Wideband Spectrum Sensing for Cognitive Radios With Correlated Subband Occupancy

Khalid Hossain and Benoît Champagne, *Senior Member, IEEE*

***Abstract*—In this letter, we consider wideband spectrum sensing in the presence of correlation between the occupancies of frequency subbands. We begin by formulating the maximum *a posteriori* (MAP) estimator of channel occupancy based on measurements from multiple frequency subbands. Since the MAP estimator's complexity grows exponentially with the number of subbands, we propose an alternative structure, in which the subband energy measurements are linearly combined according to a minimum mean-square error (MMSE) criterion to form a sufficient statistic for binary detection in each subband. Through analysis and numerical simulations, we show that the proposed frequency-coupled detector can significantly outperform the traditional decoupled one.**

***Index Terms*—Channel occupancy estimation, cognitive radio, hypothesis-testing, spectrum sensing.**

I. Introduction

IN a cognitive radio (CR) system, secondary users (SU) must have the capability to detect and opportunistically use spectrum holes allocated to other, primary users (PU). Spectrum sensing has gained further importance as CR is an integral component of the IEEE 802.22 standard. To maximize the opportunistic throughput, without interfering with existing users (EU), including the PU or other SUs, spectrum sensing must be reasonably fast and accurate.

Different approaches have been proposed to perform spectrum sensing in a CR context. Matched filtering, which requires detailed knowledge of the PU signal, is discussed in [1]. The use of special signal features (e.g., cyclostationarity) to detect and classify PU signals has been studied in [2], [3]. Many studies advocate the use of energy detection for spectrum sensing [4], [5], since it can meet the basic requirements of CR systems while offering flexibility and robustness in implementation. In this context, multiband energy detection is of particular interest as it can significantly improve the opportunistic throughput [6]. To overcome blockage effects in wireless transmissions, spectrum sensing may also be carried out by a cluster of collaborating CRs. This approach, considered in [7]–[9], is generally expected to outperform single-user detection.

Manuscript received August 09, 2010; accepted November 03, 2010. Date of publication November 11, 2010; date of current version December 02, 2010. This work was supported by InterDigital Canada Ltée., the Natural Sciences and Engineering Research Council of Canada, and the Government of Québec under the PROMPT program. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Kostantinos N. Plataniotis.

The authors are with the Department of Electrical and Computer Engineering, McGill University, Montréal, QC H3A 2A7, Canada (e-mail: khalid.hossain@mcgill.ca; benoit.champagne@mcgill.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LSP.2010.2091405

The current literature on wideband energy detection for spectrum sensing focuses on a decoupled multiband processing structure in which energy detection in any given subband is based on a sufficient statistic computed from observed data in that subband, i.e., independently of other subbands' data. Even sophisticated multiband detection schemes such as [6], while jointly optimizing the detection thresholds used in individual subbands from a wideband perspective, make use of this decoupled structure. Although the latter is indeed optimal under the assumption that the occupancies of the frequency subbands are independent of each other, this assumption is generally not true, especially in the presence of wideband PU/EU signals, e.g., WLAN and broadcast television [10]. Hence, more recently, spectrum sensing in the presence of correlated subband occupancy has been gaining attention [11].

In this letter, we investigate the problem of joint multiband spectrum sensing in the presence of correlation between subband occupancies. We introduce a vector of binary random variables to model the multiple subband occupancy. Considering a Bayesian framework, we formulate the MAP estimator of the wideband channel occupancy vector based on measurements from multiple subbands. The MAP estimator reduces to a decoupled structure when the subband occupancies are independent of each other, but, in the general case, its complexity grows exponentially with the number of subbands. We therefore propose an alternative structure in which energy measurements from multiple subbands are linearly combined, with weights derived from a minimum mean-square error (MMSE) criterion, to form a sufficient statistic for binary detection in each subband. Through both analysis and numerical simulations, we show that the proposed detector can significantly outperform the traditional decoupled detector.

