+1)M\left(-\frac{\beta}{2}, 1; -\upsilon_k\right)\right]^{1/\beta}$	β	0	0
WE [3]	$\mathcal{X}_k^p(\mathcal{X}_k - \hat{\mathcal{X}}_k)^2$	$\frac{\sqrt{\upsilon_k}}{\gamma_k}\frac{\Gamma\left(\frac{p+1}{2}+1\right)}{\Gamma\left(\frac{p}{2}+1\right)}\frac{M\left(-\frac{p+1}{2},1;-\upsilon_k\right)}{M\left(-\frac{p}{2},1;-\upsilon_k\right)}$	1	$-p/2$	0
Wβ-SA [4]	$\left(\frac{\mathcal{X}_k^\beta - \hat{\mathcal{X}}_k^\beta}{\mathcal{X}_k^\alpha}\right)^2$	$\frac{\sqrt{\upsilon_k}}{\gamma_k}\left(\frac{\Gamma\left(\frac{\beta-2\alpha}{2}+1\right)M\left(-\frac{\beta-2\alpha}{2},1;-\upsilon_k\right)}{\Gamma(-\alpha+1)M(\alpha,1;-\upsilon_k)}\right)^{1/\beta}$	β	α	0
WCOSH [3]	$\left(\frac{\mathcal{X}_k}{\hat{\mathcal{X}}_k} + \frac{\hat{\mathcal{X}}_k}{\mathcal{X}_k} - 1\right)\mathcal{X}_k^q$	$\frac{\sqrt{\upsilon_k}}{\gamma_k}\sqrt{\frac{\Gamma\left(\frac{q+3}{2}\right)M\left(-\frac{q+1}{2},1;-\upsilon_k\right)}{\Gamma\left(\frac{q+1}{2}\right)M\left(-\frac{q-1}{2},1;-\upsilon_k\right)}}$	1	$(1-q)/2$	0.5

the cost function in (14) features a new parameter, η, that acts only on the estimated clean speech STSA, $\hat{\mathcal{X}}_k$.

The Bayesian estimator corresponding to the cost function in (14) is obtained by finding the $\hat{\mathcal{X}}_k$ that minimizes the expectation of that given cost function as per (2). Using the conditional expectation, we have that

$$E\Big\{C_{\text{GWSA}}(\mathcal{X}_k, \hat{\mathcal{X}}_k)\Big\} = \int f(y_k) E\Big\{C_{\text{GWSA}}\Big(\mathcal{X}_k, \hat{\mathcal{X}}_k(y_k)\Big) \, |Y_k = y_k\Big\} dy_k \quad (15)$$

$$\geq \int f(y_k) \min E\Big\{C_{\text{GWSA}}\Big(\mathcal{X}_k, \hat{\mathcal{X}}_k(y_k)\Big)|Y_k = y_k\Big\} dy_k \quad (16)$$

where $y_k \in \mathbb{C}$ and we made explicit the dependency of $\hat{\mathcal{X}}_k$ on the observation Y_k. For a given y_k, we therefore only need to minimize the inner expectation in (16) to obtain the desired estimator. We thus find the first derivative of the inner expectation with respect to $\hat{\mathcal{X}}_k$ and set the result equal to zero; we notice that we have a quadratic form in $\hat{\mathcal{X}}_k^\beta$

$$\hat{\mathcal{X}}_k^{-1-2\eta}\left(a\hat{\mathcal{X}}_k^{2\beta} + b\hat{\mathcal{X}}_k^\beta + c\right) = 0 \quad (17)$$

where

$$a = (\beta - \eta) E\left\{\mathcal{X}_k^{-2\alpha}|Y_k\right\} \quad (18)$$

$$b = (2\eta - \beta) E\left\{\mathcal{X}_k^{\beta-2\alpha}|Y_k\right\} \quad (19)$$

$$c = -\eta E\left\{\mathcal{X}_k^{2\beta-2\eta}|Y_k\right\}. \quad (20)$$

Equation (17) has trivial solutions at $\hat{\mathcal{X}}_k = 0$ or $\hat{\mathcal{X}}_k \to \infty$, depending on the values of η and β, and two non-trivial solutions obtained as the roots of $a\hat{\mathcal{X}}_k^{2\beta} + b\hat{\mathcal{X}}_k^\beta + c$. We discard the trivial solutions, which are not interesting for the current problem, and consider the other solutions.

Using the Gaussian statistical model described in Section II, we know that (see [7])

$$E\left\{\mathcal{X}_k^m|Y_k\right\} = \frac{\upsilon_k^{m/2}}{\gamma_k^m}\Gamma\left(\frac{m}{2}+1\right) M\left(-\frac{m}{2}, 1; -\upsilon_k\right)|Y_k|^m \quad (21)$$

with $m > -2$ (see Appendix A in [3]). Using (21) in (18)–(20) and solving for the two non-trivial solutions in (17), we obtain the following family of estimators which we will term the Generalized Weighted family of STSA estimators (GWSA):

$$\hat{\mathcal{X}}_k = G_k|Y_k| \quad (22)$$

$$G_k = \frac{\sqrt{\upsilon_k}}{\gamma_k}\left(\frac{-b' \pm \sqrt{b'^2 - 4a'c'}}{2a'}\right)^{\frac{1}{\beta}} \quad (23)$$

where the parameters

$$a' = (\beta - \eta)\Gamma(-\alpha + 1) M(\alpha, 1; -\upsilon_k) \quad (24)$$

$$b' = (2\eta - \beta)\Gamma\left(\frac{\beta - 2\alpha}{2} + 1\right) \times M\left(-\frac{\beta - 2\alpha}{2}, 1; -\upsilon_k\right) \quad (25)$$

$$c' = -\eta\Gamma(\beta - \alpha + 1) M(\alpha - \beta, 1; -\upsilon_k). \quad (26)$$

From the restriction on m in (21), we have that $\alpha < 1$ and $\beta > 2(\alpha - 1)$, and from a' that $\beta \neq \eta$. Moreover, we have that $b'^2 - 4a'c' \geq 0$ to avoid complex gains. Similar restrictions may also apply to the term inside the parenthesis in (23) depending on the value of β.