II. Problem Formulation

Let $r(n)$ denote the wideband signal observed by the SU (i.e., CR detector) after down-conversion and uniform sampling. This signal can be expressed as

$$r(n) = \sum_{l=0}^{L-1} h(l)s(n-l) + v(n) \tag{1}$$

where $s(n)$ is the EU signal, $h(n)$ is the impulse response of the wireless channel between the EU and SU (assumed to be time-invariant), L is the length of $h(n)$ and $v(n)$ is an additive noise term. We consider a frequency-domain detector structure in which a K-point discrete Fourier Transform (DFT) is used to decompose successive frames of $r(n)$ into narrow-band discrete frequency components, i.e.:

$$R_k(m) = \sum_{n=0}^{K-1} r(mK+n)e^{-j2\pi nk/K}, \quad k = 0, 1, \ldots, K-1 \tag{2}$$

where k is the frequency index, $m = 0, 1, \ldots, M-1$ is the frame index and M is the number of frames available. In a similar fashion, we let H_k, $S_k(m)$ and $V_k(m)$ denote the kth DFT coefficients of $h(n)$, $s(mK+n)$ and $v(mK+n)$, respectively. Under the assumption of a large time-bandwidth product $(K > L)$, the convolution in (1) can be approximated by the product of the corresponding DFT coefficients. The mth sample of the observed signal in the kth subband can be represented as

$$R_k(m) = H_k S_k(m) + V_k(m), \quad k = 0, 1, \ldots, K-1. \tag{3}$$

The EU signal samples, $\{S_k(m)\}$, and noise samples, $\{V_k(m)\}$, are modeled as independent random processes. Given a particular state of occupancy of the wideband channel, samples from each process are assumed to be independent across frequency and frame indices and to obey a zero-mean complex circular symmetric Gaussian distribution. The noise variance, $E[|V_k(m)|^2] = \sigma_v^2$, and the channel squared magnitude response, $G_k \triangleq |H_k|^2$, are assumed to be known from *a priori* estimation. We set $E[|S_k(m)|^2] = 1$, if the kth subband is occupied and 0 otherwise. Note that this corresponds to the occupancy model currently being used in literature, e.g., [4], [6], and [8]. Indeed, in the absence of an EU, $E[|S_k(m)|^2] = 0$ and (3) reduces to $R_k(m) = V_k(m)$.

In this work, we adopt a Bayesian framework and model the occupancy of the kth subband as a binary random variable, B_k, with realization $b_k \in \{0, 1\}$. As above, 0 and 1 respectively indicate an empty and occupied subband; accordingly, we have $E[|S_k(m)|^2] = B_k$. The wideband spectrum occupancy may then be described by the random vector

$$\mathbf{B} = [B_0, B_1, \ldots, B_{K-1}]^T \tag{4}$$

with realizations $\mathbf{b} = [b_0, b_1, \ldots, b_{K-1}]^T \in \{0, 1\}^K$. We denote the joint probability mass function (PMF) of $\mathbf{B}$ by $P_{\mathbf{B}}(\mathbf{b}) = \Pr(B_0 = b_0, \ldots, B_{K-1} = b_{K-1})$. We also define the mean vector $\boldsymbol{\mu} = E[\mathbf{B}]$, with entries $\mu_i = E[B_i]$, and the correlation matrix $\boldsymbol{\Lambda} = E[\mathbf{B}\mathbf{B}^T]$, with entries $\lambda_{ij} = E[B_i B_j]$. The occupancy vector $\mathbf{B}$ is assumed to be independent of $\{V_k(m)\}$ and to remain unchanged during the detection interval.

Given the above signal model, we seek an efficient detector structure that will enable the SU to determine the state of occupancy of the wideband channel, as represented by the unknown vector $\mathbf{B}$. We consider the general situation in which the occupancy variables B_i and B_j may not be independent for $i \neq j$. For instance, because of spectral allocation plan, an EU (such as WLAN or broadcast television) may be transmitting over a wideband spectrum, which maps to multiple subbands for the SU; or, as discussed in [11], a contiguous section of the wireless spectrum licensed to the EU may be deeply faded due to multipath fading effects. In this situation, EU detection in the faded subbands is difficult, but there exists a correlation between the occupancies of the faded and unfaded subbands. We show that by exploiting such *a priori* knowledge of correlation, significant gain in detection performance can be achieved.

III. Estimators for the Channel Occupancy

A. Bayesian Estimation of Channel Occupancy Vector

Given the probability model in Section II, the mean and variance of $R_k(m)$ conditioned on $B_k = b_k$ are given by

$$E\left[R_k(m) | B_k = b_k\right] = 0 \tag{5}$$

$$Var\left[R_k(m) | B_k = b_k\right] = b_k G_k + \sigma_v^2 \tag{6}$$

Let us denote by $\mathbf{R}$ a random vector containing the complete set of observed data in (3), i.e.: $\mathbf{R} = \left[\mathbf{R}_0^T, \ldots, \mathbf{R}_{K-1}^T\right]^T$, where $\mathbf{R}_k = [R_k(0), \ldots, R_k(M-1)]^T$, with corresponding realizations $\mathbf{r} = \left[\mathbf{r}_0^T, \ldots, \mathbf{r}_{K-1}^T\right]^T$, where $\mathbf{r}_k = [r_k(0), \ldots, r_k(M-1)]^T$. Consequently, the conditional probability density function (PDF) of $\mathbf{R}$ given $\mathbf{B} = \mathbf{b}$ can be written as

$$f_{\mathbf{R}|\mathbf{B}}(\mathbf{r}|\mathbf{b}) = \prod_{m=0}^{M-1} \prod_{k=0}^{K-1} \frac{1}{\pi\left(b_k G_k + \sigma_v^2\right)} \exp\left\{-\frac{|r_k(m)|^2}{b_k G_k + \sigma_v^2}\right\} \tag{7}$$

Using (7), the MAP estimator of $\mathbf{B}$ given the observation $\mathbf{R} = \mathbf{r}$ can be formulated as

$$\hat{\mathbf{B}}_{\mathrm{MAP}} = \arg\max_{\mathbf{b}} L_{\mathrm{MAP}}(\mathbf{b}|\mathbf{r}) \tag{8}$$

The associated log-likelihood function is given by

$$L_{\mathrm{MAP}}(\mathbf{b}|\mathbf{r}) = \ln P_{\mathbf{B}}(\mathbf{b}) - M \sum_{k=0}^{K-1} \ln\left(b_k G_k + \sigma_v^2\right) - \sum_{k=0}^{K-1} \frac{y_k}{b_k G_k + \sigma_v^2} \tag{9}$$

where $y_k = \sum_{m=0}^{M-1} |r_k(m)|^2$ is the energy measured in the kth subband over M frames.

It is interesting to consider the special case where the subband occupancies are independent of each other, i.e., $P_{\mathbf{B}}(\mathbf{b}) = \prod_{k=0}^{K-1} P_{B_k}(b_k)$ where $P_{B_k}(b_k) = \Pr(B_k = b_k)$ denotes the marginal probability of occupancy of the kth subband. The maximization in (9) can then be done independently for each k, i.e., $\hat{B}_k = \arg\max_{b_k} L_k(b_k|r_k)$ where

$$L_k(b_k|r_k) = \ln P_{B_k}(b_k) - M \ln\left(b_k G_k + \sigma_v^2\right) - \frac{y_k}{b_k G_k + \sigma_v^2}$$

This leads to an independent binary hypothesis test for each subband. Present literature largely deals with this special case where the optimal multiband detector decouples into K parallel narrowband ones.

However, in the general case where the subband occupancies B_k are not independent of each other, (9) leads to a non-linear integer optimization in K-dimensional space. This is a computationally challenging problem with intricate decision regions whose complexity grows exponentially with the number of subbands K. To solve the multiband energy detection problem within reasonable limits of processing time and implementation complexity, therefore, necessitates a simpler detector structure, which is addressed below.