We analyzed the second derivative of the inner expectation in (16) with respect to $\hat{\mathcal{X}}_k$ to verify that the solutions in (23) are minimums. The result showed that the positive sign solution is a minimum if $\beta > 0$ and the negative sign solution is a minimum if $\beta < 0$. The chosen value of β therefore determines which of the positive or negative sign solution is appropriate.

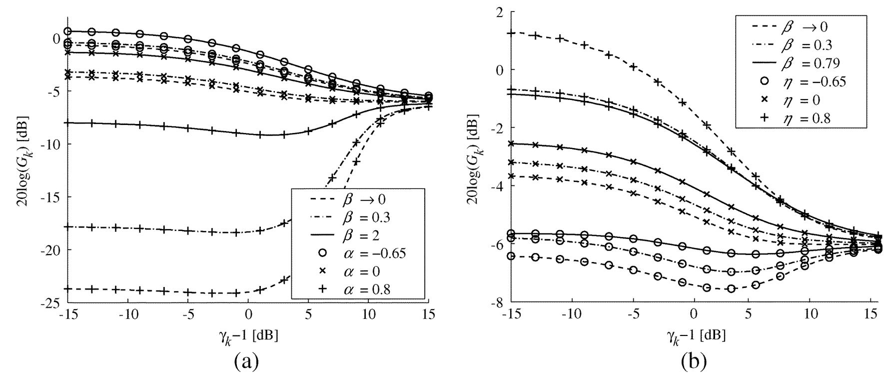

Fig. 1. Estimator gain $(20\log(G_k))$ versus instantaneous SNR $(\gamma_k - 1)$ for (a) β and α values with $\eta = 0$ and (b) β and η values with $\alpha = 0$ $(\xi_k = 0\ \mathrm{dB})$.

B. Analysis of the GWSA Family of Estimators

1) Gain Versus Instantaneous SNR: The GWSA gain depends on the parameters of the cost function (i.e. β, α and η), as well as on the parameters common to the previous STSA estimators, namely the *a posteriori* SNR γ_k and the *a priori* SNR ξ_k.

Fig. 1 presents gain $(20\log(G_k))$ curves as a function of the instantaneous SNR $(\gamma_k - 1)$ for a fixed $\xi_k = 0\ \mathrm{dB}$. In Fig. 1(a) we set $\eta = 0$ and show the gain curves for several β and α values (therefore corresponding to the $\mathrm{W}\beta - \mathrm{SA}$ estimator) while in (b) we set $\alpha = 0$ and show the gain curves for several β and η values. Different β values were chosen between Fig. 1(a) and 1(b) to avoid complex gains.

As can be observed in Fig. 1, the gain decreases when α increases, increases when η increases and generally increases when β increases. For example, we see that for the case $\alpha = 0$ and $\eta = 0.8$, the gain rather decreases as β increases. Contrary to the existing estimators discussed in this paper, which all lead to a similar set of gain curves, the GWSA family of estimators provide with more flexibility in terms of achievable gain curves. In fact, with carefully chosen parameters, a steeper transition from high to low instantaneous SNR (e.g., $\alpha = 0, \beta \to 0$, $\eta = 0.8$) or an increase in the gain between $\gamma_k - 1 = -5\ \mathrm{dB}$ and 10 dB (e.g. $\alpha = 0, \beta = 0.79, \eta = -0.65$) can be obtained.

2) High Instantaneous SNR Gain: It was shown previously that the $\mathrm{W}\beta - \mathrm{SA}$ estimator tends to the Wiener filter when the *a posteriori* SNR tends to infinity [4]. In fact, all estimators belonging to the GWSA family converge to the Wiener filter when the a posteriori SNR tends to infinity. As $\gamma_k \to \infty$, we have from (13.1.5) in [8] that

$$M\left(-\frac{m}{2}, 1; -\upsilon_k\right) = \frac{\upsilon_k^{m/2}}{\Gamma\left(\frac{m}{2}+1\right)}. \tag{27}$$

Using (27) in (24)–(26), we can show that both the positive sign solution (with $\beta > 0$) and negative sign solution (with $\beta < 0$, i.e. $\beta = -|\beta|$) of (23) simplify to

$$G_k = \frac{\xi_k}{1+\xi_k} \tag{28}$$

which is a Wiener filter gain.

3) Experimental Results: In order to show one possible advantage of this new family of estimators, we chose the parameters to obtain a gain identical to the that of LSA for low and high instantaneous SNR but higher than LSA for intermediate SNR, i.e. $\gamma_k - 1$ between -5 dB and 10 dB ($\alpha = -0.15$, $\beta = 1$, $\eta = -0.64$). Objective results using PESQ [9] (60 sentences, white noise, $\mathrm{SNR} = 0\ \mathrm{dB}$) were found to be slightly better with the chosen parameters than with the LSA estimator; this advantage was found to be statistically significant within a 95% confidence interval when using a two-tailed paired t-test.

V. Conclusion

In this letter, we reported that several existing Bayesian STSA cost functions for speech enhancement were similarly structured. We therefore proposed a generalization of the corresponding estimators under the GWSA family of estimators. It is shown that the latter yields an added flexibility in terms of achievable gain curves when compared to existing estimators. Furthermore, all estimators belonging to that family tend to a Wiener filter at high instantaneous SNR.

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