B. Optimum Linear Energy Combiner

From (9), we note that the set of measured energies $\{y_k\}_{k=0}^{K-1}$ define a sufficient statistics for the MAP estimator and that only linear processing of these quantities is needed. However, the weights applied to the energies y_k depend on the hypothesis being tested so that, in theory, 2^K different linear combiners are needed to obtain the MAP estimator. Here, driven by these considerations, we propose to investigate a simplified detector structure in which the subband energies are linearly combined, with a single combiner per subband, before being fed to a binary hypothesis test. Specifically, let

$$\hat{B}_k = \boldsymbol{\xi}_k^T \mathbf{Y} + \alpha_k \tag{10}$$

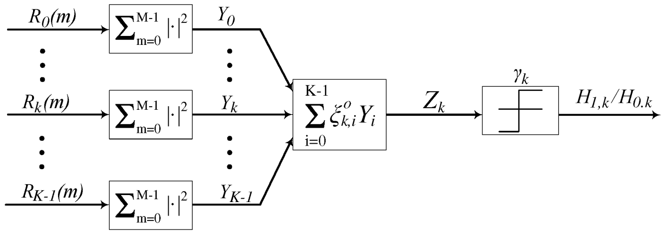

Fig. 1. Single block of the multiband detector using information from adjacent subbands.

denote an estimate of the unknown occupancy B_k of the kth subband, as obtained by an affine transformation on the random energy vector $\mathbf{Y} = [Y_0, Y_1, \ldots, Y_{K-1}]^T$, where

$$Y_k = \sum_{m=0}^{M-1} |R_k(m)|^2 \tag{11}$$

We propose to obtain the weight vector $\boldsymbol{\xi}_k$ and constant α_k in (10) as the minimizer of the MSE:

$$J(\boldsymbol{\xi}_k, \alpha_k) = E\left[(\hat{B}_k - B_k)^2\right] \tag{12}$$

The MMSE weight vector solution is obtained as

$$\boldsymbol{\xi}_k^o = E[\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T]^{-1} E[\tilde{\mathbf{Y}}\tilde{B}_k] \tag{13}$$

where we define the centered quantities $\tilde{\mathbf{Y}} = \mathbf{Y} - E[\mathbf{Y}]$ and $\tilde{B}_k = B_k - \mu_k$. The optimum value of α_k, given by $\alpha_k^o = \mu_k - \boldsymbol{\xi}_k^{oT} E[\mathbf{Y}]$, is needed in the MMSE estimation of B_k since the latter has nonzero mean.

Substituting (3) into (11) and making use of the modeling assumptions in Section II, we can show that:[1]

$$\begin{aligned} E[\tilde{Y}_i \tilde{B}_k] &= M(\lambda_{ik} - \mu_i \mu_k) G_i & (14)\\ E[\tilde{Y}_i \tilde{Y}_j] &= M^2(\lambda_{ij} - \mu_i \mu_j) G_i G_j \\ &\quad + M\left(\mu_i G_i^2 + 2\mu_i G_i \sigma_v^2 + \sigma_v^4\right)\delta_{ij} & (15) \end{aligned}$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. The resulting joint multiband detector structure, illustrated in Fig. 1 for the kth subband, can be summarized as

$$Z_k = \boldsymbol{\xi}_k^{oT}\mathbf{Y} \underset{\mathcal{H}_{0,k}}{\overset{\mathcal{H}_{1,k}}{\gtrless}} \gamma_k \quad k = 0, 1, \ldots, K-1 \tag{16}$$

We note that the bias term α_k^o is not shown in Fig. 1 as it is absorbed in the detection threshold γ_k. Also, $\boldsymbol{\xi}_k^o$ can be normalized such that $\xi_{kk}^o = 1$.

To exploit spatial diversity, linear combination of energies measured within a given subband by spatially distributed CRs has been applied to distributed sensing, e.g., in [8]. The proposed approach here is different in that it attempts to optimally combine the energies measured by a single CR across multiple subbands based on the correlation between the subband occupancies of EUs. In fact, our experimental results show that the detector in (16) can be used to improve the performance of the collaborative detection scheme shown in [6] and [8]. These results, however, will be reported separately.

For models of subband occupancies where the degree of correlation between B_j and B_k decreases as the frequency separation $|k - i|$ increases, e.g., in [11], we generally find that the relative weights ξ_{ki}^o given to Y_i in the computation of Z_k decreases in a similar way. That is, the greatest amount of weight is placed on the current and adjacent subbands, with indices k and $k \pm 1$. This suggests the consideration of a simplified, sub-optimal linear combiner structure in which the sufficient statistic used by the binary detector in the kth subband is obtained from:[2]

$$Z_k^{\pm} = Y_k + \eta_- Y_{k-1} + \eta_+ Y_{k+1} \tag{17}$$

where the gains $\eta_\pm$ can be chosen in different ways, such as truncation of the optimum solution (13), solution of a simplified (3-dimensional) version of the MMSE problem, or through minimization of the probability of missed detection for a given probability of false alarm.

C. Performance of the Multiband Energy Detector

Conditioned on $B_k = b_k$, the mean and variance of the test statistic (16) can be expressed as

$$\begin{aligned} E[Z_k|B_k = b_k] &= \boldsymbol{\xi}_k^{oT} E[\mathbf{Y}|B_k = b_k] & (18)\\ Var[Z_k|B_k = b_k] &= \boldsymbol{\xi}_k^{oT} E[\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T|B_k = b_k]\boldsymbol{\xi}_k^o & (19) \end{aligned}$$

It can be shown that

$$\begin{aligned} E[Y_i|B_k = b_k] &= M\left(\mu_{i|k} G_i + \sigma_v^2\right) & (20)\\ E[\tilde{Y}_i\tilde{Y}_j|B_k = b_k] &= M^2(\lambda_{ij|k} - \mu_{i|k}\mu_{j|k}) G_i G_j \\ &\quad + M\left(\mu_{i|k} G_i^2 + 2\mu_{i|k} G_i \sigma_v^2 + \sigma_v^4\right)\delta_{ij} & (21) \end{aligned}$$

where notations $\mu_{i|k} \equiv E[B_i|B_k = b_k]$ and $\lambda_{ij|k} \equiv E[B_i B_j|B_k = b_k]$ are used. According to the central limit theorem, for M sufficiently large, it is reasonable to assume that Z_k is normally distributed under each one of the hypotheses. Consequently, the probability of false alarm and the probability of missed detection associated with the test in (16) are given by

$$P_f^{(k)}(\gamma_k, \boldsymbol{\xi}_k^o) = Q\left(\frac{\gamma_k - E[Z_k|B_k = 0]}{\sqrt{Var[Z_k|B_k = 0]}}\right) \tag{22}$$

$$P_m^{(k)}(\gamma_k, \boldsymbol{\xi}_k^o) = 1 - Q\left(\frac{\gamma_k - E[Z_k|B_k = 1]}{\sqrt{Var[Z_k|B_k = 1]}}\right) \tag{23}$$

where the conditional mean and variance are as calculated in (18)–(19). Note that the above expressions hold true for hypothesis tests carried out on $Z_k^{\pm}$ defined in (17) if $\boldsymbol{\xi}_k^o$ is replaced by $\boldsymbol{\xi}_k^{\pm} = [0, \ldots, 0, \eta_-, 1, \eta_+, 0, \ldots, 0]^T$. To evaluate the probabilities in (22) and (23), knowledge of the conditional moments $\mu_{i|k}$ and $\lambda_{ij|k}$ is needed. These quantities can be obtained from measurements of subband occupancies by EUs, or from a suitable occupancy model (see below).

IV. Numerical Results

In this section, selected results based on analysis and Monte Carlo simulation experiments are presented to support the developments in Section III-B. In the simulations, instances of the random occupancy vector $\mathbf{B}$ are generated using a homogeneous Markov chain defined over the discrete frequency index k. The initial state of the chain, B_0, is set to 1 with probability

[1]The derivation of (15) makes use of a standard formula for the 4th moment of jointly Gaussian complex circular random variables [12]. Further details can be found in [13].

[2]With obvious modifications for edge frequencies $k = 0$ and $K - 1$.

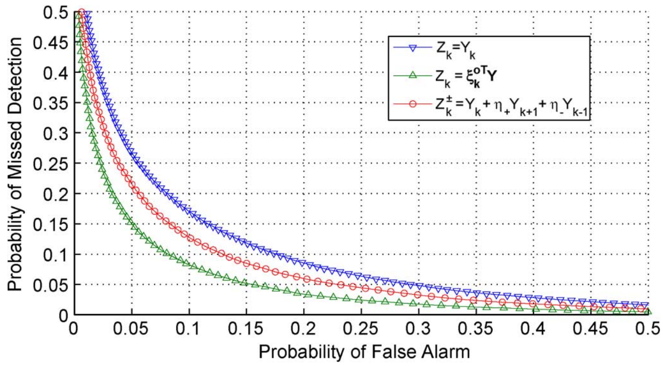

Fig. 2. $P_m^{(k)}$ versus $P_f^{(k)}$ for $\rho = 0.7$ ($k = 2, p = 0.15$).

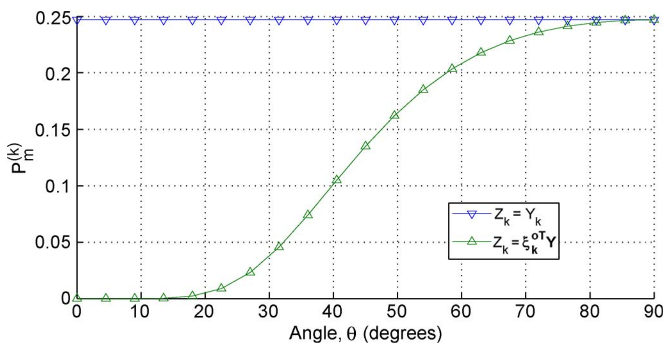

Fig. 3. $P_m^{(k)}$ versus θ for $P_f^{(k)} = 0.05$ ($k = 2$).

$P_{B_0}(1) \equiv 0.5$, while the states at frequencies $k = 1, \ldots, K-1$ are generated by means of a binary symmetric transition model with parameter p denoting the probability of a change in occupancy, that is: $P_{B_{k+1}|B_k}(1|0) = P_{B_{k+1}|B_k}(0|1) = p$. Use of this model allows the computation of the moments μ_i, $\mu_{i|k}$, λ_{ij} and $\lambda_{ij|k}$ introduced in Section III. In particular, the correlation coefficient between random variables B_{k+1} and B_k can be calculated exactly as $\rho = 1 - 2p$.

The noise samples, $V_k(m)$, and EU signal samples, $S_k(m)$, are independently generated complex circular Gaussian random variables with variances $\sigma_v^2 = 1$ and $E[|S_k(m)|^2] = B_k$. For each realization of $\mathbf{B}$, M data frames, $\{R_k(m)\}$, are generated as per (3). Here, we set $K = 8$, $M = 100$ and use: $\mathbf{G} = [G_0, \ldots, G_7] = [0.39, 0.11, 0.25, 0.10, 0.42, 0.37, 0.20, 0.29]$. For this set of parameters, and focusing our evaluation on subband $k = 2$, the optimum weight vector is found to be

$$\boldsymbol{\xi}_2^o = [0.203, 0.249, 1.0, 0.227, 0.176, 0.047, 0.013, 0.007]^T$$

For the sub-optimal linear combiner in (17), a simplified (3-dimensional) version of the MMSE problem for $k = 2$ gives $\eta_- = 0.311$ and $\eta_+ = 0.291$. Detection is performed on the simulated data samples $\{R_k(m)\}$ using hypothesis tests based on the various choices of test statistics, i.e., Y_k (11), $Z_k = \boldsymbol{\xi}_k^{oT}\mathbf{Y}_k$ (16) and $Z_k^{\pm}$ (17). In order to obtain reliable estimates of the probabilities of false alarm and missed detection, 10^5 trials are used for each choice of threshold value.

Fig. 2 shows the receiver operating characteristic (ROC) curve of the detectors, obtained by plotting $P_m^{(k)} \equiv P_m^{(k)}(\gamma_k, \boldsymbol{\xi}_k)$ against $P_f^{(k)} \equiv P_f^{(k)}(\gamma_k, \boldsymbol{\xi}_k)$ over a range of threshold parameter values γ_k, for subband $k = 2$ in the case $p = 0.15$, corresponding to a correlation of $\rho = 0.7$. For all values of $P_f^{(k)}$, use of the proposed Z_k as a test statistic, instead of the conventional Y_k, significantly reduces $P_m^{(k)}$. In this case, the use of the simplified test statistic $Z_k^{\pm}$ also provides significant improvement, but in general its performance is not as good as that of Z_k. The trend seen in Fig. 2 holds true for other choices of the transition probability p. To illustrate this point, we show in Fig. 3 the analytically computed probability of missed detection, $P_m^{(k)}$ as a function of the angle $\theta = \arccos(\rho)$ between random variables B_k and B_{k+1}, under the (Neyman–Pearson) constraint $P_f^{(k)} = 0.05$. The curves clearly demonstrate the potential advantages of exploiting *a priori* knowledge of correlation across subband occupancies in the design of a detector structure for spectrum sensing.

V. Conclusion

We presented new DFT-based energy detectors for wideband channel occupancy estimation in spectrum sensing applications. In contrast to previous works, which assume a frequency-decoupled detector structure, the proposed detectors exploit the correlation between adjacent subband occupancies within a Bayesian framework to improve the quality of the estimation. In particular, through analysis and simulations, we showed that the new detectors significantly outperform the traditional decoupled structure. They can also be used in conjunction with existing distributed detection schemes, such as [6], to obtain further performance gains. Finally, although the problem at hand differs from classical linear detection due to the squaring operation in (11), the Bayesian formalism adopted in this work enables the introduction of modern ideas from the field of digital communications (e.g., multi-user detection and decision feedback), in wideband channel sensing.

References

[1] Z. Quan, S. Cui, H. V. Poor, and A. H. Sayed, "Collaborative wideband sensing for cognitive radios," *IEEE Signal Process. Mag.*, vol. 25, no. 6, pp. 60–73, Nov. 2008.

[2] K. Kim, I. A. Akbar, K. K. Bae, J. Um, C. M. Spooner, and J. H. Reed, "Cyclostationary approaches to signal detection and classification in cognitive radio," in *Proc. IEEE Int. Symp. New Frontiers in Dynamic Spectrum Access Networks*, Apr. 2007, pp. 212–215.

[3] Z. Tian and G. B. Giannakis, "A wavelet approach to wideband spectrum sensing for cognitive radios," in *Proc. Int. Conf. Cognitive Radio Oriented Wireless Networks and Communications*, Jun. 2006, pp. 1–5.

[4] A. Ghasemi and E. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE Int. Symp. New Frontiers in Dynamic Spectrum Access Networks*, Nov. 2005, pp. 131–136.

[5] R. Chen, J.-M. Park, and K. Bian, "Robust distributed spectrum sensing in cognitive radio networks," in *Proc. 27th Conf. Computer Communications (INFOCOM)*, Apr. 2008, pp. 1876–1884.

[6] Z. Quan, S. Cui, A. H. Sayed, and H. V. Poor, "Optimal multiband joint detection for spectrum sensing in cognitive radio networks," *IEEE Trans. Signal Process.*, vol. 57, no. 3, pp. 1128–1140, Mar. 2009.

[7] C. da Silva, B. Choi, and K. Kim, "Distributed spectrum sensing for cognitive radio systems," in *Proc. Information Theory and Applications Workshop*, Jan. 2007, pp. 120–123.

[8] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

[9] J. A. Bazerque and G. B. Giannakis, "Distributed spectrum sensing for cognitive radio networks by exploiting sparsity," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1847–1862, Mar. 2010.

[10] B.-J. Kang, "Spectrum sensing issues in cognitive radio networks," in *Int. Symp. Communications and Information Technology*, Jan. 2009, pp. 824–828.

[11] C.-H. Hwang, G.-L. Lai, and S.-C. Chen, "Spectrum sensing in wideband OFDM cognitive radios," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 709–719, Feb. 2010.

[12] R. A. Monzingo and T. Miller, *Introduction to Adaptive Arrays*. Raleigh, NC: SciTech, 2004.

[13] K. S. Hossain, "Wideband Spectrum Sensing for Cognitive Radios in the Presence of Correlation Between Subband Occupancy," M.Eng. thesis, McGill Univ., Montréal, QC, Canada, Aug. 2010.